



# MATH AND SCIENCE @ WORK

AP\* PHYSICS Educator Edition



## TRAINING FOR A NEW SPACECRAFT Center of Gravity

### Instructional Objectives

Students will

- determine the center of gravity (CG) of the Water Egress and Survival Trainer (WEST);
- construct summation of moment expressions for WEST in static equilibrium; and
- explain the engineering behind the experimental set-up designed to measure the CG of WEST.

### Degree of Difficulty

For the average student in AP Physics C, this problem is at an advanced level of difficulty. Knowledge of the cross product function and the right hand rule as applied to torque will be required. This problem extends the concept of torque through the understanding of bending moments in an engineering application to determine the center of gravity of a three-dimensional object.

### Class Time Required

This problem requires 55–65 minutes.

- Introduction: 5–10 minutes
  - Read and discuss the background section with the class before students work on the problem. This background is identical to *Training for a New Spacecraft: Moment of Inertia*.
- Student Work Time: 45 minutes
- Post Discussion: 5–10 minutes

### Background

*This problem is part of a series of problems that apply Math and Science @ Work in NASA's research facilities.*

Orion is the most advanced spacecraft ever built and will carry up to four astronauts further into space than ever before. When paired with additional propulsion and life support systems, Orion can be reconfigured to take humans to asteroids or Mars.

### Grade Level

11–12

### Key Topic

Center of Gravity

### Degree of Difficulty

Advanced

### Teacher Prep Time

10 minutes

### Class Time Required

55–65 minutes

### Learning Objectives for AP Physics

Newtonian Mechanics:

- Newton's laws of motion
- Systems of particles, linear momentum
- Circular motion and rotation

### NSES

#### Science Standards

- Physical Science
- Unifying Concepts and Processes
- Science and Technology

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Spacecraft shape must be considered when designing for the speed and heat of reentry encountered while returning from a deep-space mission. The laws of physics are no different now than in the twentieth century when NASA first designed for a mission to the Moon. For this reason, the shape of the Orion spacecraft (Figure 1) is very similar to the Apollo Command Module (Figure 2), that took astronauts to the Moon in the 1960's and 70's.



Figure 1: The Orion Multi-Purpose Crew Vehicle being assembled and tested. Photo credit: Lockheed Martin



Figure 2: Apollo Crew Module

Then and now, astronauts must undergo a tremendous amount of training before traveling in a new spacecraft. Egress, or exiting the vehicle, is one activity that astronauts must practice and master in order to have a safe return from space. Although the capsule concept is not new, many physical parameters are quite different than ones used in previous vehicles. These differences require new simulators to be engineered so astronauts can train effectively.

The Water Egress and Survival Trainer (WEST) is a new simulator that will replicate the geometry and mass properties of the Orion flight capsule and will be used exclusively for egress training. Once WEST has been constructed and egress procedures have been developed, astronaut training will be conducted in NASA's Neutral Buoyancy Laboratory (NBL) located in Houston, Texas.

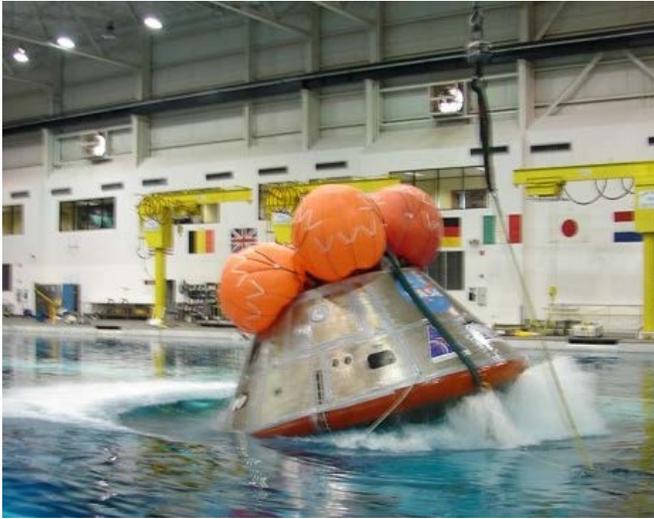


Figure 3: Crew Module Uprighting System Test being performed in the NBL with a full-scale Orion test article



Figure 4: Apollo 9 astronauts training with a mockup of the Apollo crew capsule in a pool

Like Apollo, Orion will return to Earth for an ocean water landing, but egress training methods between these missions will be drastically different. Apollo training was performed before the NBL was constructed. Training was conducted statically in pools (Figure 4) and at sea with unpredictable sea states and weather. In contrast, WEST will be attached to hydraulic actuators on the pool floor of the NBL to emulate variable sea states. This capability will provide economic, scheduling, and training benefits—great advantages for astronaut training in preparation for Orion missions.

## Learning Objectives for AP Physics

### Newtonian Mechanics

- Newton's laws of motion
  - Static equilibrium (first law)
- Systems of particles, linear momentum
  - Center of mass
- Circular motion and rotation
  - Torque and rotational statics

## NSES Science Standards

### Physical Science

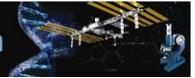
- Motions and forces

### Unifying Concepts and Processes

- Change, constancy, and measurement
- Form and function

### Science and Technology

- Abilities of technological design



**Problem and Solution Key (One Approach)**

WEST is essentially a hollow cone. Inside, it will contain four astronauts, life support systems, fuel, control systems, and other flight and on-orbit equipment. Its combined mass, and the location of its mass, will affect the stability of the capsule.

One describing property of stability is the center of gravity (CG). Once the CG of the capsule is determined, engineers can then add and remove weight, as well as relocate weight in order to facilitate different capsule orientations and egress scenarios, including rough seas that may result in a capsized craft.

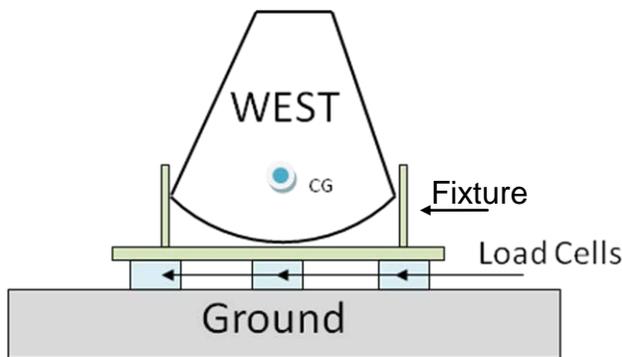


Figure 5: Vertical set-up of WEST

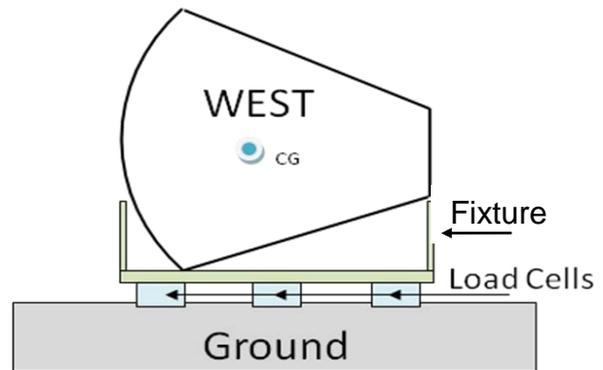


Figure 6: Horizontal set-up of WEST

Noting that WEST is hollow, one anticipates the CG to be located within the three-dimensional volume of the craft. Identifying the CG requires the ability to engineer an experimental set-up, from which to measure the location of the CG for x, y, and z-axes ( $x_{CG}$ ,  $y_{CG}$  and  $z_{CG}$ ). Figure 5 shows the vertical set-up for measuring the  $x_{CG}$  and  $z_{CG}$  and Figure 6 shows the horizontal set-up for measuring the  $y_{CG}$ .

WEST will sit in a fixture that can support the capsule either vertically or horizontally. Both WEST and the fixture will rest on three load cells. The load cells are individual balances that can accurately measure the load (weight) at their specified locations. Figure 7 and Figure 8 show the vertical and horizontal set-ups with load cell locations relative to the axes.

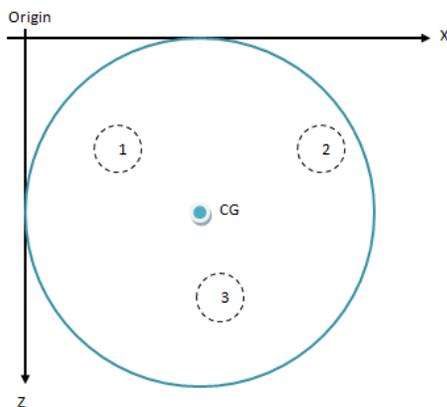


Figure 7: Vertical set-up of WEST

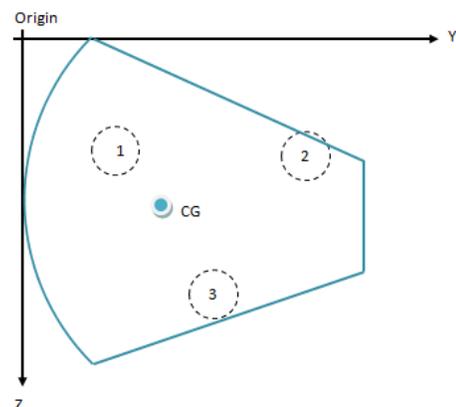
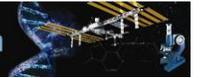


Figure 8: Horizontal set-up of WEST



- A. In order to determine the CG for the  $x$ ,  $y$ , and  $z$ -axes, certain physical measurements must be known (see Figure 9).

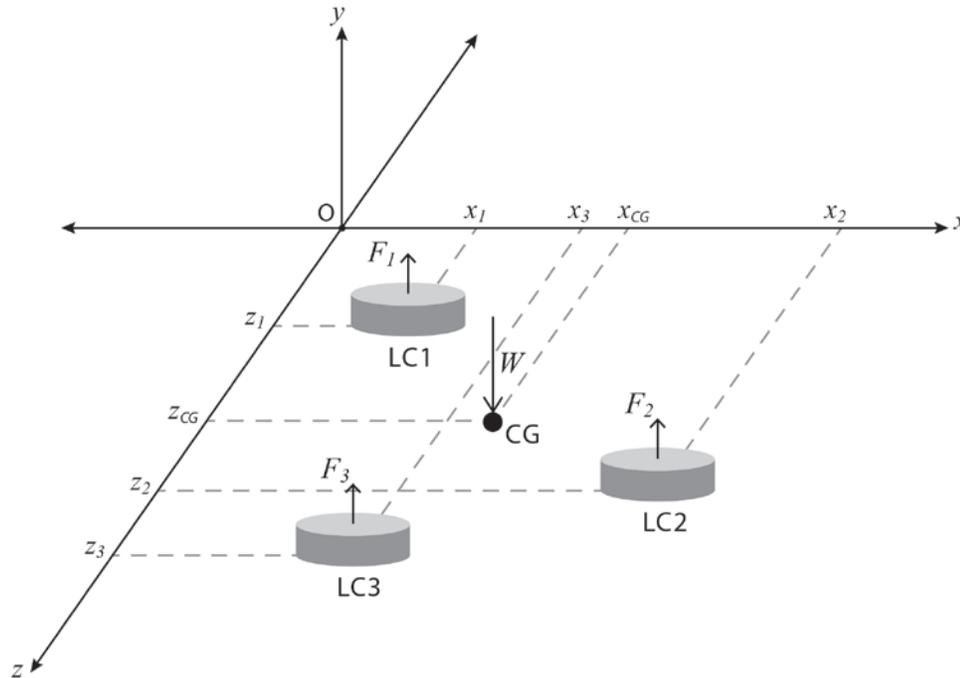


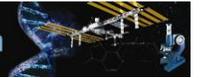
Figure 9: Vertical set-up of WEST

From the experimental set-up, engineers can physically measure the location of the load cells (LC) relative to the origin (O). The weight of WEST plus the fixture are represented by  $W$  in Figure 9 and is known from independent measuring.  $W$  is directed in the  $-y$ -direction as it acts in the same direction as gravity.  $F_1$ ,  $F_2$ , and  $F_3$  are the reaction forces of the load cells and are directed in the  $+y$ -direction to satisfy equilibrium conditions producing the counter forces on the WEST+ fixture system. The vertical set-up will allow the  $x_{CG}$  and  $z_{CG}$  to be determined by measuring the bending moments of the load cells relative to the origin.

Bending moment is an engineering term used to communicate a torque that results in a horizontal or vertical movement versus a traditional torque which results in a rotational motion. The term bending moment is more accurate given that WEST in its fixture will tend to tip up or down on the load cells. Bending moments ( $M$ ) are determined in the same manner as torque—using the cross product,  $r \times F$ , and the right hand rule to establish direction.  $M_{O,x}$ , therefore, denotes that the moment is taken with respect to point O about the  $x$ -axis.

- I. Creating a summation of moments expression in the  $x$ ,  $y$ , or  $z$ -direction means that the thumb of your right hand will point along the  $x$ ,  $y$ , or  $z$ -axis. Referring to Figure 9, explain why, to determine  $z_{CG}$ , one must evaluate the summation of moments in the  $x$ -direction.

*Having your thumb along the  $x$ -axis is a result of your fingers being directed along the  $z$ -axis.  $z_{CG}$  can therefore be determined because it lies along the  $z$ -axis.*



- II. Construct an equation for the summation of moments in the x-direction ( $\sum M_{O,x}$ ). Note that the WEST/fixture remains in equilibrium.

$$\sum M_{O,x} = (z_1 \times F_1) + (z_2 \times F_2) + (z_3 \times F_3) + (z_{CG} \times W) = 0, \text{ the "x" represents cross product}$$

- III. Determine  $z_{CG}$ .

The cross product approaches 1 as  $\theta$  approaches  $90^\circ$ . The right hand rule determines the direction along the x-axis as positive or negative. (Multiplication is represented by a dot.)

$$-z_1 \cdot F_1 - z_2 \cdot F_2 - z_3 \cdot F_3 + z_{CG} \cdot W = 0$$

$$z_{CG} = \frac{z_1 \cdot F_1 + z_2 \cdot F_2 + z_3 \cdot F_3}{W}$$

- IV. Determine  $x_{CG}$ .

$$\begin{aligned} \sum M_{O,z} &= (x_1 \times F_1) + (x_2 \times F_2) + (x_3 \times F_3) + (x_{CG} \times W) = 0 \\ +x_1 \cdot F_1 + x_2 \cdot F_2 + x_3 \cdot F_3 - x_{CG} \cdot W &= 0 \end{aligned}$$

$$x_{CG} = \frac{x_1 \cdot F_1 + x_2 \cdot F_2 + x_3 \cdot F_3}{W}$$

- B. Determining  $y_{CG}$  now requires the horizontal set-up. Recall the orientation of WEST on the fixture by referring to Figures 6 and 8. Figure 10 shows the same experimental set-up of the load cells with different axes.

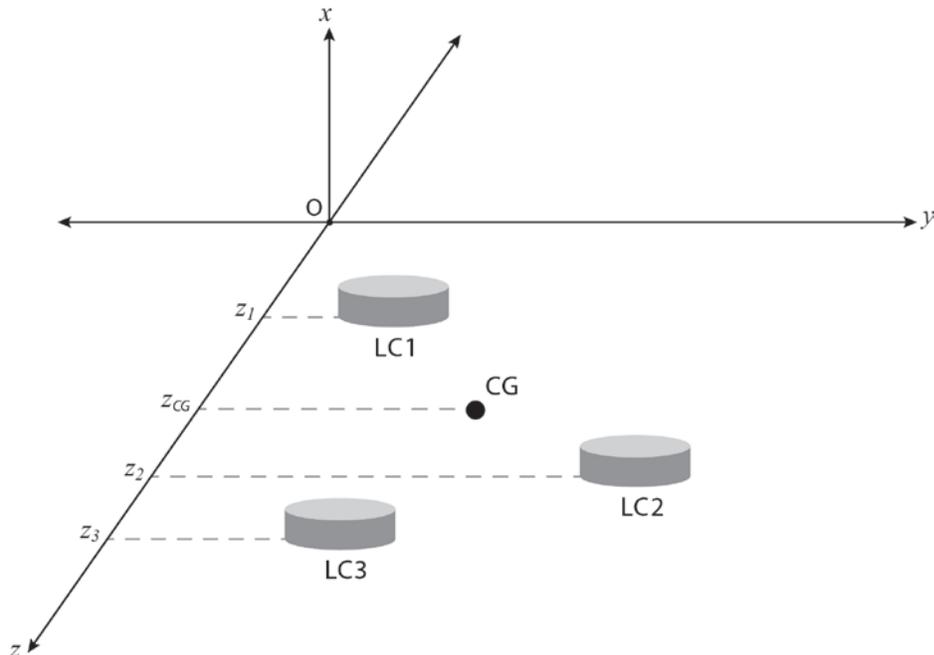
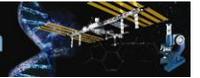
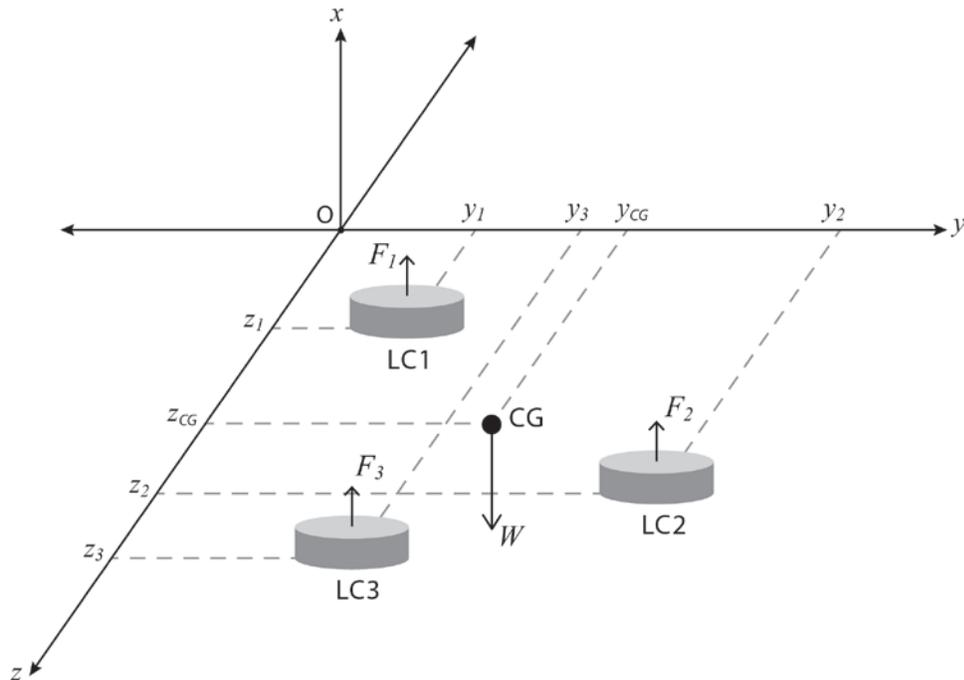


Figure 10: Horizontal set-up of WEST



I. On Figure 10, draw in the y-axis locations of the load cells and the force vectors.

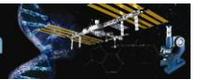


II. Determine the  $y_{CG}$ .

$$\begin{aligned} \sum M_{O,z} &= (y_1 \times F_1) + (y_2 \times F_2) + (y_3 \times F_3) + (y_{CG} \times W) = 0 \\ +y_1 \cdot F_1 + y_2 \cdot F_2 + y_3 \cdot F_3 - y_{CG} \cdot W &= 0 \end{aligned}$$

$$y_{CG} = \frac{y_1 \cdot F_1 + y_2 \cdot F_2 + y_3 \cdot F_3}{W}$$

**Note to Instructor:** The results from this work are coupled with further calculations to determine the attitude (or orientation) of WEST when resting in water. The mass of WEST will determine the amount of water displaced. When the center of buoyancy (determined by the geometry of the displaced water) is coincident with the CG of WEST, static equilibrium will have been achieved.



**Scoring Guide**

Suggested 15 points total to be given.

Question	Distribution of points
<b>A</b> <i>9 points</i>	2 points for part I: 1 point for acknowledging thumb along x-axis 1 point for acknowledging fingers along z-axis 2 points for part II: 1 point for including all four terms 1 point for setting the summation equation equal to zero 3 points for part III: 1 point for negative direction sign for $z_1$ , $z_2$ , and $z_3$ 1 point for positive direction sign for $z_{CG}$ 1 point for correct $z_{CG}$ 2 points for part IV: 2 points for correct $x_{CG}$ -1 point for any incorrect signs
<b>B</b> <i>6 points</i>	4 points for part I: 1 point for acknowledging y-axis measurement labels 1 point for correct subscripts 1 point for correct force vector directions 1 point for correct force vector labels 2 points for part II: 2 points for correct $y_{CG}$ -1 point for any incorrect signs

**Contributors**

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school AP Physics instructors.

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