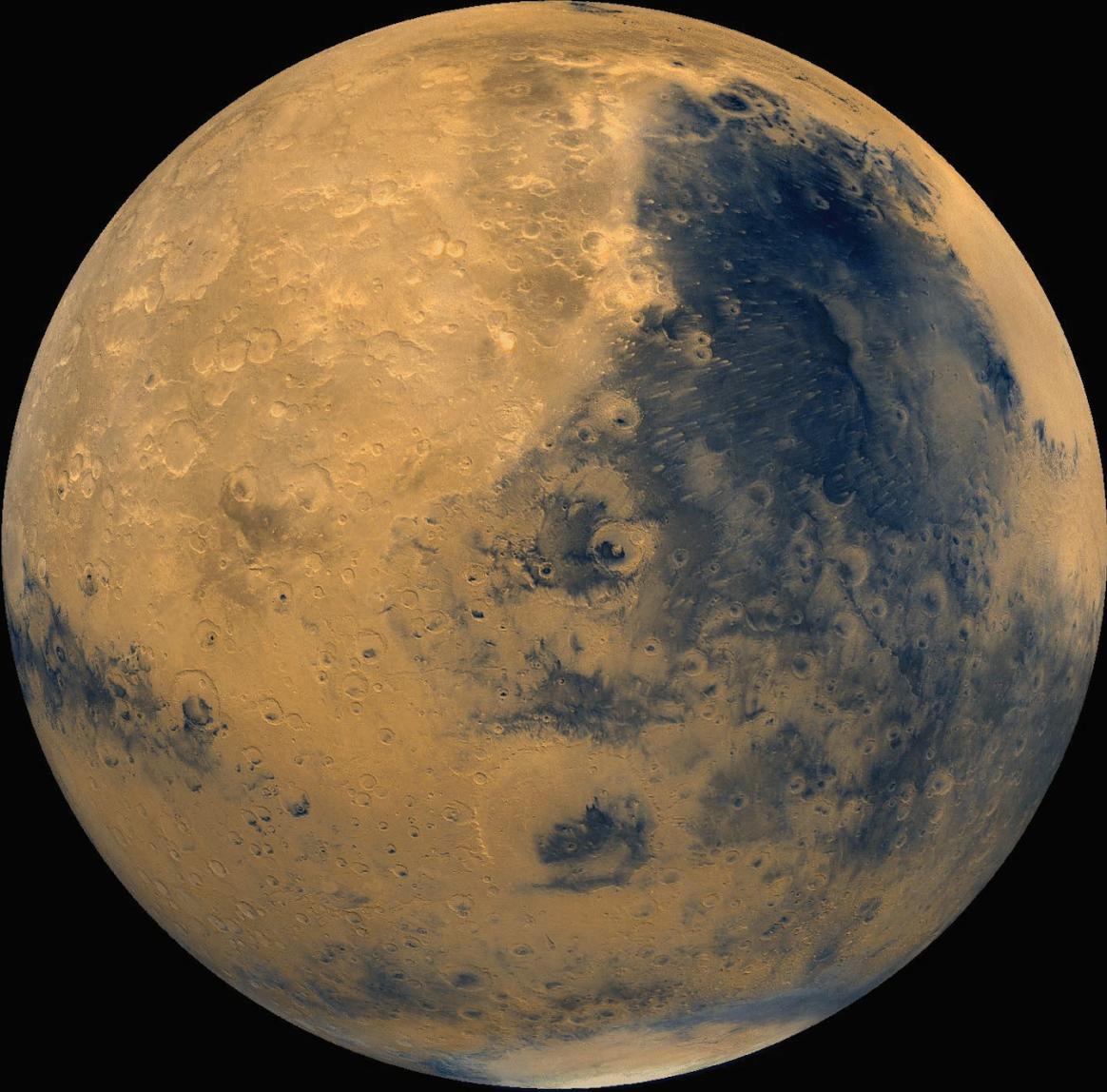
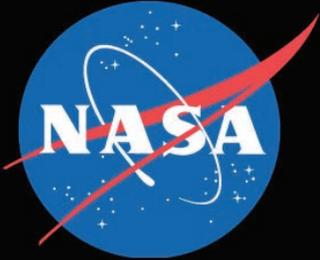


National Aeronautics and Space Administration



# Mars Math

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during 2005-2012. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 5 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

For more weekly classroom activities about astronomy and space visit the NASA website,

<http://spacemath.gsfc.nasa.gov>

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

**Front and back cover credits:** Front) Main-Composite image of Mars obtained by NASA orbiting spacecraft: Mars Observer; Mars Reconnaissance Orbiter. Top-Left: Image of Mars moon Phobos taken by ESA Mars Express. Top-right: Image of Deimos taken by Mars Reconnaissance Orbiter. Back) View of surface of Mars obtained inside Gale Crater by the Curiosity Rover (Courtesy NASA/JPL)

This booklet was created through an education grant NNH06ZDA001N-EPO from NASA's Science Mission Directorate.

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# Mathematics Topic Matrix

Topic	Problem Numbers																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Inquiry																								
Technology, rulers			X	X	X	X	X	X			X	X	X											
Numbers, patterns, percentages																								
Averages								X			X	X				X	X				X			
Time, distance, speed								X	X	X	X	X												
Mass, density, volume													X	X										
Areas and volumes									X				X	X							X			
Scale drawings			X	X	X	X	X	X	X		X	X	X			X								
Geometry												X				X								
Scientific Notation														X	X	X			X	X				
Unit Conversions																					X	X	X	
Fractions	X	X																						
Graph or Table Analysis																X	X				X	X	X	
Coordinate plane, points									X							X								
Solving for X																								
Evaluating Fns																				X	X			
Modeling																X	X	X	X		X			
Probability																								
Rates/Slopes												X				X					X	X		
Logarithmic Fns																								
Polynomials																								
Power Fns																			X	X				
Conics																					X			
Piecewise Fns																								
Trigonometry																X								
Integration																					X			
Differentiation																								
Limits																								

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. *SpaceMath@NASA* offers math applications through one of the strongest motivators-Space. Technology (e.g rulers, calculators, computers) makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for all of the Space Math applications.

This book is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Mars Math**. Read the scenario that follows:

Ms. Green decided to pose a new activity using Space Math for her students. She challenged each student team with math problems from the **Mars Math** book. She copied each problem for student teams to work on. She decided to have the students develop a fictitious space craft to explore Mars. Each team was to develop a set of criteria that included reasons for the research, timeline and budget. The student teams had to present their findings and compete for the necessary funding for their space craft. The students were to use the facts available in the **Mars Math** book and create their own mission, but to use mathematical reasoning to justify their choices.

**Mars Math** can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does demonstrate, regardless of how it is used in the classroom, is the need to be proficient in math to create accurate models of the world, and successful missions to advance our knowledge. It is needed especially in our world of advancing technology and physical science.

# Alignment with Standards

## **AAAS: Project:2061 Benchmarks**

**(3-5)** - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1 **(6-8)** Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1 **(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

## **NCTM: Principles and Standards for School Mathematics**

### **Grades 6–8 :**

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; .
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- use graphs to analyze the nature of changes in quantities in linear relationships.
- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system.

### **Grades 9–12 :**

- judge the reasonableness of numerical computations and their results.
- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- draw reasonable conclusions about a situation being modeled.

# Teacher Comments

"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

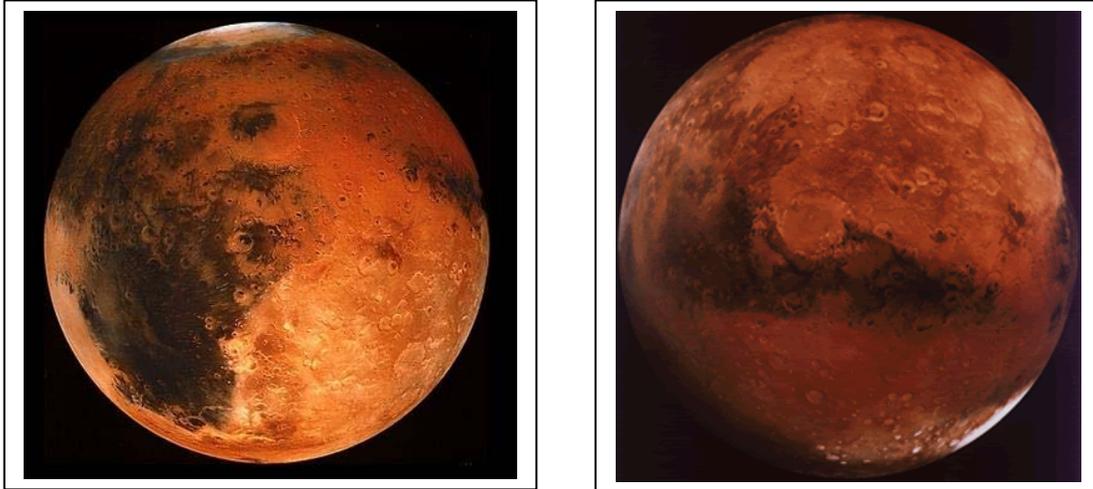
"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that..." (Associate Professor of Physics)



# How Big is Mars?

1



Inside the orbit of Jupiter, our solar system has five large objects; four of these are the planets Mercury, Venus, Earth and Mars. The fifth object is our own moon! These objects are almost perfectly round, but they are not all the same size.

From the clues below, can you figure out just how large the planet Mars is in kilometers?

Clue 1 – Mercury is  $\frac{7}{5}$  the diameter of the Moon.

Clue 2 – The Moon is  $\frac{7}{25}$  the diameter of Earth

Clue 3 – Mars is  $\frac{7}{5}$  the diameter of Mercury

Clue 4 – The diameter of Earth is 13,000 kilometers ( 8,000 miles).

# Answer Key

1

Students can begin by drawing circles that approximately match the proportions given:

The Mars circle will be bigger than the Mercury circle by an amount  $\frac{7}{5}$  times the diameter of Mercury. The Mercury circle will be bigger than the Moon circle by an amount  $\frac{7}{5}$  times the diameter of the Moon. Students can then see that Mars will be  $\frac{7}{5} \times \frac{7}{5} = \frac{49}{25}$  bigger than the Moon circle – almost twice the diameter of the Moon!

When students draw the Moon and Earth circles, they will see from Clue 2 that the Moon is  $\frac{7}{25}$  or about  $\frac{1}{4}$  the diameter of Earth. Four Moon circles fit across the diameter of Earth. Because Mars is twice the diameter of the Moon, it must be then be about  $\frac{1}{2}$  the diameter of Earth because  $\frac{49}{25} \times \frac{7}{25} = \frac{343}{625}$  which is about  $\frac{1}{2}$ .

## By the stated proportions:

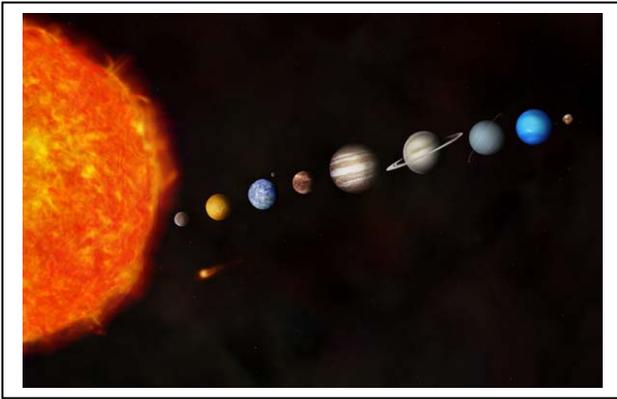
Mercury =  $\frac{7}{5}$  x Moon; Moon =  $\frac{7}{25}$  x Earth; Mars =  $\frac{7}{5}$  x Mercury

Mercury =  $\frac{7}{5}$  x ( $\frac{7}{25}$  Earth)  
Mars =  $\frac{7}{5}$  x ( $\frac{7}{5}$  x  $\frac{7}{25}$  Earth)

Mars =  $\frac{7}{5}$  x  $\frac{7}{5}$  x  $\frac{7}{25}$  x Earth diameter  
=  $\frac{343}{625}$   
= 0.55 Earth diameter  
  
= 7150 km or 4433 miles.

Mars is about  $\frac{1}{2}$  the diameter of Earth. Students can now draw Mercury, Venus, Earth and Mars in their correct relative sizes compared to each other.





Some of the planets in our solar system are much bigger than Earth while others are smaller. By using simple fractions, you will explore how their sizes compare to each other.

Image courtesy NASA/Chandra Observatory/SAO

**Problem 1** - Saturn is 10 times bigger than Venus, and Venus is  $\frac{1}{4}$  the size of Neptune. How much larger is Saturn than Neptune?

**Problem 2** - Earth is twice as big as Mars, but only  $\frac{1}{11}$  the size of Jupiter. How large is Jupiter compared to Mars?

**Problem 3** - Earth is the same size as Venus. How large is Jupiter compared to Saturn?

**Problem 4** - Mercury is  $\frac{3}{4}$  the size of Mars. How large is Earth compared to Mercury?

**Problem 5** - Uranus is the same size as Neptune. How large is Uranus compared to Earth?

**Problem 6** - The satellite of Saturn, called Titan, is  $\frac{1}{10}$  the size of Uranus. How large is Titan compared to Earth?

**Problem 7** - The satellite of Jupiter, called Ganymede, is  $\frac{2}{5}$  the size of Earth. How large is it compared to Jupiter?

**Problem 8** - The Dwarf Planet Pluto is  $\frac{1}{3}$  the diameter of Mars. How large is the diameter of Jupiter compared to Pluto?

**Problem 9** - If the diameter of Earth is 13,000 km, what are the diameters of all the other bodies?

# Answer Key

Note to teachers: The actual diameters of the planets, in kilometers, are as follows

Mercury	4,900 km	Jupiter	143,000 km
Venus	12,000 km	Saturn	120,000 km
Earth	13,000 km	Uranus	51,000 km
Mars	6,800 km	Neptune	50,000 km

Also: Titan = 5,100 km, Ganymede = 5,300 km Ceres = 950 km, and Pluto 2,300 km

Advanced students (Grades 4 and above) may use actual planetary size ratios as decimal numbers, but for this simplified version (Grades 2 and 3), we approximate the size ratios to the nearest simple fractions. **Students may also use the information in these problems to make a scale model of the solar system in terms of the relative planetary sizes.**

Problem 1 - Saturn is 10 times bigger than Venus, and Venus is  $\frac{1}{4}$  the size of Neptune. How much larger is Saturn than Neptune?

**Answer: Neptune is 4x Venus and Saturn is 10x Venus, so Saturn is  $\frac{10}{4} = \frac{5}{2}$  times as big as Neptune.**

Problem 2 - Earth is twice as big as Mars, but only  $\frac{1}{11}$  the size of Jupiter. How large is Jupiter compared to Mars?

**Answer: Jupiter is 11 x Earth, and Mars is  $\frac{1}{2}$  Earth, so Jupiter is 22x Mars.**

Problem 3 - Earth is the same size as Venus. How large is Jupiter compared to Saturn?

**Answer: If Saturn is 10 x Venus, and Jupiter is 11 x Earth, Jupiter is  $\frac{11}{10}$  times Saturn.**

Problem 4 - Mercury is  $\frac{3}{4}$  the size of Mars. How large is Earth compared to Mercury?

**Answer: Mars is  $\frac{1}{2}$  x Earth, so Mercury is  $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$  x Earth**

Problem 5 - Uranus is the same size as Neptune. How large is Uranus compared to Earth?

**Answer: Neptune was 4x Venus, but since Venus = earth and Neptune=Uranus, we have Uranus = 4x Earth.**

Problem 6 - The satellite of Saturn, called Titan, is  $\frac{1}{10}$  the size of Uranus. How large is Titan compared to Earth?

**Answer: Titan / Uranus =  $\frac{1}{10}$ , but Uranus/Earth = 4, so Titan/Earth =  $\frac{1}{10} \times 4 = \frac{2}{5}$ .**

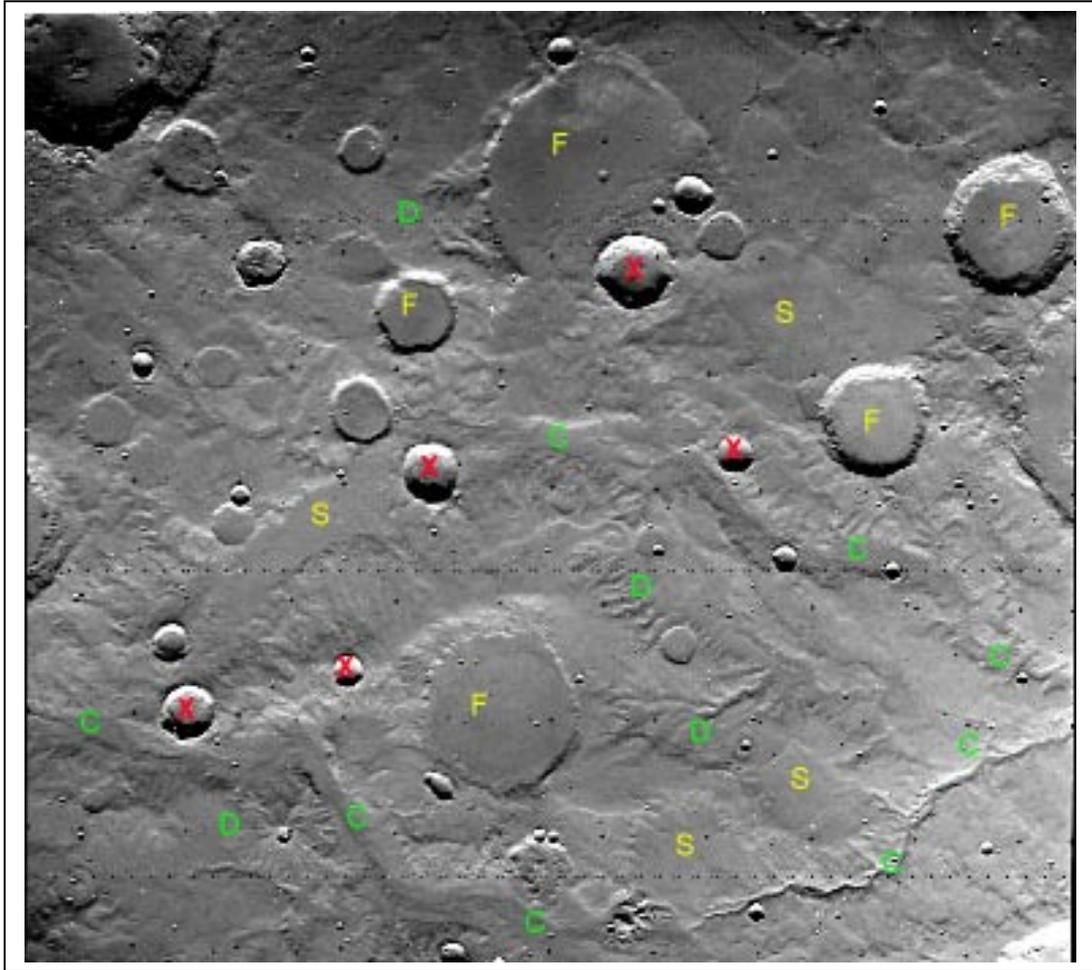
Problem 7 - The satellite of Jupiter, called Ganymede, is  $\frac{2}{5}$  the size of Earth. How large is it compared to Jupiter?

**Answer: Earth =  $\frac{1}{11}$  Jupiter so Ganymede is  $\frac{1}{11} \times \frac{2}{5} = \frac{2}{55}$  x Jupiter.**

Problem 8 - The Dwarf Planet Pluto is  $\frac{1}{3}$  the size of Mars. How large is Jupiter compared to Pluto?

**Answer: Jupiter = 11/1 Earth, Mars=  $\frac{1}{2}$  Earth, so Pluto=  $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$  Earth, and  $\frac{1}{66}$  Jupiter.**

Problem 9 – **Answer: Students should, very nearly, reproduce the numbers in the table at the top of the page.**



When the NASA Mariner IV spacecraft flew by Mars in 1965, scientists were surprised to see so many craters on the surface of this planet, which many science fiction writers had assumed was Earth-like!

The image shown above is an area on Mars called Margaritifer Sinus and was taken by the NASA Viking Orbiter in 1976. The image is 200 kilometers wide. The letters indicate different regions of interest to scientists.

**Question 1** – Using a millimeter ruler, and using proportions, what is the diameter in kilometers of the craters labeled F in the Mars image?

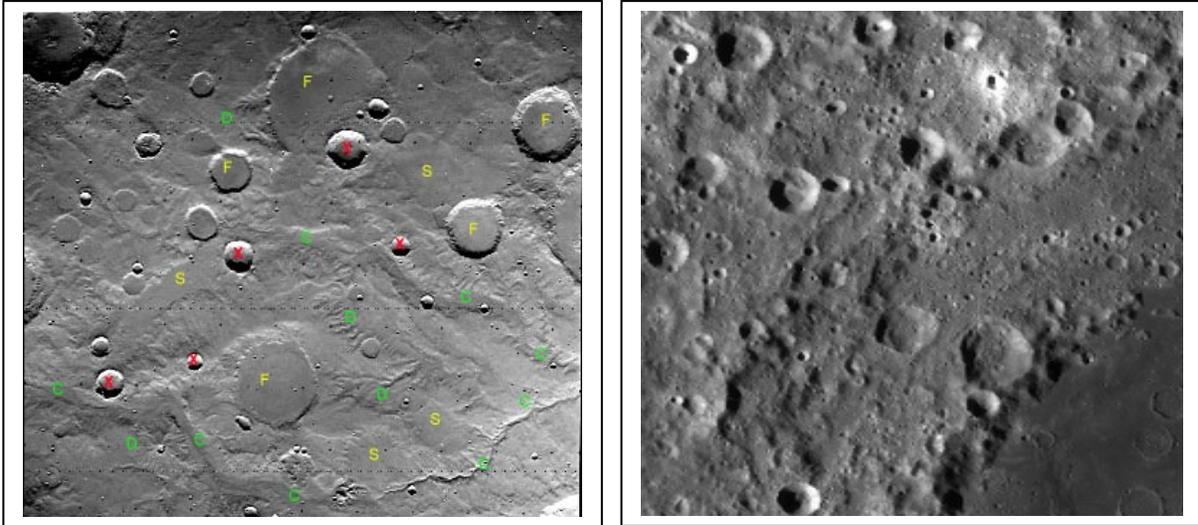
**Question 2** – How many craters can you count on the surface of Mars?

**Question 1** – Using a millimeter ruler, and using proportions, what is the diameter in kilometers of the craters labeled F in the Mars image?

Answer: The image is 200 km wide, and a millimeter ruler indicates a width of 140 mm, so the scale of the image is  $200 \text{ km}/140\text{mm} = 1.4 \text{ km/mm}$ . Then from top to bottom, the crater diameters, in millimeters, are 30mm, 15 mm, 10mm, 15mm and 20mm. The corresponding sizes in kilometers are 42 km, 21 km, 14 km, 21 km and 28 km.

**Question 2** – How many craters can you count on the surface of Mars?

Answer: Students counts will vary, but should be close to 80 craters. Students should note that most of them are very small craters less than a few millimeters (3 kilometers) across. This leads to the possibility of crater counting becoming a histogramming activity where students count the number of craters they find in specific sizes ranges.



When the NASA Mariner IV spacecraft flew by Mars in 1965, scientists were surprised to see so many craters on the surface of this planet, which many science fiction writers had assumed was Earth-like.

The image on the left is an area on Mars called Margaritifer Sinus and was taken by the NASA Viking Orbiter in 1976. The image is 200 kilometers wide. The letters indicate different regions of interest to scientists. As a comparison, the image of the Moon's surface to the right is also 200 km wide and was taken by the Lunar Reconnaissance Orbiter (LRO) in 2010.

**Question 1** – Using a millimeter ruler, and using proportions, what is the diameter in kilometers of the craters labeled F in the Mars image?

**Question 2** – How many craters can you count in the images of A) the Mars surface? B) the lunar surface?

**Question 3** – Which surface has the largest number of craters that are smaller than 5 kilometers in diameter?

**Question 4** – Mars is known to have sand storms and an atmosphere. What do you think the role of erosion has been in changing the crater counts in these images?

**Question 1** – Using a millimeter ruler, and using proportions, what is the diameter in kilometers of the craters labeled F in the Mars image?

Answer: Each image is 74 mm wide which equals 200 km, so the scale is  $200 \text{ km}/74 \text{ mm} = 2.7 \text{ km/mm}$ . The crater diameters are 15mm, 8mm, 5mm, 7mm, 10mm, which corresponds to 40 km, 22 km, 14 km, 19 km and 27 km.

**Question 2** – How many craters can you count in the images of A) the Mars surface?  
B) the lunar surface?

Answer: Students estimates will vary, but they should be able to count about A) 50 craters on the Mars image and B) about 200 craters on the moon image.

**Question 3** – Which surface has the largest number of craters that are smaller than 5 kilometers in diameter?

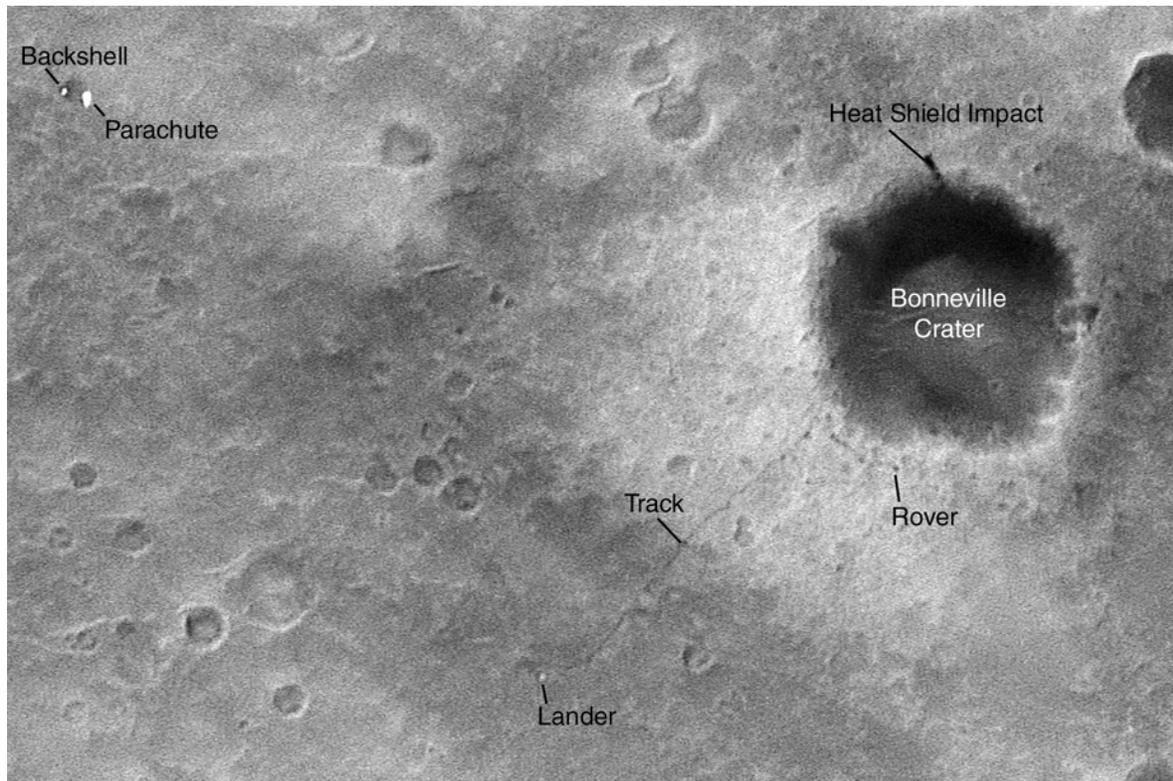
Answer: Students should convert '5 km' to millimeters using the scale of the images. A 5 km crater is then about 2 millimeters in diameter. Students should then be able to inspect the two images and deduce that the moon image has more of these craters than the Mars image.

**Question 4** – Mars is known to have sand storms and an atmosphere. What do you think the role of erosion has been in changing the crater counts in these images?

Answer: Surface erosion on Mars has removed most of the craters smaller than 5 kilometers, and left the larger craters intact. The moon has no atmosphere and sand storms so all of these smaller craters remain intact to the present day.

# The Mars Spirit Rover Landing Site

5



This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the image is exactly 895 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?

Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

**Problem 1:** About what is the diameter of Bonneville Crater rounded to the nearest ten meters?

**Problem 2:** How wide, in meters, is the track of the Rover?

**Problem 3:** How big is the Rover?

**Problem 4:** How small is the smallest well-defined crater to the nearest meter in size?

**Problem 5:** A boulder is typically 5 meters across or larger. Are there any boulders in this picture?

## Answer Key:

This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the crater is 200 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?

Answer: **157 millimeters.**

Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.

Answer: **895 meters.**

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Answer:  $895 \text{ m}/157 \text{ mm} = \mathbf{5.7 \text{ meters / millimeter.}}$

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters.

**Problem 1:** About what is the diameter of Bonneville Crater rounded to the nearest 10 meters?

Answer: Students answers for the diameter of the crater in millimeters may vary, but answers in the range from 30-40 mm are acceptable. Then this equals  $30 \times 5.7 = 170$  meters to  $40 \times 5.7 = 230$  meters. Students may average these two measurements to get  $(170+230)/2 = 200$  meters.

**Problem 2:** How wide, in meters, is the track of the Rover?

Answer:  $0.2$  millimeters = **1 meter**. Students may have difficulty estimating intervals as small as 1 millimeter, in which case they may state track widths are 1 millimeter on the photograph, or 6 meters wide or less as an answer.

**Problem 3:** How big is the Rover?

Answer:  $0.3$  millimeters =  $1.7$  meters but since the measurement is only 1 significant figure, the answer should be **2 meters**. Students may have difficulty estimating intervals as small as 1 millimeter, in which case they may state the Rover is 1 millimeter wide on the photograph, or 6 meters wide or less as an answer.

**Problem 4:** How small is the smallest well-defined crater in meters?

Answer:  $2$  millimeters  $\times 5.7 = 11.4$  meters, which to the nearest meter is **11 meters**.

**Problem 5:** A boulder is typically 5 meters across or larger. Are there any boulders in this picture?

Answer: Students answers may vary and lead to interesting discussions about what features are real, and which ones are flaws in the printing of the picture. This is an important discussion because 'image artifacts' are very common in space-related photographs. 5 meters is about 1 millimeter, and there are no obvious rounded objects this large or larger in this image.



NASA/JPL/Malin Space Science Systems

This NASA, Mars Orbiter image was taken of a crater wall in the southern hemisphere of Mars from an altitude of 450 kilometers. It shows the exciting evidence of water gullies flowing downhill from the top left to the lower right. The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the length of the dark bar is a distance of 300 meters.

Step 1: Measure the length of the bar with a metric ruler. How many millimeters long is the bar?

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

**Question 1:** What are the dimensions, in kilometers, of this image?

**Question 2:** How wide, in meters, are the streams half-way down their flow channels?

**Question 3:** What is the smallest feature you can see in the image?

**Question 4:** How wide is the top of the crater wall at its sharpest edge?

## Answer Key:

This NASA, Mars Orbiter image was taken of a crater wall in the southern hemisphere of Mars from an altitude of 450 kilometers. It shows the exciting evidence of water gullies flowing downhill from the top left to the lower right. The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the length of the dark bar is a distance of 300 meters.

Step 1: Measure the length of the bar with a metric ruler. How many millimeters long is the bar?

Answer: 13 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Answer: 300 meters

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Answer:  $300 \text{ meters} / 13 \text{ mm} = 23 \text{ meters} / \text{millimeter}$ .

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?

Answer:  $134 \text{ mm} \times 120 \text{ mm} = 3.1 \text{ km} \times 2.8 \text{ km}$

Question 2: How wide, in meters, are the streams half-way down their flow channels?

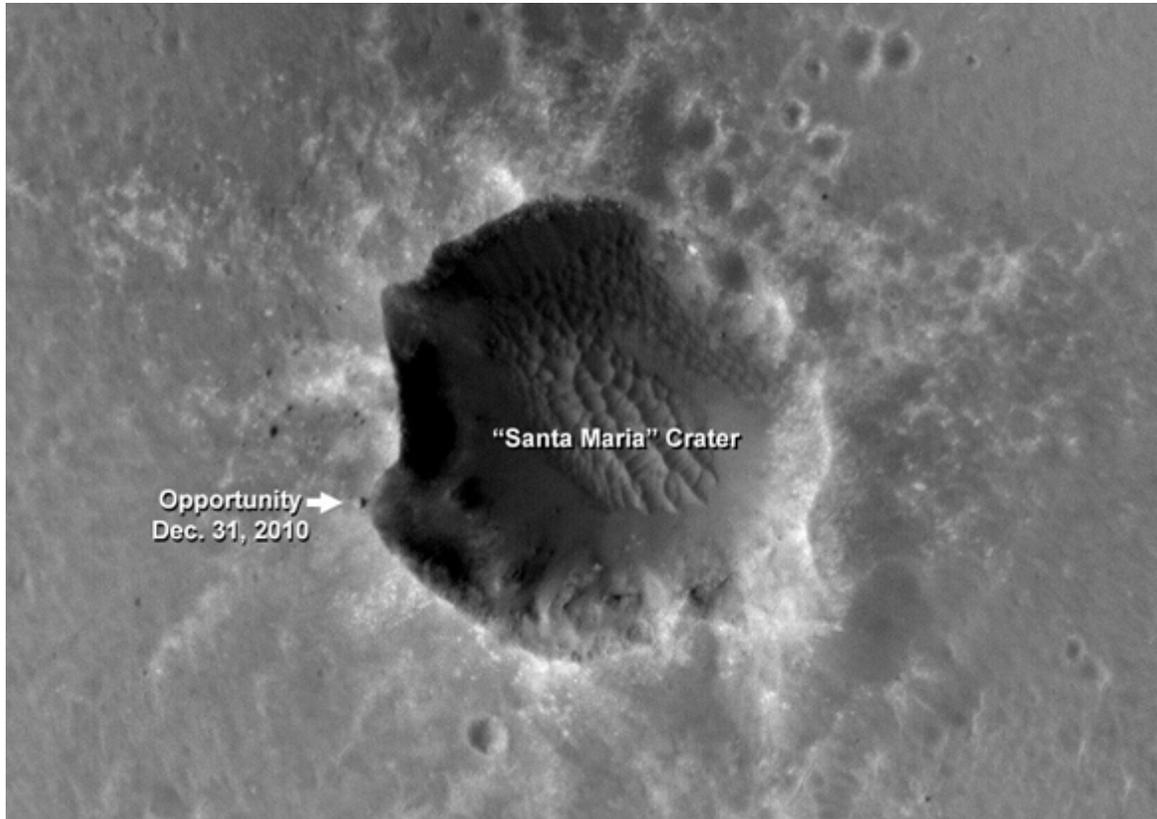
Answer: 0.5 millimeters = 12 meters. Students may have difficulty estimating intervals as small as 1 millimeter, in which case they may state the streams are less than 23 meters wide as an answer.

Question 3: What is the smallest feature you can see in the image?

Answer: Sand dunes in upper left of image = 0.3 millimeters or 7 meters wide. Students may have difficulty estimating intervals as small as 1 millimeter, in which case they may state the smallest features are less than 23 meters wide as an answer.

Question 4: How wide is the top of the crater wall at its sharpest edge?

Answer: 0.2 millimeters or 4 meters wide. Students may have difficulty estimating intervals as small as 1 millimeter, in which case they may state the crater edge is less than 23 meters wide as an answer.



The High Resolution Imaging Science Experiment (HiRISE) camera on NASA's Mars Reconnaissance Orbiter acquired this image of the Opportunity rover on the southwest rim of "Santa Maria" crater on New Year's Eve 2010. Opportunity arrived at the western edge of Santa Maria crater in mid-December, 2009 and spent about two months investigating rocks there. That investigation took Opportunity into the beginning of its eighth year on Mars. Opportunity imaged the crater interior to better understand the geometry of rock layers and the meteor impact process on Mars. Santa Maria is a relatively young impact crater, but old enough to have collected sand dunes in its interior. The width of this image corresponds to 310 meters.

**Problem 1** - About what is the circumference of the rim of this crater in meters?

**Problem 2** - If the crater can be approximated as a circle, about what is its average diameter in meters?

**Problem 3** - The rover can travel about 100 meters in one day. About how long will it take the rover to travel once around this crater?

**Problem 4** - A comfortable walking speed is about 100 meters per minute. About how long would it take a human to stroll around the edge of this crater?

**Problem 1** - About what is the circumference of the rim of this crater in meters?

Answer: The width of the image is 150 millimeters or 310 meters in actual distance. The scale of this image is about 310 meters/150 mm = 2.0 meters/millimeter. Because the crater is not a perfect circle, students may use a piece of string and obtain an answer of about 200 millimeters for the circumference, then using the scale of the image, the distance in meters around the crater is about 200 mm x 2.0 meters/mm = **400 meters**.

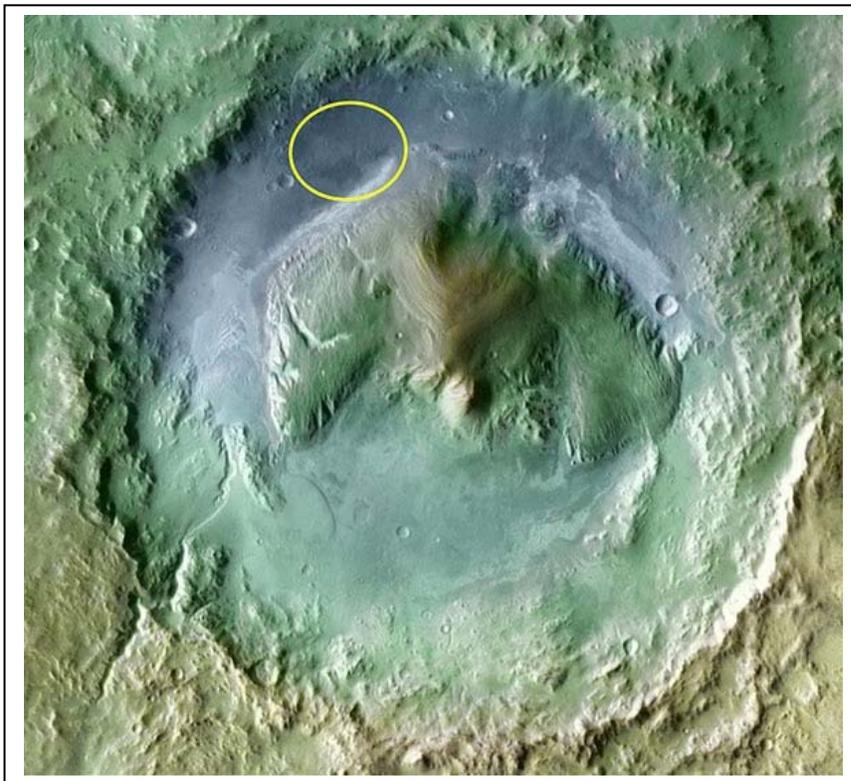
**Problem 2** - If the crater can be approximated as a circle, about what is its average diameter in meters? Answer:  $C = \pi D$ , and for  $C = 400$  meters we have  $D =$  **127 meters**.

**Problem 3** - The rover can travel about 100 meters in one day. About how long will it take the rover to travel once around this crater?

Answer: 400 meters x (1 day/100 meters) = **4 days**.

**Problem 4** - A comfortable walking speed is about 100 meters per minute. About how long would it take a human to stroll around the edge of this crater?

Answer: 400 meters x (1 minute/100 meters) = **4 minutes**.



Gale Crater has a diameter of 150 kilometers and is the landing site for the Curiosity Rover, which arrived on August 6, 2012.

The image to the left was taken by the Odyssey spacecraft from its orbit around Mars. The yellow ellipse shows the expected landing area for Curiosity in a flat region of the crater floor.

The central mountain shows layered rock strata, which will let geologists explore the early history of Mars. It will also let scientists see if conditions favoring life may have existed when Mars was a younger planet.

**Problem 1** - Print this page on a standard printer and measure the width of this image in millimeters. If the actual width of this image covers a distance of 150 kilometers, what is the scale of this photograph in kilometers per millimeter?

**Problem 2** – From your measurement of the length and width of the landing ellipse, how many kilometers long and wide is the targeted landing ellipse on the floor of Gale Crater?

**Problem 3** – Curiosity can travel at a typical speed of 150 meters per day. How many days will it take Curiosity to travel the length of the landing ellipse?

**Problem 4** – Create a possible path that could be taken by Curiosity to explore five interesting things that you see in the crater floor. Make sure that Curiosity does not cross over rough areas (hills and mountains) or fall off a cliff. If the planned mission lasts only 2 years, can you complete your journey in the required time?

**Problem 1** - Print this page on a standard printer and measure the width of this image in millimeters. If the actual width of this image covers a distance of 150 kilometers, what is the scale of this photograph in kilometers per millimeter?

Answer:  $112 \text{ mm} = 150 \text{ km}$ , scale =  $150/112 = 1.3 \text{ km/mm}$

**Problem 2** – From your measurement of the length and width of the landing ellipse, how many kilometers long and wide is the targeted landing ellipse on the floor of Gale Crater?

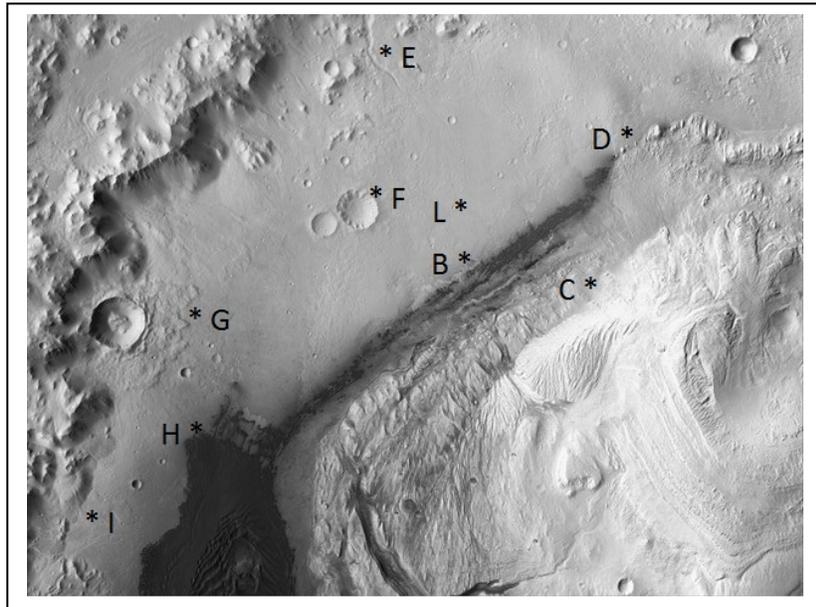
Answer: Length = 15 mm , width = 12 mm, so Length =  $15 \times 1.3 = 20 \text{ km}$ , width =  $12 \times 1.3 = 16 \text{ km}$ .

**Problem 3** – Curiosity can travel at a typical speed of 150 meters per day. How many days will it take Curiosity to travel the length of the landing ellipse?

Answer: Distance = 20 km = 20,000 meters. Time = Distance/speed =  $20,000/150 = 133 \text{ days}$ .

**Problem 4** – Create a possible path that could be taken by Curiosity to explore five interesting things that you see in the crater floor. Make sure that Curiosity does not cross over rough areas (hills and mountains) or fall off a cliff. If the planned mission lasts only 2 years, can you complete your journey in the required time?

Answer: Students answers will vary. They will select five points, then use a millimeter ruler to measure the distances between them .They will convert these into kilometer units, then estimate the travel time between the points. By adding up the travel times they will check that the sum is less than 2 years (730 days).



The table below gives the coordinates for the locations to be visited by the Curiosity Rover shown in the figure above. The X and Y coordinates are given in kilometers. Although Curiosity is free to travel between most points on the map, Point C is at a much higher elevation than the other points located in the crater floor, and a steep and impassible cliff wall exists between points B and C and runs diagonally to the lower left.

Label	Name	(X,Y)	Label	Name	(X,Y)
L	Landing Area	(45,40)	F	Crater Wall	(38,43)
B	Layered Wall	(50,35)	G	Mudslide	(17,30)
C	Alluvial Fan	(60,32)	H	Dark Sands	(17,19)
D	Summit Access	(65,50)	I	Mystery Valley	( 5,10)
E	River Bed	(37,58)			

**Problem 1** – Curiosity can travel at a top speed of 300 meters/hr. As soon as it lands, Curiosity will be instructed to travel to the highest priority location first, just in case the mission prematurely fails. To the nearest kilometer, what is the distance traveled, and to the nearest hour, how long will it take to travel between Point L and Point B?

**Problem 2** – To the nearest kilometer, what is the distance from Point D to Point I, and to the nearest hour, how long will it take Curiosity to travel this far?

**Problem 3** – One possible path Curiosity might take that connects all of the points is represented by the sequence L-B-D-C-D-E-F-B-G-H-I. To the nearest kilometer, what is the total distance traveled, and to the nearest tenth, how many days will this journey take?

## Answer Key

**Problem 1** – Curiosity can travel at a top speed of 300 meters/hr. As soon as it lands, Curiosity will be instructed to travel to the highest priority location first, just in case the mission prematurely fails. To the nearest kilometer, what is the distance traveled, and to the nearest hour, how long will it take to travel between Point L and Point B?

Answer: L (45,40) and B (50,35). Using the Pythagorean Theorem and distance formula for Cartesian points  $D = ((50-45)^2 + (35-40)^2)^{1/2} = 7 \text{ km}$ . Traveling at 300 m/hr, this will take  $7000\text{m}/300\text{m} = 23 \text{ hours}$ .

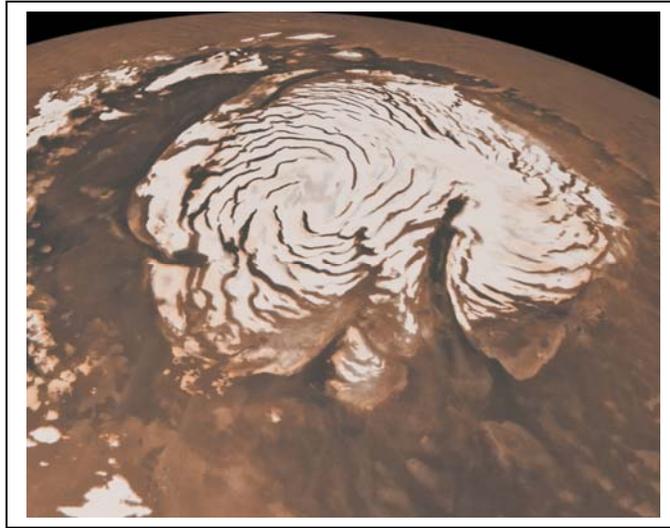
**Problem 2** – To the nearest kilometer, what is the distance from Point D to Point I, and to the nearest hour, how long will it take Curiosity to travel this far?

Answer: Point D (65,50), Point I (5,10).  $D = ((5-65)^2 + (10-50)^2)^{1/2} = 72 \text{ kilometers}$ . Traveling at 300 m/hr, this takes  $72000/300 = 240 \text{ hours}$  (or 10 days).

**Problem 3** – One possible path Curiosity might take that connects all of the points is represented by the sequence L-B-D-C-D-E-F-B-G-H-I. To the nearest kilometer, what is the total distance traveled, and to the nearest tenth, how many days will this journey take?

$$\begin{aligned} D(LB) &= ((50-45)^2 + (35-40)^2)^{1/2} = 7 \\ D(BD) &= ((65-50)^2 + (50-35)^2)^{1/2} = 21 \\ D(DC) &= ((60-65)^2 + (32-50)^2)^{1/2} = 19 \\ D(CD) &= ((65-60)^2 + (50-32)^2)^{1/2} = 19 \\ D(DE) &= ((37-65)^2 + (58-50)^2)^{1/2} = 29 \\ D(EF) &= ((38-37)^2 + (43-58)^2)^{1/2} = 15 \\ D(FB) &= ((50-38)^2 + (35-43)^2)^{1/2} = 14 \\ D(BG) &= ((17-50)^2 + (30-35)^2)^{1/2} = 33 \\ D(GH) &= ((17-17)^2 + (19-30)^2)^{1/2} = 11 \\ D(HI) &= ((5-17)^2 + (10-19)^2)^{1/2} = 15 \end{aligned}$$

Total distance traveled = **183 km**. Time =  $183,000/300 = 610 \text{ hours} = 25.4 \text{ days}$



It has been known for nearly 100 years that Mars has two polar caps, which change in size with the martian seasons. Both the North and South Polar Caps contain frozen water in different amounts. The photo above was taken by NASA's Mars Reconnaissance Orbiter and shows the details in the 1,200-km wide North Polar Ice Cap in the summer showing the deposits of frozen water ice.

Scientists estimate that, if the Polar Ice Caps were completely melted, they would produce 2.5 million cubic kilometers of liquid water covering the surface of Mars.

**Problem 1** – Mars is in the geometric shape of a sphere, with a radius of 3,400 km. If the formula for the surface area of a sphere is  $S = 4\pi R^2$ , what is the surface area of Mars?

**Problem 2** – The volume of a shell of water covering the surface of Mars to a depth of  $h$  kilometers is given by  $V = h \times A$  where  $A$  is the surface area of Mars in square kilometers. If the volume of the melted water ice from the polar caps is 2.5 million  $\text{km}^3$ , what will the depth of the water be in meters if it completely covered the surface of Mars?

**Problem 1** – Mars is in the geometric shape of a sphere, with a radius of 3,400 km. If the formula for the surface area of a sphere is  $S = 4\pi R^2$ , what is the surface area of Mars?

**Answer:**  $S = 4 \times 3.141 \times (3400)^2 = 145 \text{ million km}^2$

**Problem 2** – The volume of a shell of water covering the surface of Mars to a depth of  $h$  kilometers is given by  $V = h \times A$  where  $A$  is the surface area of Mars in square kilometers. If the volume of the melted water ice from the polar caps is 2.5 million  $\text{km}^3$ , what will the depth of the water be in meters if it completely covered the surface of Mars?

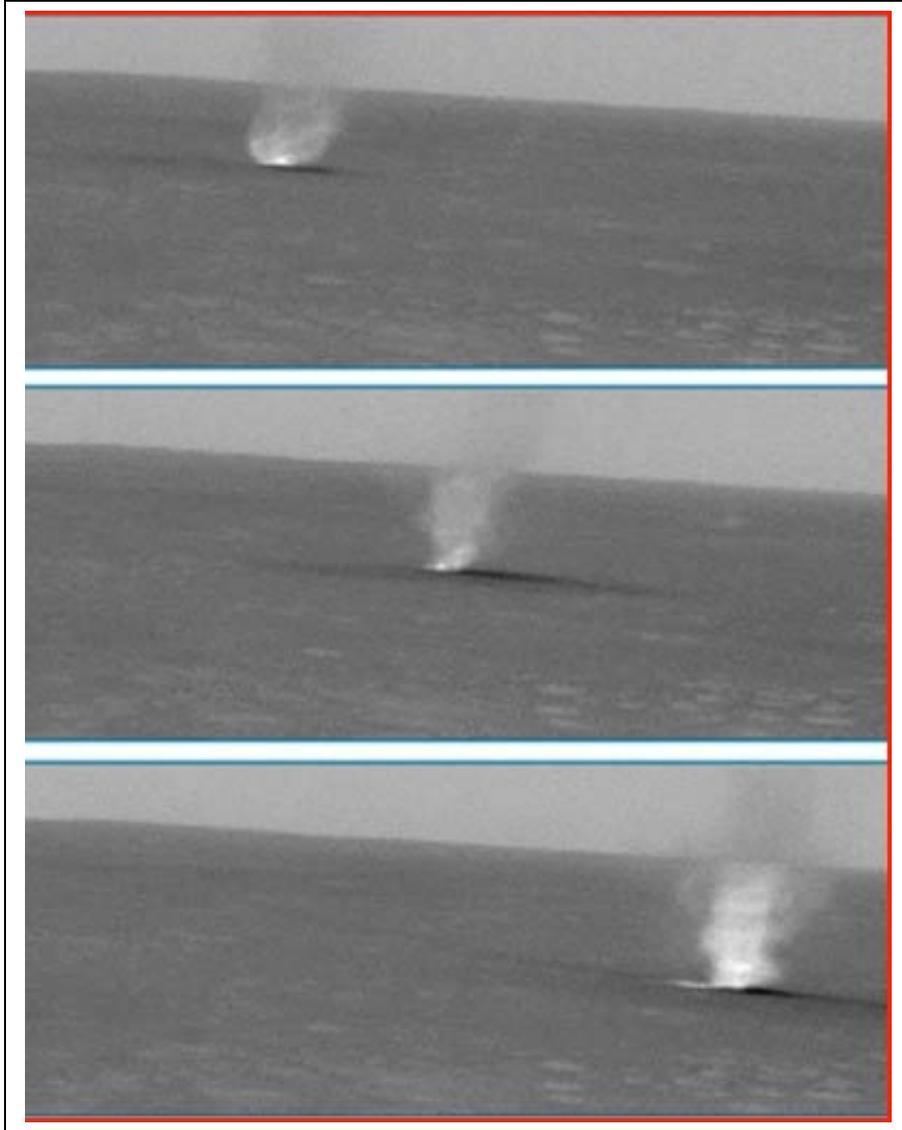
Answer:

$$2.5 \text{ million km}^3 = h \times (145 \text{ million km}^2)$$

$$\text{So } h = 2.5 \text{ million} / 145 \text{ million}$$

And so  $h = 0.017$  kilometers. Since there are 1000 meters in 1 kilometer,  
 **$h = 17 \text{ meters}$ .**

Note: Our Earth's oceans have an average depth of 3,800 meters. If the Greenland Ice Sheet completely melted, it would add an additional 7 meters.



A dust devil spins across the surface of Gusev Crater just before noon on Mars. NASA's Spirit rover took the series of images to the left with its navigation camera on March 15, 2005. The images were taken at:

11:48:00 (T=top)  
 11:49:00 (M=middle)  
 11:49:40 (B=bottom)

based upon local Mars time. The dust devil was about 1.0 kilometer from the rover at the start of the sequence of images on the slopes of the "Columbia Hills."

A simple application of the rate formula

$$speed = \frac{distance}{time}$$

let's us estimate how fast the dust devil was moving. In the following problems, we will assume the dust devil remains at the same distance from the Spirit Rover in each of the images.

**Problem 1** - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top (T) and bottom (B) frames?

**Problem 2** - What was the time difference, in seconds, between the images T-M, M-B and T-B?

**Problem 3** - What was the distance, in meters, traveled between the images T-M and M-B?

**Problem 4** - What was the average speed, in meters/sec, of the dust devil between T-B. If an astronaut can briskly walk at a speed of 120 meters/minute, can she out-run a martian dust devil?

**Problem 5** - What were the speeds during the interval from T-M, and the interval M-B?

**Problem 6** - Was the dust devil accelerating or decelerating between the times represented by T-B?

# Answer Key

Problem 1 - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top and bottom frames? Answer: The location of the dust devil in frame B when placed in image T is a shift of about 65 millimeters, which at a scale of 7.4 meters/mm equals about **480 meters**.

Problem 2 - What was the time difference between the images T-M, M-B and T-B? Answer: T-M = 11:49:00 - 11:48:00 = 1 minute or 60 seconds. For M-B the time interval is 40 seconds. For T-B the time interval is **100 seconds**.

Problem 3 - What was the distance traveled between the images T-M and M-B? Answer: T-M = about 30 mm or 222 meters; M-B = about 35 mm or **259 meters**.

Problem 4 - What was the average speed, in meters/sec, of the dust devil between T-B? If an astronaut can briskly walk at a speed of 120 meters/minute, can she out-run a martian dust devil?

Answer: Speed = distance/time so  $480 \text{ meters} / 100 \text{ seconds} = \mathbf{4.8 \text{ meters/sec}}$ . This is about twice as fast as an astronaut can walk, so running would be a better option.

Problem 5 - What were the speeds during the interval from T-M, and the interval M-B? Answer: Speed(T-M) =  $222 \text{ meters} / 60 \text{ seconds} = 3.7 \text{ meters/sec}$ . Speed(M-B) =  $259 \text{ meters} / 40 \text{ seconds} = \mathbf{6.5 \text{ meters/sec}}$ .

Problem 6 - Was the dust devil accelerating or decelerating between the times represented by T-B? Answer: because the speed increased from 4.8 meters/sec to 6.5 meters/sec, the dust devil was **accelerating** during this time interval from 11:48:00 to 11:49:40.



This image was taken by NASA's Mars Reconnaissance Orbiter on February 19, 2008. It shows an avalanche photographed as it happened on a cliff on the edge of the dome of layered deposits centered on Mars' North Pole. From top to bottom this impressive cliff is over 700 meters (2300 feet) tall and reaches slopes over 60 degrees. The top part of the scarp, to the left of the image, is still covered with bright (white) carbon dioxide frost which is disappearing from the polar regions as spring progresses. The upper mid-toned (pinkish-brownish) section is composed of layers that are mostly ice with varying amounts of dust. The dust cloud extends 190 meters from the base of the cliff.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the cloud extends 190 meters from the base of the cliff.

Step 1: Measure the length of the dust cloud with a metric ruler. How many millimeters long is the cloud?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

**Question 1:** What are the dimensions, in meters, of this image?

**Question 2:** What is the smallest detail you can see in the ice shelf?

**Question 3:** What is the average thickness of the red layers on the cliff?

**Question 4:** What is the total width of the reddish rock cliff?

**For experts:** Two sides of the right triangle measure 700 meters, and your answer to Question 4. What is the angle of the cliff at the valley floor?

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the cloud extends 190 meters from the base of the cliff.

Step 1: Measure the length of the dust cloud with a metric ruler. How many millimeters long is the cloud?  
Answer: 60 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length.  
Answer: 190 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.  
Answer:  $190 \text{ meters} / 60 \text{ mm} = 3.2 \text{ meters} / \text{mm}$

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in meters, of this image?

Answer:  $140 \text{ mm} \times 86 \text{ mm} = 448.0 \text{ meters} \times 275.2 \text{ meters}$ , but to two significant figures this becomes  $450 \text{ meters} \times 280 \text{ meters}$ .

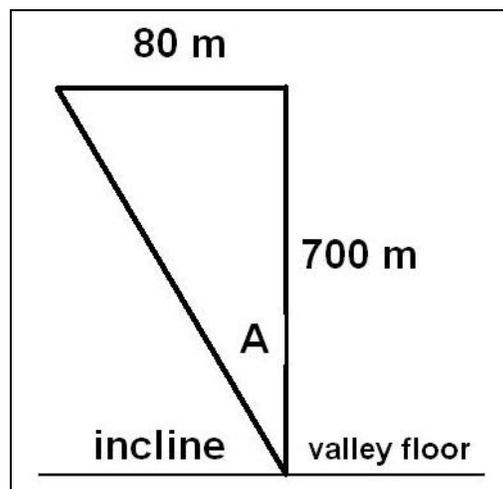
Question 2: What is the smallest detail you can see in the ice shelf?  
Answer:  $0.2 \text{ mm} = 0.6 \text{ meters}$

Question 3: What is the average thickness of the red layers on the cliff?  
Answer:  $1.0 \text{ millimeter} = 3.2 \text{ meters}$ .

Question 4: What is the total width of the reddish rock cliff?  
Answer:  $25 \text{ millimeters} = 80 \text{ meters}$ .

**For experts:** Two sides of the right triangle measure 700 meters, and your answer to Question 4. What is the angle of the cliff at the valley floor?

Answer: Draw the triangle with the 700-meter side being vertical and the 80 meter side being horizontal. The tangent of the angle is  $(80 / 700) = 0.11$  so the angle is 6.5 degrees. This is the angle from the vertical, so the incline angle from the floor of the valley is  $90 - 6.5 = 84$  degrees. This is a nearly vertical wall!



This sequence of stills was obtained from a YouTube.com video of the launch of MSL by United Space Alliance available at <http://www.youtube.com/watch?v=0cxsvVBemHY>

This sequence shows the launch of the MSL mission from the Kennedy Space Center Launch Complex 49 on November 27, 2011 at 10:02 EST. The four images were taken, from bottom to top, at times 10:02:48 EST, 10:02:50 EST, 10:02:51 EST and 10:02:52 EST. At the distance of the launch pad, the width of each image is 400 meters.

**Problem 1** - With the help of a millimeter ruler, what is the scale of each image in meters/mm?

**Problem 2** - For each image, what is the distance between the bottom of the image and the base of the rocket nozzle for the Atlas V rocket in each scene?

**Problem 3** - What is the estimated distance from the base of the launch pad to the rocket nozzle in each image?

**Problem 4** - From the time information, what is the average speed of the rocket between A) Image 1 and 2? B) Image 2 and 3? C) Image 3 and 4?

**Problem 5** - From the speed information in Problem 4, what is the average acceleration between A) Image 1 and Image 3? B) Image 2 and Image 4?

**Problem 6** - Graph the height of the rocket versus the time in seconds since launch.

**Problem 7** - Graph the speed of the rocket versus time in seconds after launch. For the time, use the midpoint time for each speed interval.

**Problem 8** - Graph the acceleration of the rocket versus time in seconds after launch. For the time, use the midpoint time for each acceleration interval.

**Problem 1** - Answer: Width = 69 mm, so scale = 400 m/69 mm = **5.8 meters/mm**

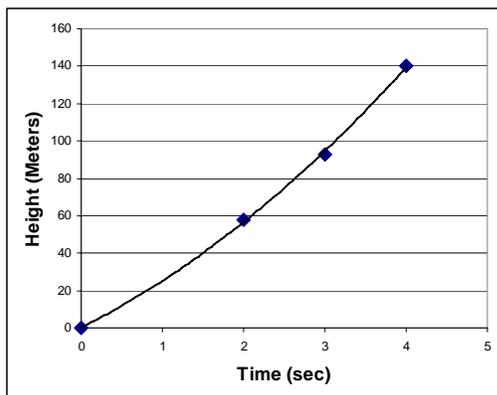
**Problem 2** - Answer: 8mm, 18mm, 24mm and 32mm so using the scale of the image, the actual distances are **46m, 104m, 139m and 186 meters**.

**Problem 3** – Answer: Take the differences in the measurements relative to the first image at the moment of launch to get  $h_1 = 46m - 46m = 0m$ ,  $h_2 = 104m - 46m = 58m$ ,  $h_3 = 139m - 46m = 93m$  and  $h_4 = 186m - 46m = 140m$ .

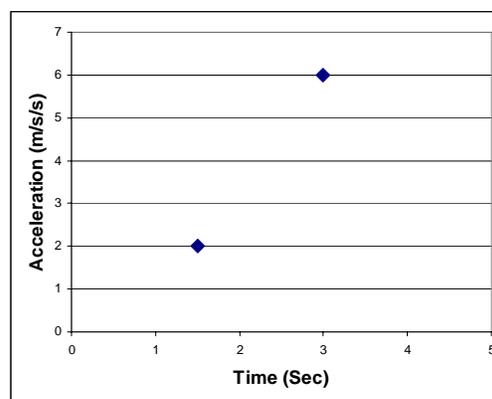
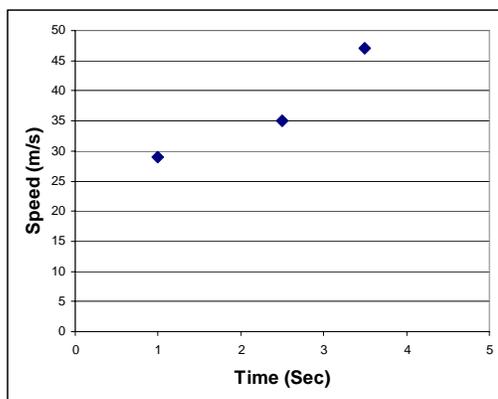
**Problem 4** – Answer: A)  $v = \text{distance}/\text{time}$ ,  $v_1 = (58m - 0m)/2\text{sec} = 29\text{m/sec}$  B)  $v_2 = (93m - 58m)/1\text{sec} = 35\text{ m/sec}$ , C)  $v_3 = (140m - 93m)/1\text{sec} = 47\text{ m/sec}$ .

**Problem 5** – Answer: A)  $a_1 = (v_2 - v_1)/3\text{sec} = (35 - 29)/3 = 6/3 = 2\text{ m/sec}^2$ . B)  $a_2 = (v_3 - v_2)/2\text{sec} = (47 - 35)/2\text{sec} = 6\text{ m/sec}^2$ .

**Problem 6** – Graph the height of the rocket versus the time in seconds since launch.



**Problem 7** – Graph the speed of the rocket versus time in seconds after launch. For the time value, use the midpoint time for each speed interval. Answer: Left Above. For the first speed, the two height measurements are made at  $T=0$  and  $T=2$ , so the speed  $V_1$  will be plotted at the midpoint time:  $T = (2-0)/2 = 1\text{ sec}$



**Problem 8** – Graph the acceleration of the rocket versus time in seconds after launch. For the time value, use the midpoint time for each acceleration interval. Answer: Right Above.



How does the mass of Mars compare to the mass of Earth? We know that the average density of Earth is about  $4000 \text{ kg/m}^3$  and that

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

So this means that

$$\text{Mass} = \text{Density} \times \text{Volume}$$

Because Mars is a round 'spherical' planet, the volume of Mars is given by the formula for the volume of a sphere:

$$\text{Volume} = \frac{4}{3} \pi R^3$$

**Problem 1** - The radius of Mars is 3,400 kilometers. What is the volume of Mars in cubic meters?

**Problem 2** - If the density of Mars is similar to Earth, what is your estimate for the total mass of Mars in kilograms?

**Problem 3** - The mass of Earth is  $5.97 \times 10^{24}$  kg. About how many planets like Mars could you make from the mass of Earth?

**Problem 1** - The radius of Mars is 3,400 kilometers. What is the volume of Mars in cubic meters?

Answer: Volume =  $\frac{4}{3} \pi R^3$

$$\text{so } V = 1.333 \times 3.141 \times (3.4 \times 10^6)^3$$

$$\text{and so } V = 1.65 \times 10^{20} \text{ meters}^3$$

**Problem 2** - If the density of Mars is similar to Earth, what is your estimate for the total mass of Mars in kilograms?

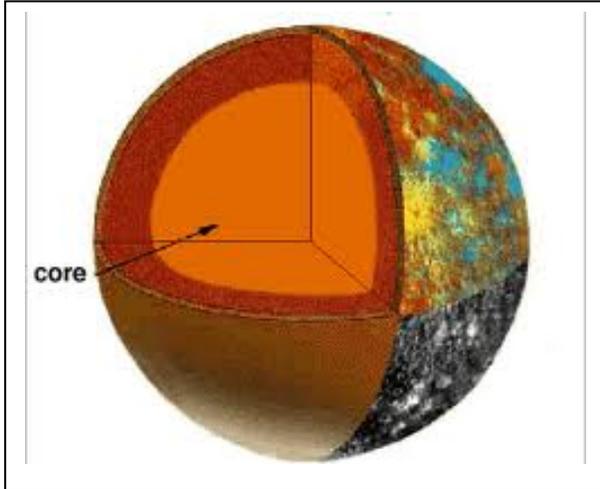
Answer: Mass = Density x Volume so

$$M = (4000) \times (1.65 \times 10^{20})$$

$$\text{and so } M = 6.6 \times 10^{23} \text{ kg.}$$

**Problem 3** - The mass of Earth is  $5.97 \times 10^{24}$  kg. About how many planets like Mars could you make from the mass of Earth?

$$\text{Answer: } 5.97 \times 10^{24} \text{ kg} / 6.6 \times 10^{23} \text{ kg} = \mathbf{9 \text{ planets like Mars.}}$$



In 2012, NASA selected the InSight mission to place a seismographic station on the surface of Mars in order to directly study the interior of Mars. The InSight mission will use seismographic data from 'mars quakes' to probe the interior of Mars.

With the exception of Earth and the Moon, scientists have only been able to deduce what the interior of a planet like Mars looks like by a detailed measurement of its mass, radius and its gravity field.

We can create a simple model of the interior of Mars by dividing it into a spherical core region, and an overlying shell of matter that reaches to the observed surface of the planet. Here is how we do this!

**Problem 1** - The mass of Mars is  $6.4 \times 10^{23}$  kilograms. If the planet is a perfect sphere with a radius of 3,340 kilometers, what is the average density of the planet Mars in  $\text{kg}/\text{meter}^3$  defined as density = mass/volume?

**Problem 2** - Astronomers believe that the crust of Mars has an average density of  $3,000 \text{ kg}/\text{m}^3$  and there may be a denser core with a density of  $7,800 \text{ kg}/\text{m}^3$  similar to iron

A) What is the formula that gives the total mass of the core of Mars, if the core has a radius of  $R_c$  in meters?

B) What is the formula for the outer shell 'crust' of Mars if the density equals the density of the crust, and its inner radius is  $R_c$  and its outer radius is the actual radius of the planet of 3,340 kilometers?

**Problem 3** - If the sum of the core and shell masses must equal the mass of the planet, what is the value for  $R_c$ , the radius of the core in kilometers, that leads to a solution for this simple model?

**Problem 1** - The mass of Mars is  $6.4 \times 10^{23}$  kilograms. If the planet is a perfect sphere with a radius of 3,340 kilometers, what is the average density of the planet Mars in  $\text{kg/meter}^3$  defined as density = mass/volume?

Answer: Volume =  $\frac{4}{3}\pi R^3$

so Volume =  $\frac{4}{3}(3.141)(3,340,000)^3 = 1.56 \times 10^{20} \text{ meters}^3$

Density =  $6.4 \times 10^{23} \text{ kilograms} / 1.56 \times 10^{20} \text{ meters}^3 = \mathbf{4,100 \text{ kg/m}^3}$ .

**Problem 2** - Astronomers believe that the crust of Mars has an average density of  $3,000 \text{ kg/m}^3$  and the iron-rich core has a density of  $7,800 \text{ kg/m}^3$ . A) What is the formula that gives the total mass of the core, if the core has a radius of  $R_c$  in kilometers? B) What is the formula for the outer shell 'crust' of Mars if the density equals the density of the crust, and its inner radius is  $R_c$  and its outer radius is the actual radius of the planet of 3,340 kilometers?

Answer A)  $M(\text{core}) = \frac{4}{3}\pi(1000R_c)^3(7800) = 3.26 \times 10^{13} R_c^3 \text{ kilograms}$

B) Volume of the spherical shell =  $\frac{4}{3}\pi(3,340,000)^3 - \frac{4}{3}\pi(1000R_c)^3 \text{ cubic meters}$

Then  $M(\text{shell}) = 3,000 \times V(\text{shell})$

=  $3000 \left( \frac{4}{3}\pi(3,340,000)^3 - \frac{4}{3}\pi(1000R_c)^3 \right) \text{ kilograms}$

=  $4.68 \times 10^{23} - 1.25 \times 10^{13} R_c^3 \text{ kilograms}$

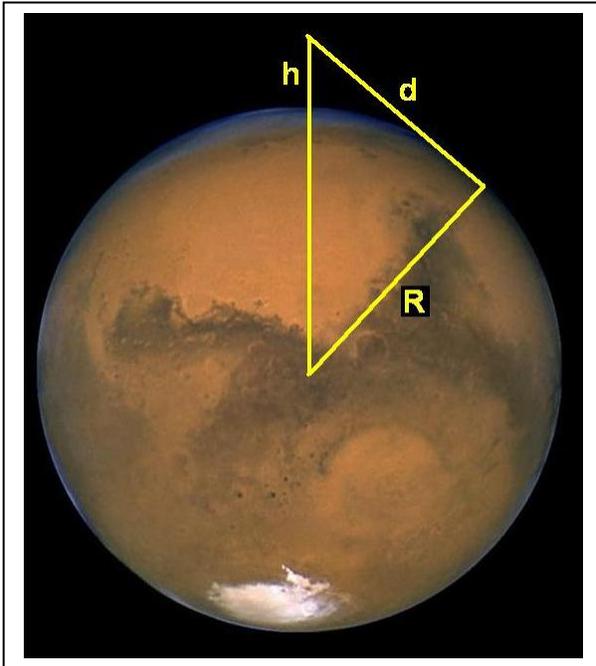
**Problem 3** - If the sum of the core and shell masses must equal the mass of the planet, what is the value for  $R_c$ , the radius of the core in kilometers, that leads to a solution for this simple model?

Answer:  $6.4 \times 10^{23} = 3.26 \times 10^{13} R_c^3 + 4.68 \times 10^{23} - 1.25 \times 10^{13} R_c^3$

So  $1.72 \times 10^{23} = 2.01 \times 10^{13} R_c^3$

And so  $R_c = \mathbf{2,043 \text{ kilometers!}}$

So the hypothetical dense iron core of Mars occupies  $100\% \times (2043/3340) = 61\%$  of the radius of Mars! By comparison, Earth's core is only 30% of its radius.



Suppose you are a camera perched on the top of the Curiosity Rover, or you are an astronaut walking on the surface of Mars, who wants to remain within eyesight of Home Base.

An important quantity in planetary exploration is the distance to the horizon. This will, naturally, depend on the radius of the planet and the height of the observer's eyeballs above the ground.

For small bodies such as asteroids, the horizon may only be a hundred meters, but for planets like Earth it can be many kilometers.

**Problem 1** - If the radius of the planet is given by  $R$ , and the height above the surface is given by  $h$ , use the figure above, and the Pythagorean Theorem to derive the formula for the distance,  $d$ , to the horizon tangent point, assuming that the triangle is a right triangle.

**Problem 2** - For a typical human height of 2 meters, what is the horizon distance on:

A) Earth ( $R=6,378$  km);

B) Mars ( $R=3,340$  km)

Answer Key:

**Problem 1 -** By the Pythagorean Theorem  $d^2 = (R+h)^2 - R^2$

so  $d = (R^2 + 2Rh + h^2 - R^2)^{1/2}$

and so  $d = (h^2 + 2Rh)^{1/2}$

Note: so long as the height,  $h$ , is much less than the radius of the planet, the formula simplifies to

$$d = (2Rh)^{1/2}$$

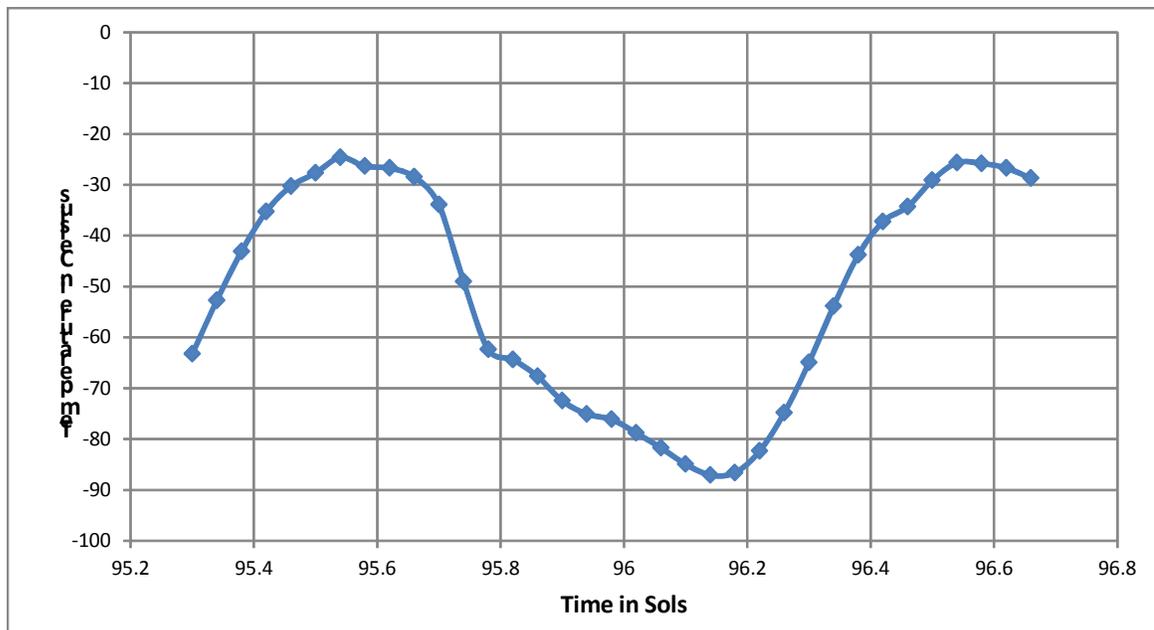
**Problem 2 -** Use the equation from Problem 1.

A) For Earth,  $R=6378$  km and  $h=2$  meters so

$$d = (2 \times 2 \text{ meters} \times 6.378 \times 10^6 \text{ meters})^{1/2}$$

= 5051 meters or **5.1 kilometers**.

B) For Mars,  $R = 3,340$  km so  $d =$  **3.7 kilometers**



In 1976, the NASA Viking 1 lander recorded the daily temperature of the air located 1.5 meters above the surface of Mars as shown in the graph above. The data were taken during the local Mars summer time. The horizontal scale is in local Mars days, called sols, which are slightly longer than Earth days (23 hours and 56 minutes) and last 24 hours and 37 minutes.

**Problem 1** – What is the range of the Mars temperatures to the nearest degree in Celsius?

**Problem 2** - What is the average temperature in Celsius for this period of time?

**Problem 3** – To the nearest tenth of a sol, how soon after sol 95.0 was the lowest temperature in Celsius recorded?

**Problem 4** – What is the average period of the temperature changes?

**Problem 5**– From the information in Problems 1 to 4, create a model for the Celsius temperature based on a sine function that approximates the temperature data for the Viking 1 landing area using the basic form

**Problem 6** – What would you predict as the temperature for the Viking 1 landing site at sol 98.0?

**Problem 1** – What is the range of the Mars temperatures to the nearest degree in Celsius?  
 Answer: From the tabulated data, **Viking 1: (-87 , -25)**

**Problem 2** - What is the average temperature in Celsius for this period of time? Answer:  
 The average is (Maximum + Minimum)/2 so for Viking 1 this is **-56.1 C**.

**Problem 3** – To the nearest tenth of a sol, about what time after sol 95.0 was the average temperature in Celsius computed in Problem 2 recorded for Viking 1? Answer: From the tabulated data, for Viking 1 this happened between sols 95.34 and 95.3 or to the nearest tenth, at sol **95.3**.

**Problem 4** – What is the average period of the temperature changes?  
 Answer: From maximum to maximum is about, **P = 1.0 sols**.

**Problem 5** – From the information in Problems 1 to 3, create a model for the Celsius temperature for the Viking 1 landing site based on a sine function that approximates the temperature data using the basic form  $T = T_{ave} + A \sin(2\pi(t-t_0)/P)$  where  $t$  is the local time in sols.

Answer: Viking 1:  **$T_{ave} = -58.4 \text{ C}$**

The amplitude of the sine is just the average difference between  $T_{ave}$  and the maximum and minimum.  $A_1 = -58.4 - (-87.4) = +29\text{C}$  and  $A_2 = -58.4 - (-24.7) = 33.7$  so  $A = (A_1+A_2)/2$  and so the amplitude of the temperature change is just  $A = 31.4 \text{ C}$ .

From Problem 3,  $t_0=95.3$  sols, then  **$T = -58.4 + 31.4 \sin(2\pi(t-95.3))$**

**Problem 6** – What would you predict as the temperature for the Viking 1 landing site at sol 98.0?

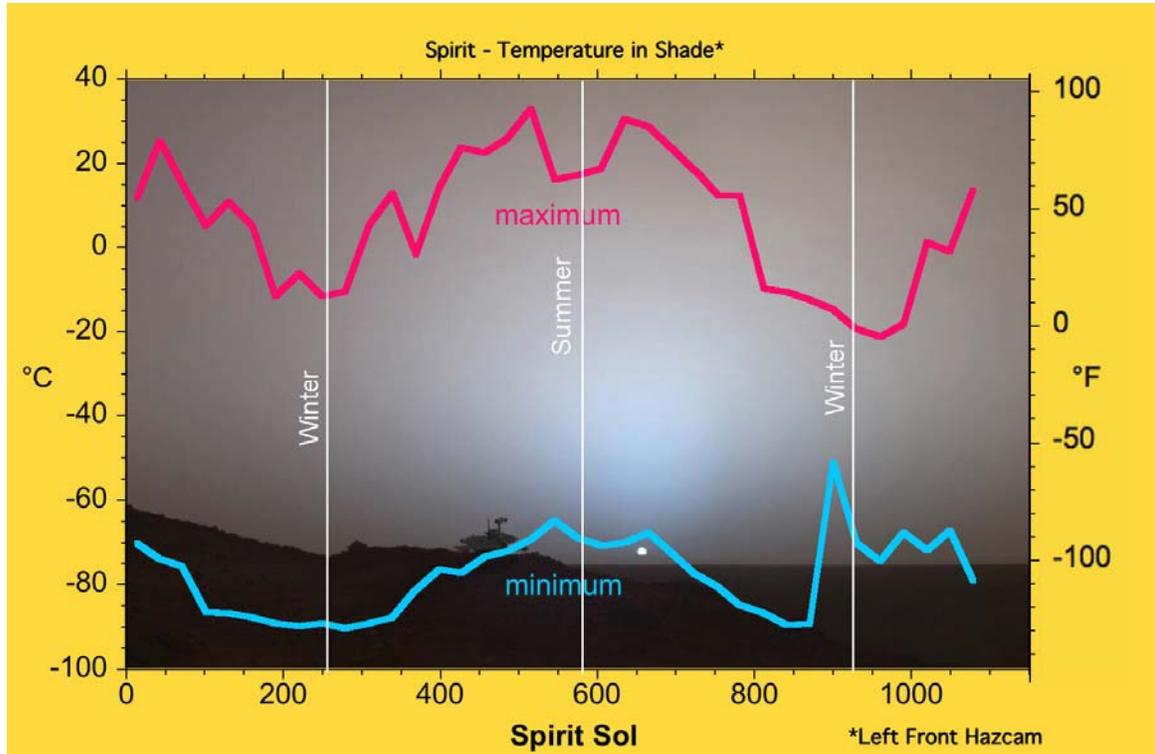
Answer:  $T = -58.4 + 31.4 \sin(2\pi(98-95.3)/1.0)$  so  **$T = -88.2 \text{ C}$** .

## Temperature Data for Viking 1 and 2

Sol	Viking 1		Viking 2	
	F	C	F	C
95.3	-81.92	-63.29	-87.39	-66.33
95.34	-63.01	-52.79	-67.83	-55.46
95.38	-45.63	-43.13	-60.56	-51.42
95.42	-31.56	-35.31	-61.56	-51.98
95.46	-22.6	-30.33	-52.62	-47.01
95.5	-17.9	-27.72	-47.72	-44.29
95.54	-12.37	-24.65	-31.45	-35.25
95.58	-15.46	-26.37	-34.04	-36.69
95.62	-16.14	-26.74	-28.34	-33.52
95.66	-19.27	-28.48	-29.22	-34.01
95.7	-29.11	-33.95	-37.57	-38.65
95.74	-56.31	-49.06	-43.24	-41.8
95.78	-80.33	-62.41	-57.87	-49.93
95.82	-83.92	-64.4	-73.03	-58.35
95.86	-89.85	-67.69	-82.61	-63.67
95.9	-98.44	-72.47	-88.46	-66.92
95.94	-103.18	-75.1	-95.12	-70.62
95.98	-105.07	-76.15	-100.57	-73.65
96.02	-109.86	-78.81	-105.97	-76.65
96.06	-115.15	-81.75	-110.11	-78.95
96.1	-120.88	-84.93	-112.02	-80.01
96.14	-124.75	-87.09	-110.94	-79.41
96.18	-123.93	-86.63	-120.3	-84.61
96.22	-116.3	-82.39	-105.12	-76.18
96.26	-102.76	-74.87	-96.68	-71.49
96.3	-84.96	-64.98	-79.55	-61.97
96.34	-65	-53.89	-69.56	-56.42
96.38	-46.88	-43.82	-59.64	-50.91
96.42	-35.06	-37.26	-53.39	-47.44
96.46	-29.8	-34.33	-53.48	-47.49
96.5	-20.46	-29.15	-40.58	-40.32
96.54	-14.22	-25.68	-32.67	-35.93
96.58	-14.52	-25.84	-25.91	-32.17
96.62	-16.1	-26.72	-25.24	-31.8



# The Annual Temperature of Mars



During its six years on the surface of Mars between 2004 and 2010, the Spirit Rover traveled 7.7 kilometers (4.8 miles) and took over 124,000 pictures of the martian surface. Its discovery of silica-rich soils at many locations helped scientists confirm that Mars had liquid water on its surface and there may have been conditions favorable for life to emerge. The Spirit Rover also made numerous temperature measurements during this period of time as shown in the graph above. The curve is not smooth because the local conditions near the rover changed frequently due to periods of dust storms and other weather conditions. The top curve is the highest daytime temperature recorded each day. The bottom curve is the coldest nighttime daily temperatures recorded. (See: <http://marsrover.nasa.gov/spotlight/20070612.html>)

**Problem 1** – About what were the highest and lowest temperatures recorded by Spirit?

**Problem 2** – What is the range of daytime temperatures at the Spirit location?

**Problem 3** – What is the range of the nighttime temperatures?

**Problem 4** – Between the winter and summer maximum temperatures, draw a straight line that begins at (250, -10 C) and ends at (600, +30 C). What is the average rate in degrees C per day, at which the temperature is increasing during this period of time?

**Problem 1** – About what were the highest and lowest temperatures recorded by Spirit?

**Answer:** The highest temperatures are shown on the top curve and reach a value of about **highest = +35 Celsius ( +95 Fahrenheit)** on martian day number 520 during the local martian summer at noon. The lowest temperatures are presented on the lower curve and the minimum occurred during the winter near martian day 275 at **lowest = -90 Celsius (-130 Fahrenheit)**.

**Problem 2** – What is the range of daytime temperatures at the Spirit location?

**Answer:** The minimum and maximum values for the top curve are **( -20 C, +35 C) or (-5 F, +95 F)**.

**Problem 3** – What is the range of the nighttime temperatures?

**Answer:** The range is found from the lower curve **(-90 C, -50 C) or (-130 F, -60 F)**.

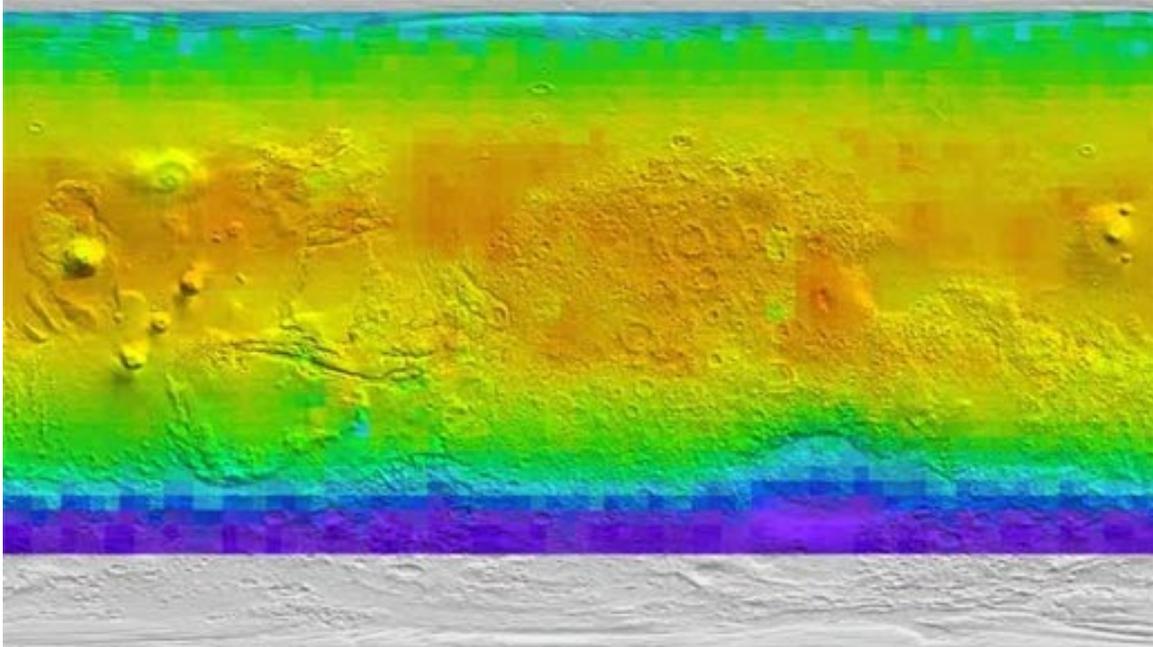
**Problem 4** – Between the winter and summer maximum temperatures, draw a straight line that begins at (250, -10 C) and ends at (600, +30 C). What is the average rate in degrees C per day, at which the temperature is increasing during this period of time?

**Answer:** The problem is asking for the slope of the line connecting the points (250,-10) and (600, +30). The slope is the change in the vertical units (C) divided by the change in the horizontal units (days), which can be written as

$$\text{Slope} = \frac{(30 - (-10))}{(600-250)}$$

or **+0.011 Celsius/day**.

At this location, the daytime temperature increases by about 0.1 Celsius each day between the middle of winter and the middle of summer.



The average surface temperature on Mars is  $-63^{\circ}\text{C}$  and can fall to as low as  $-140^{\circ}\text{C}$  and as high as  $+20^{\circ}\text{C}$ . In contrast, the coldest temperature ever recorded on Earth was  $-89.2^{\circ}\text{C}$  and the hottest was  $70.7^{\circ}\text{C}$ . The figure above shows the surface of Mars colored to show the highest (red) and lowest (blue) temperatures measured on a single day.

The temperature of a planet's surface depends on how far it is from the sun,  $R$ , and how much sunlight the planet reflects, which is given by a quantity called its albedo,  $A$ . These quantities can be connected together in a mathematical relationship as shown in the following equation:

$$T(R) = 1.1 \times 10^9 \left( \frac{(1-A)}{R^2} \right)^{\frac{1}{4}} \quad \text{Kelvin (K)}$$

where  $R$  is the distance from the sun to Mars in meters; and  $A$  is the albedo of Mars. The albedo of Mars is about  $A = 0.15$ , which means that it reflects back into space only 15% of the light it receives from the sun.

**Problem 1** – For the given values for the quantities, and for a Mars distance of  $R = 2.3 \times 10^{13}$  meters, what does the formula predict as the average temperature of Mars in Kelvin temperature units?

**Problem 2** – The distance to the sun from Mars varies from  $2.0 \times 10^{13}$  meters to  $2.5 \times 10^{13}$  meters and its albedo can change from 0.10 to as high as 0.40 during a dust storm. What is the minimum and maximum temperature you would predict using this model?

**Problem 1** – For the given values for the quantities, and for a Mars distance of  $R = 2.3 \times 10^{13}$  meters, what does the formula predict as the average temperature of Mars in Kelvin temperature units?

$$\text{Answer: } T(R) = 1.1 \times 10^9 \left( \frac{(1 - 0.15)}{(2.3 \times 10^{13})^2} \right)^{\frac{1}{4}}$$

So  $T = +220$  Kelvin.

**Problem 2** – The distance to the sun from Mars varies from  $2.0 \times 10^{13}$  meters to  $2.5 \times 10^{13}$  meters and its albedo can change from 0.10 to as high as 0.40 during a dust storm. What is the minimum and maximum temperature you would predict using this model?

Answer: The minimum temperature will occur when R is large and A is large, so for  $R = 2.5 \times 10^{13}$  meters and  $A = 0.40$ , we have **T = 193 Kelvin**. The maximum temperature will occur when R is small and A is small, so  $R = 2.0 \times 10^{13}$  meters and  $A = 0.1$  and so **T = +239 Kelvins**.

Note: Students can also convert the Kelvin units to Celsius using  $C = K - 273$  and to Fahrenheit using  $F = 9/5 K - 459.7$ . In Celsius units the temperature range is  $-80$  C to  $-34$  C. In Fahrenheit degrees the range is  $-112$  F to  $-29$  F.



Kepler's Third Law of Planetary Motion says that the square of the orbit period of a planet is proportional to the cube of the orbit radius, which we can write as:

$$R^3 = cT^2$$

The constant of proportionality,  $c$ , is related to the mass of the sun by the formula

$$c = \frac{G}{4\pi^2} M$$

What this also means is that, if we can measure the orbit period of a satellite of a planet and the radius of the satellite's orbit, we can determine the mass of the planet!

The exact formula relating  $T$ ,  $R$  and  $M$  is given by

$$M = \frac{5.88 \times 10^{11} R^3}{T^2}$$

where  $T$  is expressed in seconds,  $R$  is in meters and  $M$  is in kilograms.

**Note: In the following problems, give all answers to two significant figures.**

**Problem 1** – The martian moon Phobos, shown in the picture above, orbits Mars with a period  $T = 7$  hours 39 minutes, and  $R = 9,380$  km. What is the mass of Mars using Phobos?

**Problem 2** – Mars has a second satellite, Deimos, which orbits Mars with a period of  $T = 30$  hours 30 minutes, and  $R = 23,460$  kilometers. What is the mass of Mars using Deimos?

**Problem 3** – The NASA Mars Reconnaissance Orbiter spacecraft orbits Mars with a period of  $T = 112$  minutes, and has an orbit radius of  $R = 3679$  kilometers. What is the mass of Mars using this artificial 'moon'?

**Problem 4** – Suppose you discovered a captured asteroid orbiting Mars at a distance of 85,000 kilometers. What would you predict as the orbit period of the asteroid-moon in days? (Hint: Average the masses for Mars determined in Problems 1, 2 and 3).

**Problem 1** – The martian moon Phobos, shown in the picture above, orbits Mars with a period  $T = 7$  hours 39 minutes, and  $R = 9,380$  km. What is the mass of Mars using Phobos?

Answer: First convert the time and distances into seconds and meters to match the units used in the formula. 39 minutes  $\times 60$  sec/1 minute = 2340 seconds. 7 hours  $\times 60$  min/1 hr  $\times 60$  sec/1 min = 25,200 seconds, so  $T = 25200 + 2340 = 27,540$  seconds.  $R = 9380$  km  $\times 1000$  meters/1km = 9,380,000 meters. Then from the formula:

$$M = \frac{5.88 \times 10^{11} (9.38 \times 10^6)^3}{(2.754 \times 10^4)^2} = 6.4 \times 10^{23} \text{ kilograms}$$

**Problem 2** – Mars has a second satellite, Deimos, which orbits Mars with a period of  $T = 30$  hours 30 minutes, and  $R = 23,460$  kilometers. What is the mass of Mars using Deimos?

Answer:  $T = 1.098 \times 10^5$  seconds and  $R = 2.346 \times 10^7$  meters so

$$M = \frac{5.88 \times 10^{11} (2.346 \times 10^7)^3}{(1.098 \times 10^5)^2} = 6.3 \times 10^{23} \text{ kilograms}$$

**Problem 3** – The NASA Mars Reconnaissance Orbiter spacecraft orbits Mars with a period of  $T = 112$  minutes, and has an orbit radius of  $R = 3679$  kilometers. What is the mass of Mars using this artificial ‘moon’?

Answer:  $T = 6.72 \times 10^3$  seconds and  $R = 3.679 \times 10^6$  meters so

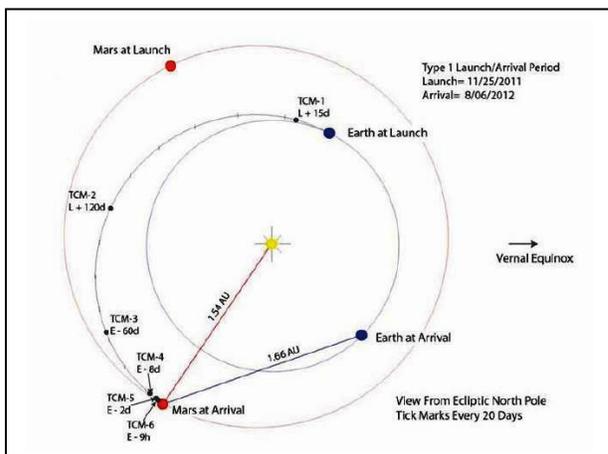
$$M = \frac{5.88 \times 10^{11} (3.679 \times 10^6)^3}{(6.72 \times 10^3)^2} = 6.5 \times 10^{23} \text{ kilograms}$$

**Problem 4** – Suppose you discovered a captured asteroid orbiting Mars at a distance of 85,000 kilometers. What would you predict as the orbit period of the asteroid-moon in days?

Answer: From the answers for the mass of Mars,  $M$ , take the average to obtain  $M = 6.4 \times 10^{23}$  kg, then solve the formula for  $T$  and use  $R = 8.5 \times 10^7$  meters to get

$$T^2 = \frac{5.88 \times 10^{11} R^3}{M} = \frac{5.88 \times 10^{11} (8.5 \times 10^7)^3}{(6.4 \times 10^{23})}$$

Then  $T^2 = 5.64 \times 10^{11}$  and then after taking the square-root you get  $T = 7.51 \times 10^5$  seconds. Since there are 86,400 seconds in 1 day, the orbit period will be **8.7 days**.



The Mars Science Laboratory was launched from Cape Canaveral on November 26, 2011 for a 251-day journey to Mars along an orbital path 567 million kilometers in length. The path is along a portion of an elliptical orbit called a Hohmann Transfer Orbit.

Hohmann Transfer Orbits require the least amount of fuel to transfer a spacecraft from Earth's orbit around the sun to the orbit of Mars.

A Hohmann Transfer Orbit to Mars has a perihelion distance of 1.0 Astronomical Units, and an aphelion distance equal to the distance of Mars from the sun at the time of interception, which for August 6, 2012 is 1.5 Astronomical Units. One Astronomical Units (1 AU) equals 149 million km. One focus of the transfer orbit is located on the sun, as are the elliptical orbits of all the other planets. The relationship between the aphelion and perihelion distances, A and P, and the semi-major axis, A, and eccentricity, e, of the corresponding ellipse is given by

$$P = a - c \quad A = a + c$$

where a is the semi-major axis and also the semi-minor axis is  $b = (a^2 - c^2)^{1/2}$

**Problem 1** – For the desired transfer orbit, what is the equation of the required elliptical orbit in standard form?

**Problem 2** – From the diagram above, where would Earth be in its orbit if the spacecraft could complete its original transfer orbit and attempt to return to Earth?

**Problem 3** - To the proper number of significant figures, what is the average speed of the Mars Science Laboratory spacecraft in its journey to Mars A) in kilometers/hr? B) miles/hour?

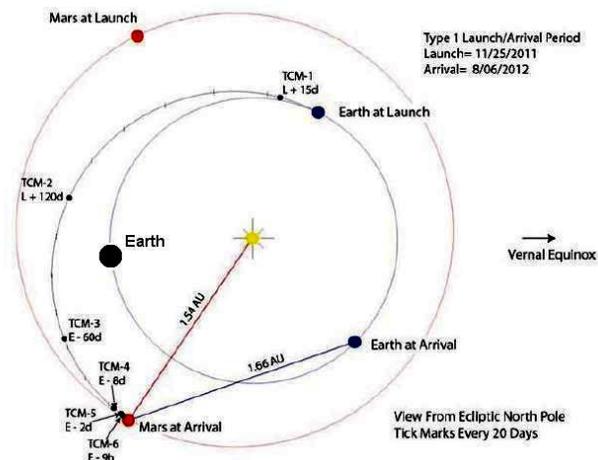
**Problem 1** – For the desired transfer orbit, what is the equation of the required elliptical orbit in standard form?

Answer: The major axis has a total length of  $1.0 + 1.0 + 0.5 = 2.5$  AU, so  $a = 2.5/2 = 1.25$  AU. Then from  $P = a - c$  and  $A = a + c$  we have  $1.0 = 1.25 - c$  and  $1.5 = 1.25 + c$  so that  $c = 0.25$  AU. Then  $b = (1.25^2 - 0.25^2)^{1/2} = 1.2$  AU. Then from the standard form for an ellipse we get:

$$1 = \frac{x^2}{(1.25)^2} + \frac{y^2}{(1.2)^2}$$

**Problem 2** – From the diagram above, where would Earth be in its orbit if the spacecraft could complete its original transfer orbit and attempt to return to Earth?

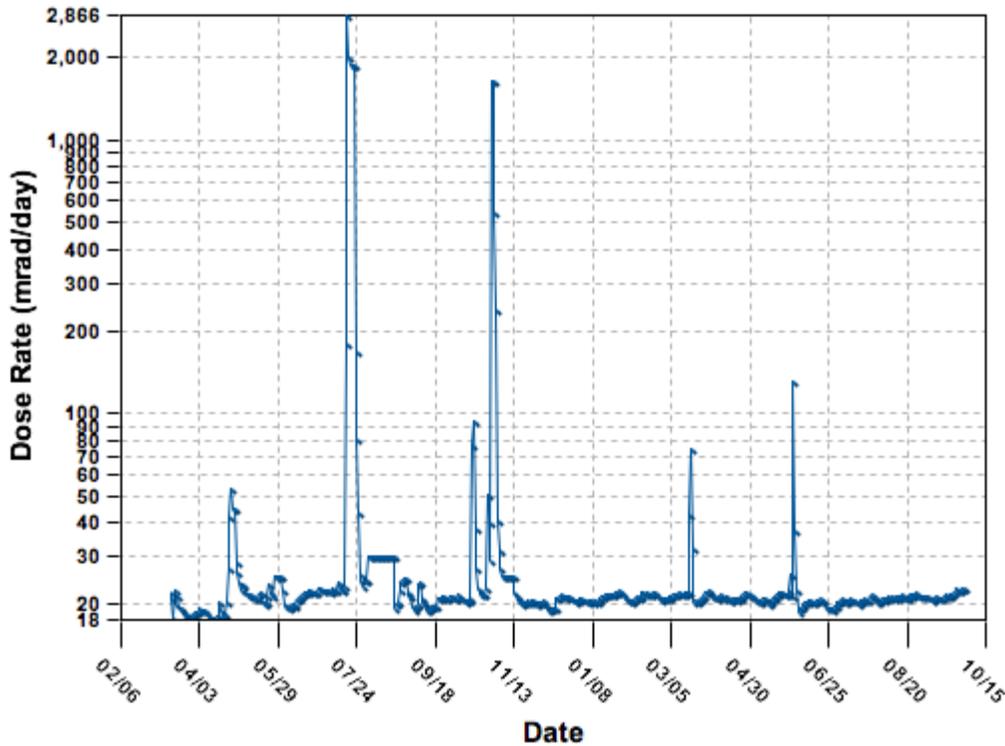
Answer: By symmetry, the dot on the orbit for Earth on August 6, 2012 would have to move an equal distance to its journey since November 25, 2011, which would place it near the indicated point in the diagram below:



**Problem 3** - To the proper number of significant figures, what is the average speed of the Mars Science Laboratory spacecraft in its journey to Mars A) in kilometers/hr? B) miles/hour?

Answer: The 567 million km journey will take 251 days, so the average speed will be  
 A) 567 million km/251 days = 2.26 million km/day or **94,200 km/hr** B) **58,400 miles/hr.** (since 1 km = 0.62 miles)

**MARIE Daily Average Dose Rates: 03/13/2002 - 09/30/2003**



The NASA, Mars Radiation Environment Experiment (MARIE) measured the daily radiation dosages from a satellite orbiting Mars between March 13, 2002 and September 30, 2003 as shown in the figure above. The dose rate is given in units of milliRads per day. (1 Rad = 2 Rems for cosmic radiation.) The six tall 'spikes' are Solar Proton Events (SPEs) which are related to solar flares, while the rest of the plotted data (the wiggly line!) is the dosage caused by galactic cosmic rays (GCRs).

1. By finding the approximate area under the plotted data, calculate the total radiation dosage in Rems for the GCRs during the observation period between 4/03/2002 and 8/20/2003.
2. Assuming that each SPE event lasted 3 days, and that its plotted profile is a simple rectangle, calculate the total radiation dosage in Rems for the SPEs during the observation period.
3. What would be the total radiation dosage for an unshielded astronaut orbiting Mars under these conditions?
4. Are SPEs more important than GCRs as a source of radiation? Explain why or why not in terms of estimation uncertainties that were used.

# Calculating Total Radiation Dosages at Mars

Teachers Note: Because students will be asked to determine the areas under a complicated curve using rectangles, please allow student answers to vary from the below estimates, by reasonable amounts! This may be a great time to emphasize that, sometimes, two scientists can get different answers to the same problem depending on how they do their calculation. Averaging together the student responses to each answer may be a good idea to improve accuracy!

1. By finding the approximate area under the plotted data, calculate the total radiation dosage in Rems for the GCRs during the observation period between 4/03/2002 and 8/20/2003.

From the graph, the average dosage rate is about 20 mRads/day. The time span is about  $365 + 4 \times 30 + 17 = 502$  days. The area of a rectangle with a height of 20 milliRads/day and a width of 502 days is  $(20 \text{ milliRads/day}) \times (502 \text{ days}) = 10040$  milliRads. This can be converted to Rems by multiplying by  $(1 \text{ Rad}/1000 \text{ milliRads})$  and by  $(2 \text{ Rem}/1 \text{ Rad})$  to get **20 Rems**.

2. Assuming that each SPE event lasted 3 days, and that its plotted profile is a simple rectangle, calculate the total radiation dosage in Rems for the SPEs during the observation period.

Peak 1 = 53 milliRads/day x 3 days = 159 millirads

Peak 2 = 2866 millirads/day x 3 days = 8598 milliRads

Peak 3 = 90 milliRads/day x 3 days = 270 milliRads

Peak 4 = 1700 milliRads/day x 3 days = 5100 milliRads

Peak 5 = 70 milliRads/day x 3 days = 210 milliRads

Peak 6 = 140 milliRads/day x 3 days = 420 milliRads

The total dosage is 14,757 milliRads.

Convert this to Rems by multiplying by  $(1 \text{ Rad}/1000 \text{ milliRads}) \times (2 \text{ Rem}/1 \text{ Rad})$   
To get **30 Rems after rounding**.

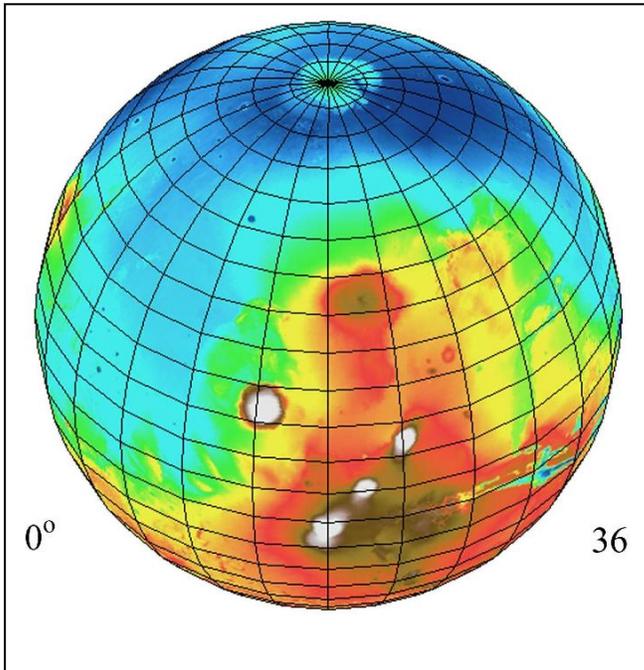
3. What would be the total radiation dosage for an unshielded astronaut orbiting Mars under these conditions?

Answer: 20 Rems + 30 Rems = **50 Rems** for a 502-day visit.

4. Are SPEs more important than GCRs as a source of radiation? Explain why or why not.

**Answer:** Solar Proton Events may be slightly more important than Galactic Cosmic Radiation for astronauts orbiting Mars.

The biggest uncertainty is in the SPE dose estimate. We had to approximate the duration of each SPE by a rectangular box with a duration of exactly three days, although the plot clearly showed that the durations varied from SPE to SPE. If the average dose rate for each SPE were used, rather than the peak, and a shorter duration of 1-day were also employed, the estimate for the SPE total dosage would be significantly lower, perhaps by as much as a factor of 5, from the above estimates, which would make the GCR contribution, by far, the largest.



Mars has virtually no atmosphere leaving its surface unprotected from solar and cosmic radiation.

This figure, created with the NASA, MARIE instrument on the Odyssey spacecraft orbiting Mars, shows the unshielded surface radiation dosages:

Color	Rem/yr
Brown	20
Orange	18
Yellow	16
Light blue	13
Dark blue	10

Astronauts landing on Mars will want to minimize their total radiation exposure during the 540 days they will stay on the surface. Assume that the Mars astronauts used improved post-Apollo spacesuit technology providing a shielding reduction of 1/8, and that the Mars Habitat provided a 1/20 radiation reduction.

**Problem 1** - The typical, unshielded radiation dose on the surface of Earth for cosmic rays is about 0.040 Rems/yr. By what factor is the unshielded, minimum radiation exposure for Mars astronauts in excess of the normal terrestrial rates?

**Problem 2** - The Mars explorers would like to spend 2 hours in spacesuits and the remaining 24-hours inside the Mars Habitat during each of the 540-days of exploration on Mars. What would be the approximate total dose for the astronauts in the 'dark blue' polar regions at the end of A) a single day? B) 1 Earth-year? C) the entire Mars visit?

**Problem 3** - The total background+lifestyle dose on Earth at ground-level is about 360 milliRem/yr. How many extra years of radiation exposure will an astronaut accumulate exploring the surface of Mars rather than 'staying home'?

**Problem 1** - The typical, unshielded radiation dose on the surface of Earth for cosmic rays is about 0.040 Rems/yr. By what factor is the unshielded, minimum radiation exposure for Mars astronauts in excess of the normal terrestrial rates?

Answer: The lowest mapped rates are about 10 Rem/yr, so this is about  $10 \text{ Rem}/0.040 \text{ Rem} = 250$ -times higher than terrestrial ground rates.

**Problem 2** - The Mars explorers would like to spend 2 hours in spacesuits and the remaining 24-hours inside the Mars Habitat during each of the 540-days of exploration on Mars. What would be the approximate total dose for the astronauts in the 'dark blue' polar regions at the end of A) a single day? B) 1 Earth-year? C) the entire Mars visit?

Answer: The dark blue region corresponds to 10 Rem/yr or 27 milliRem/day, assuming 1 Earth-year = 365 days. In terms of an hourly rate we have for 1 day = 24 hours that the dose rate is 1.1 milliRem/hr.

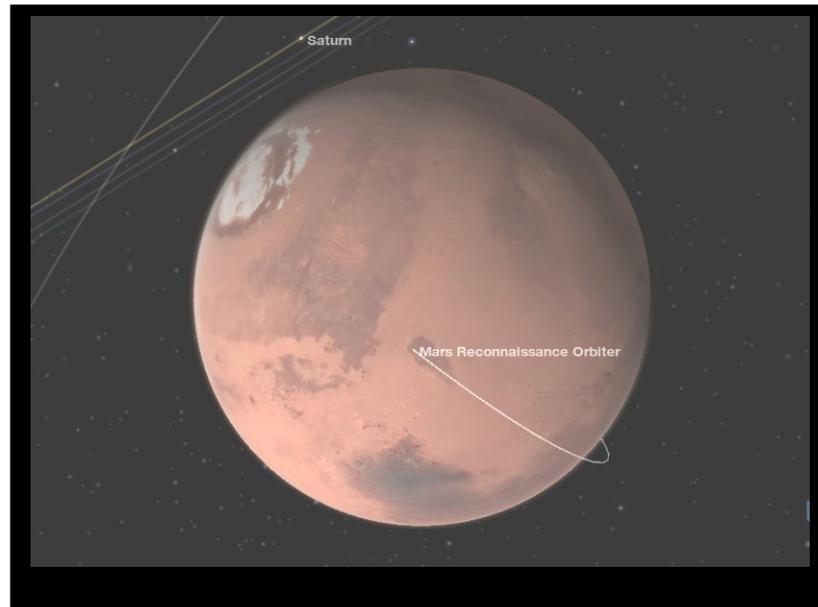
A) For the Mars Habitat, its shielding reduces the daily dose by 1/20 so for 22 hours the dose will be  $1.1 \text{ milliRem/hr} \times (1/20) \times 22 \text{ hrs} = 1.21 \text{ milliRem}$ . The spacesuit dose would be  $1.1 \text{ milliRem/hr} \times (1/8) \times 2 \text{ hrs} = 0.28 \text{ milliRem}$ , so the total daily accumulated dose would be  $1.21 + 0.28 = \mathbf{1.49 \text{ milliRem/day}}$ .

B) For one Earth-year the accumulated dose would be  $1.49 \text{ milliRem/day} \times 365 \text{ days} = \mathbf{543 \text{ milliRem/yr or } 0.54 \text{ Rem/yr}}$ .

C) For 540-days, the total dose would be  $1.49 \text{ milliRem/day} \times 540 \text{ days} = \mathbf{805 \text{ milliRem or } 0.80 \text{ Rem}}$ .

**Problem 3** - The total background+lifestyle dose on Earth at ground-level is about 360 milliRem/yr. How many extra years of radiation exposure will an astronaut accumulate exploring the surface of Mars rather than 'staying home'?

Answer: If he spent 540-days on Earth, he would have accumulated  $360 \text{ milliRem/yr} \times 540 \text{ days}/365 \text{ days} = 532 \text{ milliRem}$ . On Mars the total accumulation would have been 805 milliRem for 540 days. The excess accumulation is just  $805 \text{ milliRem} - 532 \text{ milliRem} = 273 \text{ milliRem}$ . This equals  $273/360 = \mathbf{0.76 \text{ additional years}}$ .



## Get the Data

Visit EOSS <http://1.usa.gov/GCEcT7> to recreate this exact scene. Recommended operating system: MS Windows Vista or later; Browser: Internet Explorer 8 or later.

The Mars Science Laboratory Rover called Curiosity landed in Gale Crater located 22 degrees from the North Pole of Mars. It will relay its data to the Mars Reconnaissance Orbiter (MRO), which will then transmit the data to Earth.

**Step 1** - Pick a convenient spot on the surface of Mars near the North Pole, and advance the date and time until the MRO passes over this spot.

**Step 2** - Advance the date and time until MRO once again passes over (or close by) the spot you selected.

## Answering Questions

**Problem 1** – How many hours will elapse between the times when MRO is overhead of the Curiosity Rover in Gale Crater?

**Problem 2** - During a full Earth year, how many times will MRO fly over the Curiosity Rover area?

## Math Challenge

The Mars Curiosity Rover will transmit up to 4,000 bytes of data every second. When the MRO flies overhead, Curiosity will have 20 minutes to transmit data to the MRO. How much data, in gigabytes, will Curiosity be able to transmit every Earth Year?

**Answer Key**

**Problem 1** – How many hours will elapse between the times when MRO is overhead of the Curiosity Rover in Gale Crater?

Answer: **Through trial-and-error, students should be able to determine that MRO flies over the same spot on Mars about once every 12 hours.**

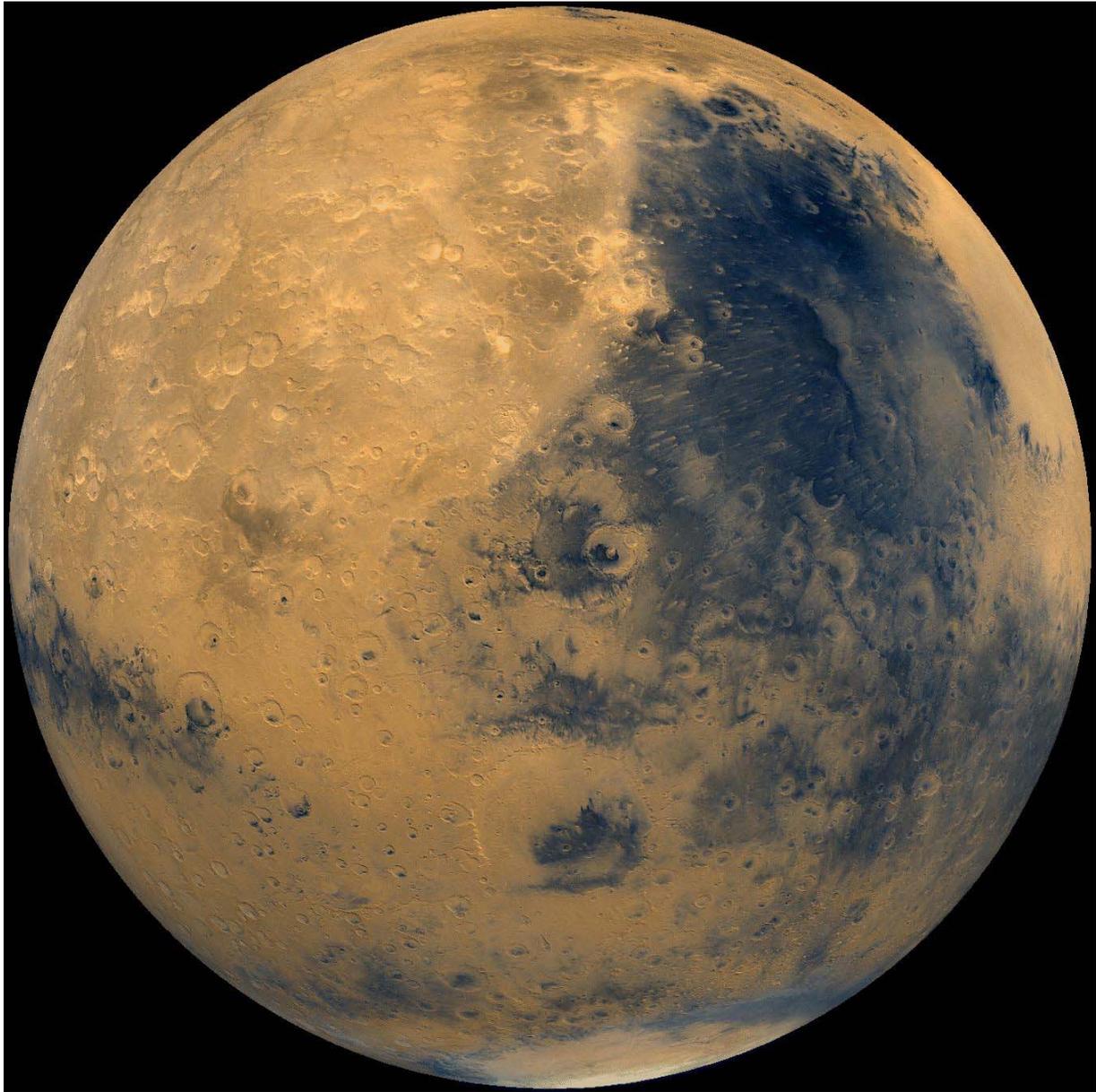
**Problem 2** - During a full Earth year, how many times will MRO fly over the Curiosity Rover area?

Answer: **At a rate of about twice a day, it will 'transit' the Gale Crater about  $2 \times 365 = 730$  times every Earth Year.**

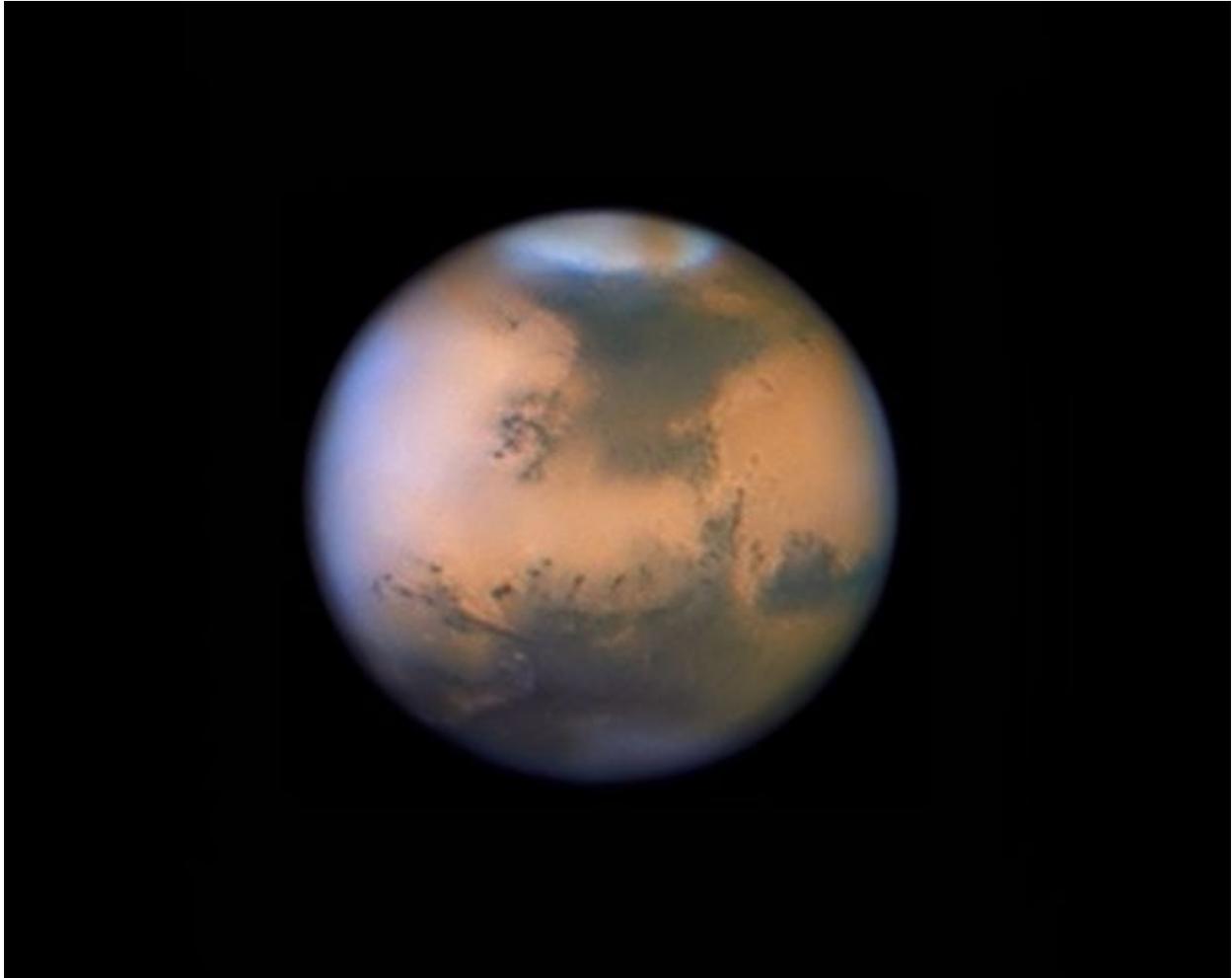
**The Mars Curiosity Rover will transmit up to 4,000 bytes of data every second. When the MRO flies overhead, Curiosity will have 20 minutes to transmit data to the MRO. How much data, in gigabytes, will Curiosity be able to transmit every Earth Year?**

Answer:  **$730 \text{ times every year} \times 20 \text{ minutes} \times (60 \text{ seconds}/1 \text{ minute}) \times (4,000 \text{ bytes}/1 \text{ minute}) = 3,504,000,000 \text{ bytes}$  or 3.504 gigabytes. Students may compare this to an equivalent number of songs on their portable player ( 1 song = 40 megabytes, so this is about 1000 songs!)**





Composite image of Mars based on individual images taken by NASA spacecraft.



This photograph of Mars was taken from Earth in 2010 using the 1-meter telescope at the Pic du Midi Observatory. For over 100 years, photos such as these were the best that astronomers could get in order to see details on the surface of Mars. Some astronomers during the 1800s even claimed to have seen 'canals', which caused much excitement over the possibilities for lakes, rivers and life existing on this planet.



Mars moon Deimos is only 15 kilometers across but its surface is filled with craters and other interesting features. This color-enhanced image of Deimos was taken by the HiRISE instrument on the Mars Reconnaissance Orbiter on February 21, 2009. Deimos orbits Mars every 30 hours at a distance of 20,000 km from the surface of Mars.



This image of Phobos was taken by the European Mars Express spacecraft. Phobos orbits Mars in just over 7 hours at a distance of 6,000 km from the surface of Mars. Because of its closeness to Mars, the orbit of Phobos is unstable. Within the next 50 million years it will crash onto the martian surface, or may even be torn apart to form a thin ring around the planet.



This photo, taken by NASA's Viking Orbiter in 1976 shows an area of Mars where ancient rivers once flowed many billions of years ago. The area on Mars is located near 20°N latitude and ~55°W longitude, at the northern edge of Chryse Planitia. The image shows impact craters and river channels. The image is about 200 kilometers across



A solid sheet of water ice lies at the bottom of a crater near the North Pole of Mars. The robotic Mars Express spacecraft took the above image in early February. The ice pocket was found in a 35-kilometer wide crater that resides 70 degrees north of the Martian equator. There, sunlight is blocked by the 300-meter tall crater wall from vaporizing the water-ice on the crater floor into the thin Martian atmosphere. The ice pocket may be as deep as 200 meters thick. Frost can be seen around the inner edge on the upper right part of the crater, while part of the lower left crater wall is bathed in sunlight.



An avalanche on Mars caught by the Mars Reconnaissance Orbiter in 2010. The High Resolution Imaging Science Experiment (HiRISE) camera is one of the instruments on the Mars Reconnaissance Orbiter. Recently this camera captured an avalanche on Mars. The white portion of the image is carbon dioxide (dry ice). The cliff there is 2,000 ft high. The plume of dust itself is probably 150 feet high.



The 800-meter-tall plume of a dust devil casts a long shadow on the surface of Mars in this image from the HiRISE camera aboard NASA's Mars Reconnaissance Orbiter. Caused by warm air near the ground rapidly rising in spinning columns, dust devils are a common sight on Mars during the northern spring season.



This image of the Victoria Crater in the Meridiani Planum region of Mars was taken by the High Resolution Imaging Science Experiment (HiRISE) camera on NASA's Mars Reconnaissance Orbiter. Victoria Crater is approximately 800 meters (about half a mile) wide. The photographer was the Mars Reconnaissance Orbiter at an altitude of 268.6 km (167.9 miles) above the surface.



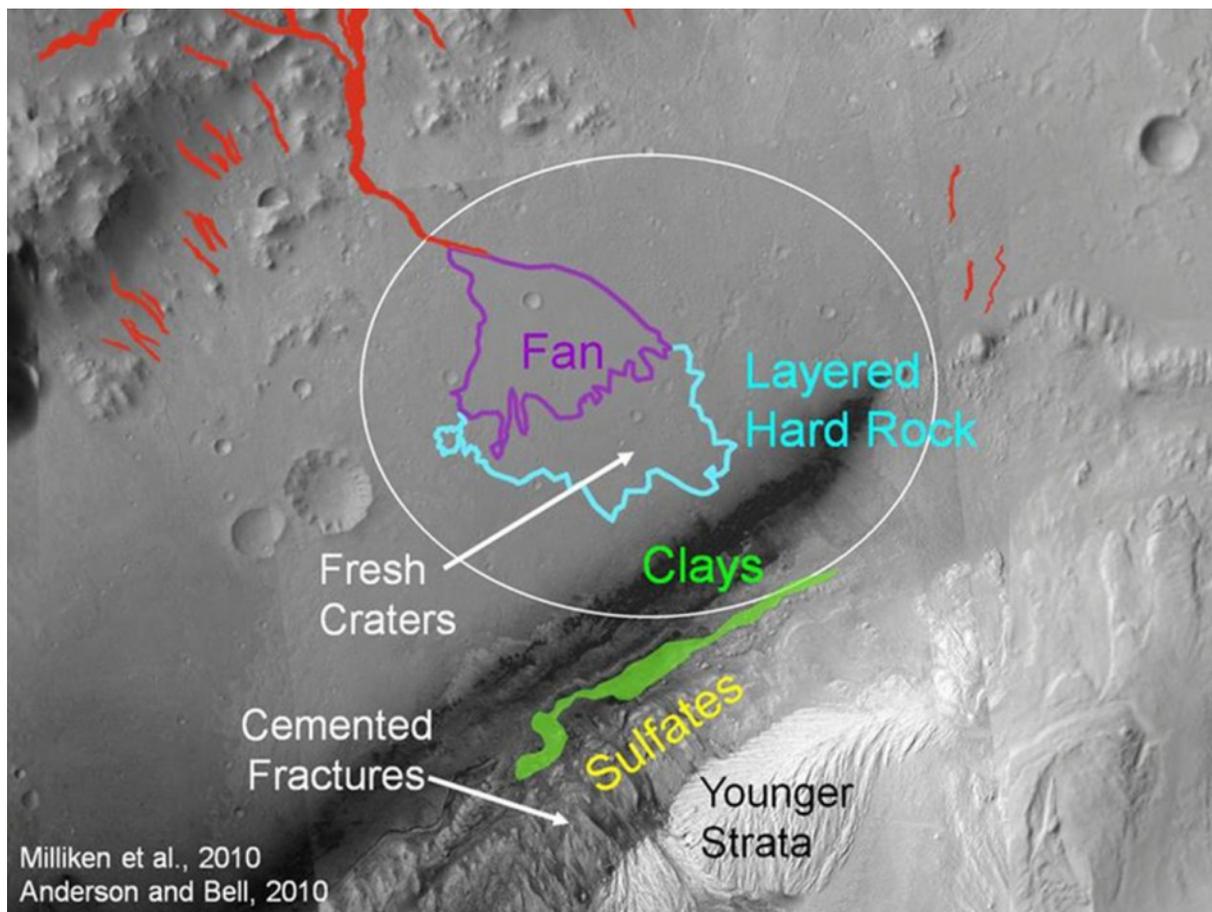
Gully channels in a crater in the southern highlands of Mars, taken by the High Resolution Imaging Science Experiment (HiRISE) camera on the Mars Reconnaissance Orbiter, are shown in this image released by NASA on Sept. 20, 2007. Once thought to have formed by liquid water, the gullies' origins have been reassessed by researchers who say these gullies are formed by wind-blown sand and other dry debris. Similar non-water features can be found on Earth.



One of the first view of the martian landscape near the Viking 2 lander in August 1976 (foot pad in lower right). Also shown are a series of trenches dug by Viking to investigate the surface and sub-surface composition. Samples from these trenches were also used to search for traces of chemical compounds needed for life.



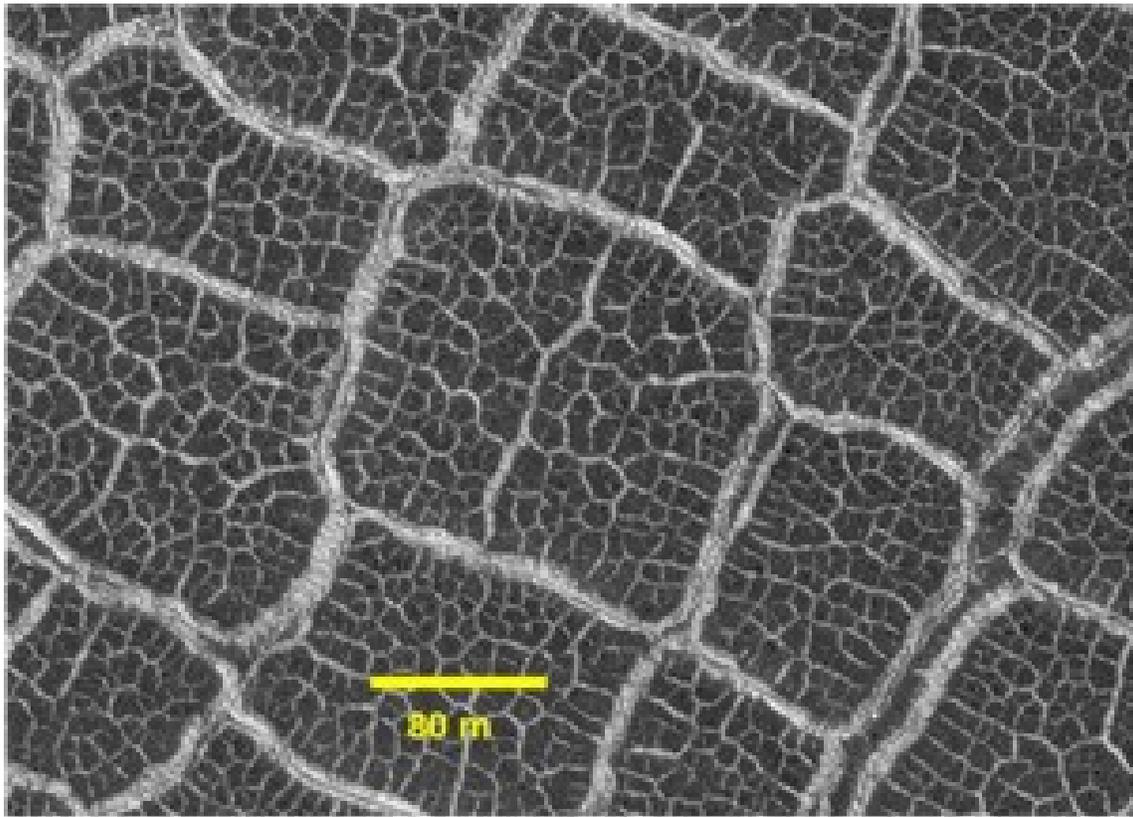
This sharp view from the Thermal Emission Imaging System camera on NASA's Mars Odyssey orbiter is centered on 154 kilometer (96 mile) wide Gale crater, near the martian equator. Within Gale, an impressive layered mountain rises about 5 kilometers (3 miles) above the crater floor. Layers and structures near its base are thought to have been formed in ancient times by water-carried sediments.



The Gale crater has low elevation and water runs downhill, so if running water did exist on Mars, a crater would be a good place to look. The Gale crater, in particular, has an alluvial fan, which is likely formed by water-carried sediments. (Alluvial fans on earth are formed that way.)

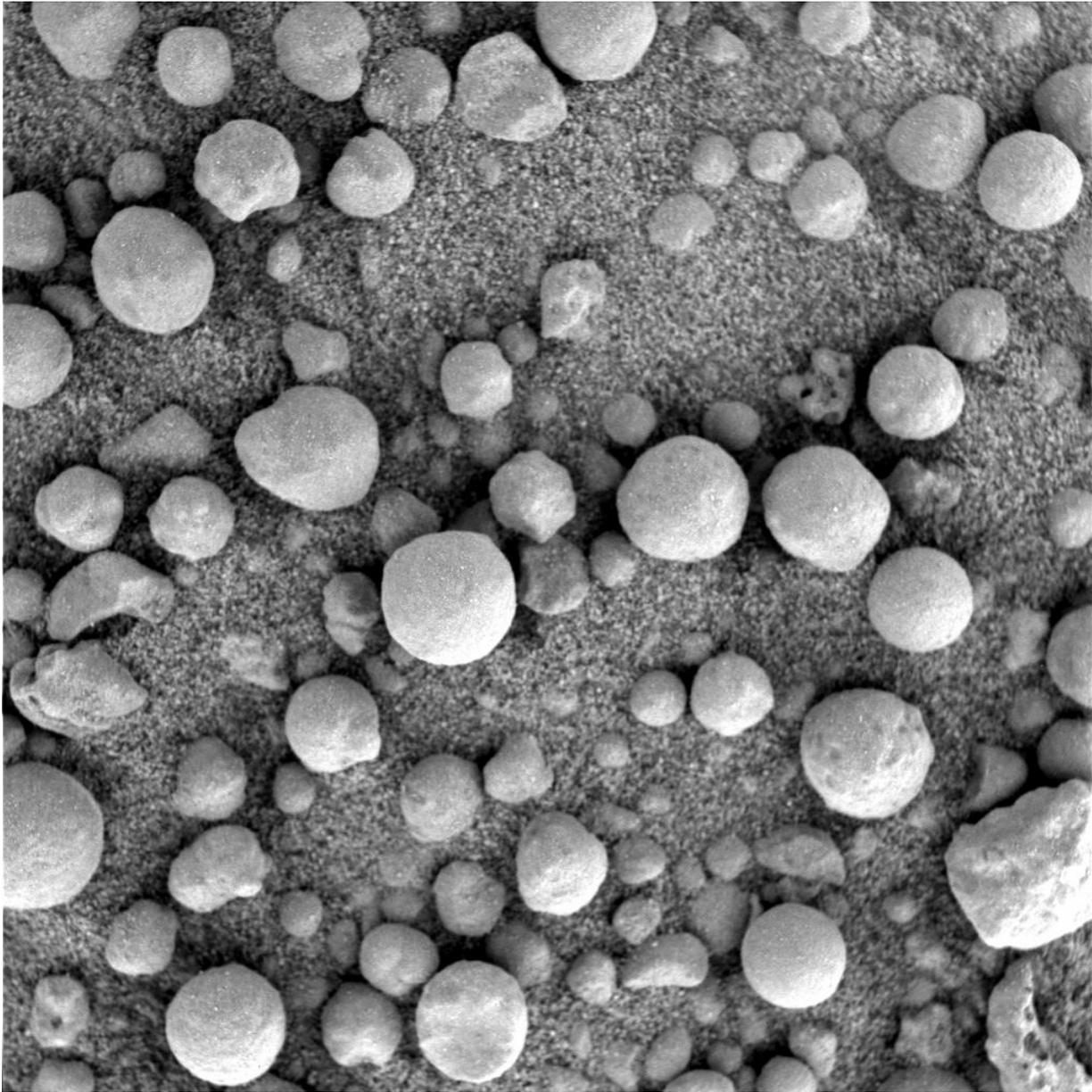
Aside from liquid water, the Curiosity Rover will look for carbon-based organic compounds, another evidence of life. Even if scientists don't discover evidence of running water and life on Mars, a crater is still an attractive location because its depth offers a diversity of features and layers for investigating changing environmental conditions, some of which could inform a broader understanding of habitability on ancient Mars.

The Gale crater is about the size of Connecticut and Rhode Island combined. Within this large area, Curiosity will specifically land near a mountain in the middle that reaches as high as 5.5 km. The layering in this mound suggests it is the surviving remnant of an extensive sequence of deposits, which makes it an attractive object to study.



This is a detailed image of large-scale crater floor polygons, caused by water evaporation, with smaller polygons inside caused by thermal contraction. The central polygon is 160 meters in diameter, smaller ones range 10 to 15 meters in width and the cracks are 5-10 meters across. Networks of cracks have been found inside 266 impact basins across the surface of Mars with polygons reaching up to 250 meters in diameter

Networks of giant polygonal troughs found in crater basins on Mars are cracks caused by evaporating lakes. These polygon-shaped cracks are too large to be caused by thermal contractions and provide further evidence of a warmer, wetter Martian past.



The mineral hematite, on Earth, requires abundant amounts of liquid water to form into round 'spherules' that are often blue in color, hence called hematite blueberries. The image above, taken by instruments on the Opportunity rover show hematite spherules on Mars. These martian 'blueberries' dot the Martian surface north of Victoria Crater. The Mars rover Opportunity found the spherules here and at its Eagle Crater landing site. This image is about 2.5 centimeters (1-inch) across.



Meteorites can be found on the surface of Mars just as they are found on Earth, but even though the Opportunity Rover covered a minute fraction of the Mars surface, it actually found a meteorite sitting on the surface! The meteorite, dubbed "Heat Shield Rock," sits near debris of Opportunity's heat shield on the surface of Meridiani Planum, which is a cratered flatland that has been Opportunity's home since the robot landed on Mars. The meteorite is about the size of a basketball, and a study by Opportunity's instruments confirmed that it is made of iron and nickel. Opportunity would discover five more meteorites in this region of Mars during its amazing 'non-stop' mission that began in 2005.

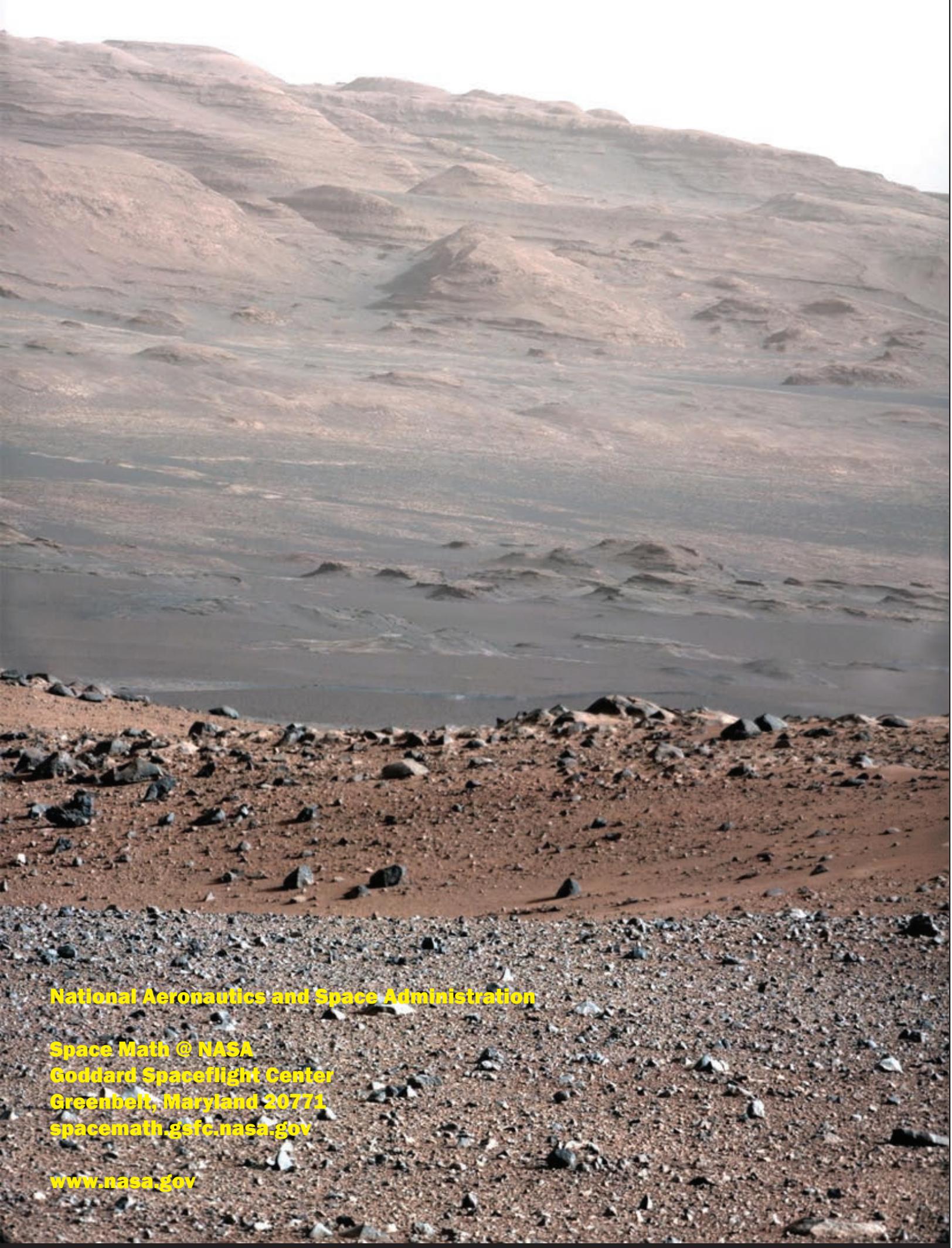


This image is from a series of test images to calibrate the 34-millimeter Mast Camera on NASA's Curiosity rover. It was taken on Aug. 23, 2012 and looks south-southwest from the rover's landing site.

The gravelly area around Curiosity's landing site is visible in the foreground. Farther away, about a third of the way up from the bottom of the image, the terrain falls off into a depression (a swale). Beyond the swale, in the middle of the image, is the boulder-strewn, red-brown rim of a moderately-sized impact crater. Farther off in the distance, there are dark dunes and then the layered rock at the base of Mount Sharp. Some haze obscures the view, but the top ridge, depicted in this image, is 10 miles (16.2 kilometers) away.



This image shows a close-up of track marks from the first test drive of NASA's Curiosity rover. The gravelly surface of this area in Gale Crater is the result of many complex erosive agencies including contemporary sand and dust storms, and ancient water flows and sedimentation. The rockets used in the landing operation also blew-away dust on the surface to reveal the more solid surface below. The pebbles in this scene range in size from a few millimeters to several centimeters across.



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