Exploring Space Through MATH
Applications in Geometry
EDUCATOR EDITION

Oh, Chute!

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will
- use scale factor to complete a table;
- use the Pythagorean Theorem;
- calculate the volume of a cylinder;
- calculate the area of a circle;
- investigate how changes in scale factor affect volume; and
- investigate how changes in scale factor affect area.

Prerequisites

Students should have prior knowledge of scale factor, the Pythagorean Theorem, volume of 3D objects, and area of circles.

Background

A new Capsule Parachute Assembly System (CPAS), currently being designed and tested by engineers at NASA Johnson Space Center in Houston, Texas, is destined to become part of the new space vehicle, which will transport humans beyond Earth's orbit to places like the Moon, Mars, or even nearby asteroids. This parachute system will include a new crew return module with a capsule shape, much like the capsule used during the Apollo program (Figure 1). The CPAS will help the capsule decelerate (or reduce its speed) once it re-enters Earth's atmosphere, enabling it to land safely.

Figure 1: Shape comparison of a new crew module (left) and the previous Apollo capsule from the 1960's (right); not to scale.

Key Concepts
Measurement, right triangles

Problem Duration
65 minutes

Technology
Projection technology

Materials
- *Oh, Chute!* Student Edition
- Cylindrical object
- Ruler

Skills
Scale factor, Pythagorean Theorem, volume, area

NCTM Standards
- Numbers and Operations
- Algebra
- Geometry
- Measurement

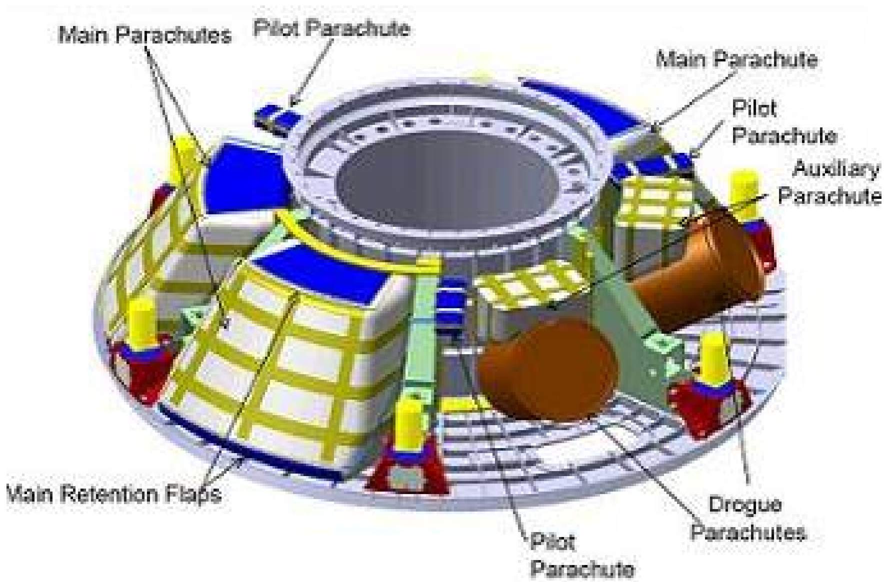

Figure 2: Artist's concept of the Capsule Parachute Assembly System (CPAS)

Figure 2 shows the configuration of CPAS as it is installed on the parachute compartment. The parachutes are stored around the transfer tunnel and it has a total of six parachute bays. Three of the bays are for the main parachutes and the other bays contain a combination of drogue, pilot, and auxiliary parachutes. Drogue parachutes are designed to slow the vehicle. Pilot parachutes help the vehicle to glide to a safe landing. The auxiliary parachutes are reserved in case other parachutes fail to deploy. A similar parachute system was used during the Apollo program in the 1960's and early 1970's to provide a safe water landing.

Figure 3: An Apollo Command Module nears splashdown in the South Pacific Ocean

As with any engineering design, the proposed CPAS will require adjustments to the design based on the results of thorough testing. Since testing the actual capsule could be very costly, engineers plan to instead attach the parachute system to a test structure. NASA has already created a test vehicle to replicate the weight, aerodynamics, and center of mass of the new capsule; however, it will be used only to test the parachute system when released from an aircraft (approximately 25,000 feet above the Earth's surface).

NCTM Principles and Standards

Numbers and Operations

- Judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities.
- Judge the reasonableness of numerical computations and their results.

Algebra

- Use symbolic algebra to represent and explain mathematical relationships.
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

Geometry

- Analyze properties and determine attributes of two-and three-dimensional objects.
- Explore relationships (including congruence and similarity) among classes of two-and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Measurement

- Make decisions about units and scales that are appropriate for problem situations involving measurement.
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (20 minutes)

- With students in small groups of three to four, place a cylindrical object on the table and ask them to 1) identify the solid and 2) calculate the volume using their rulers.
- Circulate to facilitate discussion and provide clarification in small groups.
- Choose one group to explain how they found the volume of the cylinder.
- Distribute the worksheet, *Oh, Chute*!
- Have students read the Background section in the Student Edition.
- Encourage student discussion of the Background section and ask if there are any questions.

2 – Explore (15 minutes)

- Have students remain in their small groups to complete question 1.
- Use projection technology to show Figures 4 and Figure 5 to help student gain perspective on the difference in scale. (optional)

3 – Explain (15 minutes)

- Have students remain in their small group to complete questions 2-3.
- Call on students to give their answers and discuss.

4 – Extend (5 minutes)

- Have students remain in their small group to work as a team on question 4.
- Encourage student discussion and ask if there are any questions.

5 – Evaluate (10 minutes)

- Have students work independently to complete questions 5-8. *This may be done in class or assigned as homework.*

Oh, Chute!

Solution Key

Directions: Show all work and justify your answers to questions 1-8. Discuss answers to be sure there is understanding and agreement on the solutions. Round all answers to the nearest thousandth and label them with the appropriate units.

Problem

The Systems Architecture and Integration Office at NASA Johnson Space Center is designing and testing the Capsule Parachute Assembly System (CPAS) using a test vehicle as the capsule. Figure 4 shows the parachute compartment (light-colored segment) sitting atop the test vehicle structure (dark-colored segment). Figure 5 is a picture of a small 3D model of the parachute compartment that has been scaled down to one-twelfth of the original size.

Figure 4: Parachute compartment atop the test vehicle structure

Figure 5: Parachute compartment scaled model

1. An illustration of some of the key structural components on the parachute compartment is provided in Figure 6. Table 1 contains the dimensions for some components on the actual parachute compartment and the model.

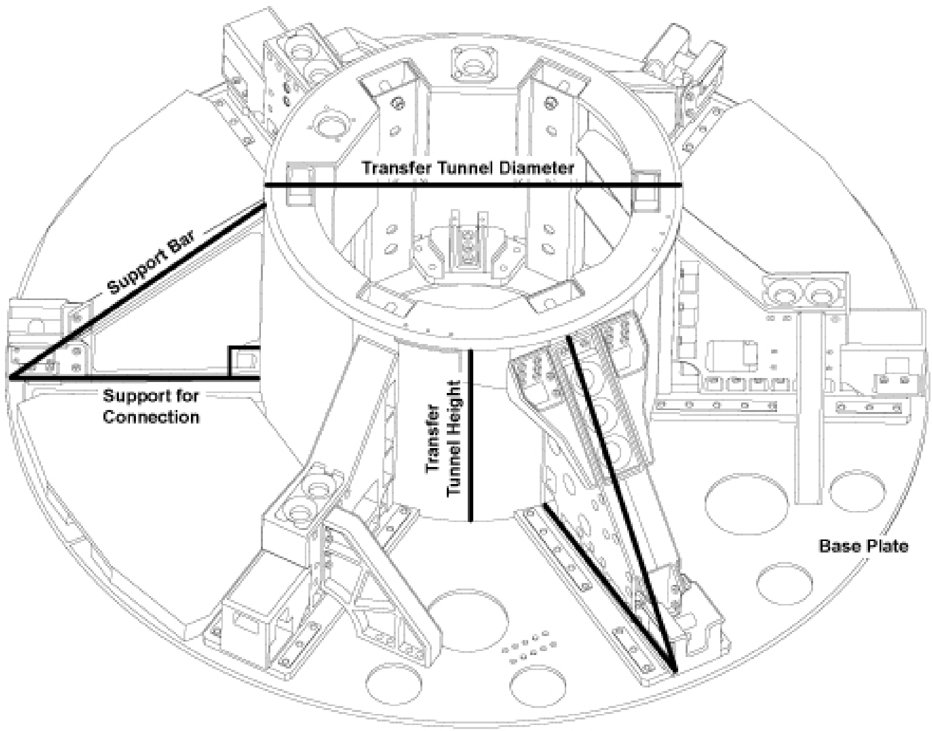

Figure 6: Engineer drawing of the parachute compartment

Table 1: Parachute compartment dimensions

Part name	**Actual (m)**	**3D model (cm)**
Base plate diameter	1.820	15.167
Transfer tunnel height	**1.493**	**12.439**
Transfer tunnel diameter	1.000	**8.333**
Support bar length	**1.548**	12.900
Support for connection length	0.410	**3.417**

a. The scale factor of the 3D model to the actual parachute compartment is $\frac{1}{12}$. Use this scale factor to complete Table 1.

Multiply actual dimensions by the scale factor to get 3D model dimensions in meters. To convert meters to centimeters, students must multiply by 100.

$$(1.82\ m)\left(\frac{1}{12}\right) = 0.151667\ \text{meters}$$

$$(0.151667\ m)100 = 15.167\ \text{centimeters}$$

Divide 3D model dimensions by 100 and multiply by the scale factor to get the actual dimensions.

$$\frac{12.900\ cm}{100} = 0.129\ m$$

$$(0.129\ m)12 = 1.548\ m$$

b. How can you use Figure 6 to find the actual height of the transfer tunnel?

Use the Pythagorean Theorem.

c. Calculate the actual height, h_0 of the transfer tunnel.

$$h_0 = \sqrt{1.548^2 - 0.41^2} = 1.493\ meters$$

2. Astronauts use the transfer tunnel on the parachute compartment to move from one module to another. Calculate the actual volume, V_0 of the transfer tunnel.

$$V_0 = \pi r_0^{\,2} h_0$$
$$V_0 = \pi (0.500\ m)^2 \cdot (1.493\ m)$$
$$V_0 = 1.173\ m^3$$

3. Engineers have determined that the volume of the tunnel needs to increase for astronauts to move through it with ease. Use a scale factor of 1.500 to increase the volume of the tunnel.

a. Calculate the new radius, r_1 of the transfer tunnel.

$$r_1 = r_0 (1.5)$$
$$r_1 = (0.500\ m)(1.5)$$
$$r_1 = 0.750\ m$$

b. Calculate the new height, h_1 of the transfer tunnel.

$$h_1 = h_0 (1.5)$$
$$h_1 = (1.493\ m)(1.5)$$
$$h_1 = 2.240\ m$$

c. Calculate the new volume, V_1 of the transfer tunnel using h_1 and r_1 from the previous questions.

$$V_1 = \pi r_1^2 h_1$$
$$V_1 = \pi (0.750\ m)^2 \cdot (2.240\ m)$$
$$V_1 = 3.958\ m^3$$

4. The parachute compartment must fit properly on the test vehicle. The recommended area of the base plate must be less than 2.7 m^2. Use the dimensions in Table 1 to determine if the area of the base plate, A_0 meets the criteria.

$$A_0 = \pi r^2$$
$$A_0 = \pi (0.91\ m)^2$$
$$A_0 = 2.602\ m^2$$

Yes, it meets the criteria.

Directions: Complete questions 5-8 independently. Round all answers to the nearest thousandth and label them using the appropriate units.

5. How can scale factor alone be used to find a change in volume? Explain.

   The change in volume can be found by cubing the scale factor, because volume is cubic.

6. How can scale factor alone be used to find a change in area? Explain.

   The change in area can be found by squaring the scale factor, because area is squared.

7. Calculate the volume of the transfer tunnel, V_2 using a scale factor of 1.25.

$$V_2 = V_0 (1.25)^3$$
$$V_2 = (1.173\ m^3)(1.25)^3$$
$$V_2 = 2.291\ m^3$$

8. Calculate the new area of the base plate using a scale factor of 1.25.

$$A_1 = A_0 (1.25)^2$$
$$A_1 = (2.602\ m^2)(1.25)^2$$
$$A_1 = 4.066\ m^2$$

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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