



Exploring Space Through MATH

Applications in Algebra 1

**STUDENT
EDITION**

Space Shuttle Ascent: Altitude vs. Time

Background

This problem is part of a series that applies algebraic principles in NASA's human spaceflight.

The Space Shuttle Mission Control Center (MCC) and the International Space Station (ISS) Control Center use some of the most sophisticated technology and communication equipment in the world. Teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle and the ISS. They work together as a powerful team, spending many hours performing critical simulations as they prepare to support each mission and crew during normal operations and any unexpected events.

Since its first flight in 1981, the space shuttle has been used to extend research, repair satellites, and help with building the ISS. NASA plans to retire the space shuttle, but until then space exploration depends on the continued success of space shuttle missions. Critical to any space shuttle mission is the ascent into space.

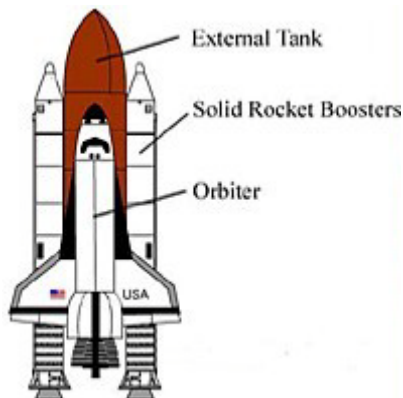


Figure 1: Main components of the space shuttle



Figure 2: Space Shuttle Discovery at liftoff

The ascent phase begins at liftoff and ends when the space shuttle reaches Earth's orbit. The space shuttle must accelerate from zero to approximately 7,850 meters per second (which is approximately 17,500 miles per hour) in eight and a half minutes to reach the minimum altitude required to orbit Earth. It takes a very unique vehicle to accomplish this task.



There are three components of the space shuttle that enable the launch into orbit (Figure 1). The main component is the orbiter. It not only serves as the crew's home in space and is equipped to dock with the ISS, but it also contains maneuvering engines for finalizing the orbital trajectory, or flight path. The External Tank (ET), the largest component of the space shuttle, supplies the propellant (liquid oxygen and liquid hydrogen) to the Space Shuttle Main Engines (SSMEs) which are liquid propellant rocket engines. The third component is a pair of Solid Rocket Boosters (SRBs) that are reusable. They are attached to the sides of the ET and provide the main thrust at launch (Figure 2).

The Flight Dynamics Officer (FDO) is the flight controller in the Space Shuttle MCC that is responsible for the planning and execution of the trajectory of the space shuttle during ascent, orbit, rendezvous, and re-entry. This requires intensive computational processing to be performed by FDO to ensure that the mission objectives are safely met. The flight controllers in the MCC monitor systems and functions of the space shuttle, and FDO manages where the space shuttle is and where it is going. Providing technical support for FDO with these duties are the engineers in the Multi-Purpose Support Room (MPSR), pronounced "Mipser" and sometimes referred to as the FDO Backroom.

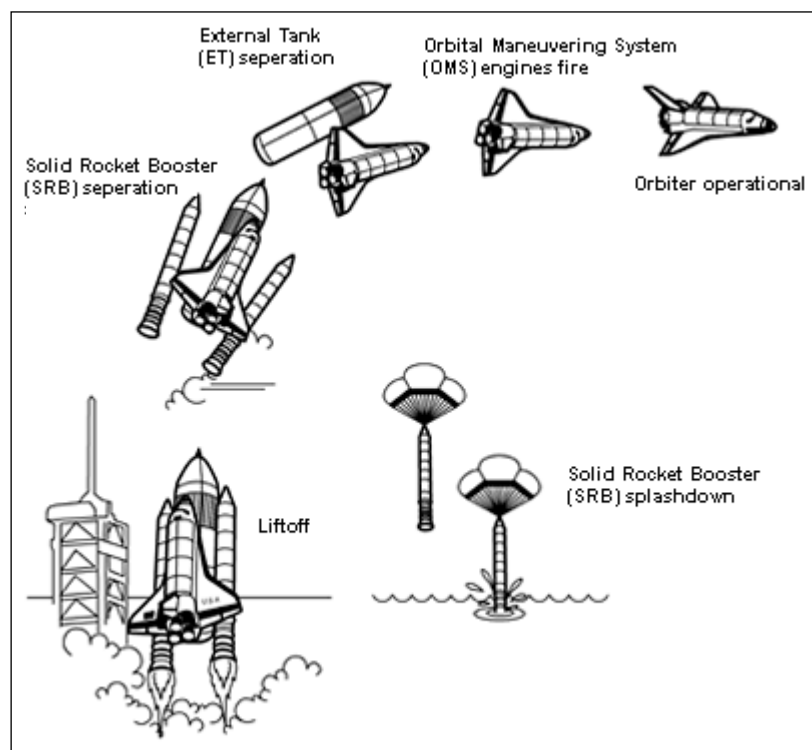


Figure 3: Space shuttle ascent process

The space shuttle experiences changes in altitude, velocity, and acceleration during the ascent into space. These changes can be seen by taking a closer look at the entire ascent process (Figure 3). The ascent process begins with the liftoff from the launch pad. As the SRB's burn their propellant and the Main Engines burn propellant from the ET, the space shuttle accelerates very quickly. This high-rate



of acceleration causes a rapid increase in dynamic pressure, known as Q in aeronautics (sometimes called velocity pressure). As the space shuttle breaks the sound barrier, its structure can only withstand a certain level of dynamic pressure before it suffers damage. Before this critical level is reached, the engines of the space shuttle are throttled down to about 67% of full power to avoid damage. About 50 seconds after liftoff, the dynamic pressure reaches its maximum aerodynamic load (Max Q). The air density then drops rapidly due to the thinning atmosphere, and the space shuttle can be throttled up to full power without fear of structural damage. The command is given, "Go at throttle up!"

As the space shuttle climbs, the velocity is increasing and the density of the air is decreasing. About two minutes after liftoff the atmosphere is so thin that the dynamic pressure drops to near zero. The SRBs, having used their propellant, are commanded by the space shuttle's onboard computer to separate from the ET. The spent SRBs fall into the ocean and are recovered, refurbished, reloaded with propellant, and reused for several missions. The jettisoning of these booster rockets marks the end of the first ascent stage and the beginning of the second. The second stage of ascent lasts about six and a half minutes, at the end of which time the space shuttle maneuvers into orbit. Just prior to orbit, ET separation occurs, and the ET re-enters the Earth's atmosphere, breaking up before impact in the ocean.

Instructional Objectives

- You will create scatter plots from a data table.
- You will determine the correlation of the data and interpret its meaning.
- You will find quadratic regression equations.
- You will use a function to find the altitude for times not shown in the table.

Regression Equations

Problem

On July 4, 2006 Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121 to begin a rendezvous with the International Space Station (ISS). Before each mission, projected trajectory data is compiled to assist in the launch of the space shuttle and to ensure safety and success during the ascent. To complete this data, flight design specialists take into consideration a multitude of factors such as space shuttle mass, propellant used, mass of payload being carried to space, and the mass of any payload returning. They must also factor in atmospheric density, which changes throughout the year. After running multiple tests and simulations, information is compiled into a table showing exactly what should happen each second of the ascent. A "state vector" is a set of data describing exactly where an object is located in space, and how it is moving. From a state vector, the object's current and future position can be determined. One of the responsibilities of the Flight Dynamics Officer (FDO) is to compute state vectors for position and velocity as the space shuttle gains altitude above Earth in order to achieve orbit.

Table 1 shows the altitude of Discovery for mission STS-121 every 10 seconds from liftoff to SRB separation.



Table 1: STS-121 Discovery Ascent data (altitude)

Time (s)	Altitude (ft)
0	7
10	938
20	4,160
30	9,872
40	17,635
50	26,969
60	37,746
70	50,548
80	66,033
90	83,966
100	103,911
110	125,512
120	147,411

Directions: Answer questions 1 – 3 with your group. Discuss answers to be sure everyone understands and agrees on the solutions. Share your answers with the class and discuss.

Use the graphing calculator to analyze the data from flight STS-121. To enter the data press the **STAT** button and select the option **1: Edit**. Enter the times in seconds into **L1** and enter the altitude values in feet in **L2**. (Directions are for a TI-84 series calculator. Consult user manual for other models.)

1. Use Table 1 to help you predict the shape of the graph of Altitude versus Time and determine the appropriate ranges and scales for the viewing window.
 - a. Study the data in Table 1 and predict the shape of the graph.
 - b. Sketch your prediction.
 - c. Look at the range of values in the Time column. What are reasonable numbers for **Xmin** and **Xmax**?
 - d. Considering the difference between any two consecutive times, what is a reasonable **Xscl** value?
 - e. Look at the range of values in the Altitude column. What are reasonable numbers for **Ymin** and **Ymax**?
 - f. Since these numbers are quite large and in order to have visible space between the tick marks on the y-axis, what is a reasonable number for **Yscl**?
2. Using graphing technology create a scatter plot of altitude vs. time. Use the scatter plot to explain why a quadratic function is a better fit than a linear function.



3. Find the regression equation of the line that best fits the data. Using function notation, write the equation as a function of altitude vs. time. Use the variable t to represent time and round coefficients and constants to the nearest whole number.

Directions: Answer question 4 with your group. Discuss answers to be sure everyone understands and agrees on the solutions. Share your answers with the class and discuss.

4. Graph the rounded function found in question 3 and compare to the scatter plot as you answer the following questions:
 - a. Does the line fit the data? How can you tell?
 - b. What is the correlation of the data (positive, negative, constant, or no correlation)? Explain what this represents with regard to the space shuttle.
 - c. What is the time in seconds for the first stage of ascent? Use this value and the function in question 3 to find the value of the function at the end of the first stage of ascent. Explain what this represents with regard to the space shuttle.
 - d. In Table 1, what is the altitude of the space shuttle at $t = 120$ seconds? Why do you think it is different from the value of the altitude which you found in part c above?

Directions: Work with a partner on the Google Earth Tour of STS-119 on the computer. In the frame on the left, click on the Altitude Pacemarks Folder to expand it. Record in Table 2 on your worksheet the Mission Elapsed Time (MET) in seconds and the altitude in feet for each Altitude Placemark. Answer questions 5-8 with your partner.



Table 2: STS-119 Google Earth Tour ascent data (altitude)

Time (s)	Google Earth Tour Altitude (ft)
0	0
31	10,000
45	20,000
54	30,000
62	40,000
71	50,000
77	60,000
83	70,000
89	80,000
95	90,000
99	100,000
123	150,000

5. Enter the data from Table 2 in your calculator. Adjust your viewing window to accommodate the range of entries in the table. Create a scatter plot of the altitude vs. time and graph it. What type of function would best fit the data? Describe the scatter plot.
6. Find the equation that best fits the data. Using function notation, write the equation as a function of altitude vs. time. Use the variable t to represent time and round coefficients and constants to the nearest whole number.
7. Graph the rounded function found in question 6 on the scatter plot. Does the line fit the data? How can you tell?
8. Use the function from question 6 to find the approximate altitude of Space Shuttle Discovery at 92 seconds after liftoff.