



Exploring Space Through MATH

Applications in Algebra 1



EDUCATOR
EDITION

Space Shuttle Ascent: Altitude vs. Time

Note: This problem is related to the Algebra 1 problem, *Space Shuttle Ascent: Mass vs. Time*, in the Exploring Space Through Math series.

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will

- create scatter plots from a data table;
- determine correlation and interpret its meaning;
- find quadratic regression equations; and
- use a function to find the altitude for times not shown in the table.

Prerequisites

Students should have prior knowledge of scatter plots, types of correlations, quadratic equations and quadratic graphs. Students should also have experience using a graphing calculator or spreadsheet application to create scatter plots and to find quadratic regression equations.

Background

This problem is part of a series that applies algebraic principles in NASA's human spaceflight.

The Space Shuttle Mission Control Center (MCC) and the International Space Station (ISS) Control Center use some of the most sophisticated technology and communication equipment in the world. Teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle and the ISS. They work together as a powerful team, spending many hours performing critical simulations as they prepare to support each mission and crew during normal operations and any unexpected events.

Since its first flight in 1981, the space shuttle has been used to extend research, repair satellites, and help with building the ISS. NASA plans to retire the space shuttle, but until then space exploration depends on the continued success of space shuttle missions. Critical to any space shuttle mission is the ascent into space.

The ascent phase begins at liftoff and ends when the space shuttle reaches Earth's orbit. The space shuttle must accelerate from zero to

Key Concept

Modeling data with quadratic regression equations

Problem Duration

90 minutes

Technology

TI-83, 84 series graphing calculator or spreadsheet application, computer with projector, computer lab, Internet, Google Earth downloaded, Google Earth Tour, video of space shuttle launch

Materials

Space Shuttle Ascent: Altitude vs. Time Student Edition

Degree of Difficulty

Moderate to Difficult

Skills

Use a graphing calculator to create scatter plots, find regression equations, interpret graphs and equations of quadratic data.

NCTM Standards

- Number and Operations
- Algebra
- Data Analysis and Probability
- Problem Solving
- Communication
- Connections



approximately 7,850 meters per second (which is approximately 17,500 miles per hour) in eight and a half minutes to reach the minimum altitude required to orbit Earth. It takes a very unique vehicle to accomplish this task.

There are three components of the space shuttle that enable the launch into orbit (Figure 1). The main component is the orbiter. It not only serves as the crew's home in space and is equipped to dock with the ISS, but it also contains maneuvering engines for finalizing the orbital trajectory, or flight path. The External Tank (ET), the largest component of the space shuttle, supplies the propellant (liquid oxygen and liquid hydrogen) to the Space Shuttle Main Engines (SSMEs) which are liquid propellant rocket engines. The third component is a pair of Solid Rocket Boosters (SRBs) that are reusable. They are attached to the sides of the ET and provide the main thrust at launch (Figure 2).

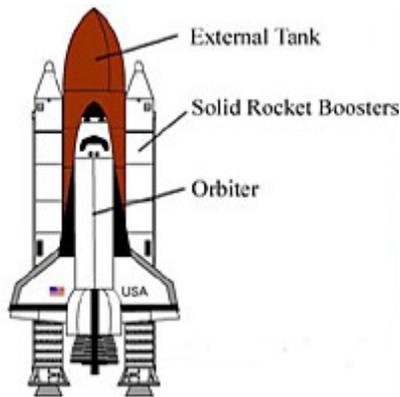


Figure 1: Main components of the space shuttle



Figure 2: Space Shuttle Discovery at liftoff

The Flight Dynamics Officer (FDO) is the flight controller in the Space Shuttle MCC that is responsible for the planning and execution of the trajectory of the space shuttle during ascent, orbit, rendezvous, and re-entry. This requires intensive computational processing to be performed by FDO to ensure that the mission objectives are safely met. The flight controllers in the MCC monitor the systems and functions of the space shuttle, and FDO manages where the space shuttle is and where it is going. Providing technical support for FDO with these duties are the engineers in the Multi-Purpose Support Room (MPSR), pronounced "Mipser" and sometimes referred to as the FDO Backroom.

The space shuttle experiences changes in altitude, velocity, and acceleration during the ascent into space. These changes can be seen by taking a closer look at the entire ascent process (Figure 3). The ascent process begins with the liftoff from the launch pad. As the SRB's burn their propellant and the Main Engines burn propellant from the ET, the space shuttle accelerates very quickly. This high-rate of acceleration causes a rapid increase in dynamic pressure, known as Q in aeronautics (sometimes called velocity pressure). As the space shuttle breaks the sound barrier, its structure can only withstand a certain level of dynamic pressure before it suffers damage. Before this critical level is reached, the engines of the space shuttle are throttled down to about 67% of full power to avoid damage. About 50 seconds after liftoff, the dynamic pressure reaches its maximum aerodynamic load (Max Q). The air



density then drops rapidly due to the thinning atmosphere, and the space shuttle can be throttled to full power without fear of structural damage. The command is given, "Go at throttle up!"

As the space shuttle climbs, the velocity is increasing and the density of the air is decreasing. About 2 minutes after liftoff the atmosphere is so thin that the dynamic pressure drops to near zero. The SRBs, having used their propellant, are commanded by the space shuttle's onboard computer to separate from the ET. The spent SRBs fall into the ocean and are recovered, refurbished, reloaded with propellant, and reused for several missions. The jettisoning of these booster rockets marks the end of the first ascent stage and the beginning of the second. The second stage of ascent lasts about six and a half minutes, at the end of which, the space shuttle maneuvers into orbit. Just prior to orbit, ET separation occurs, and the ET re-enters Earth's atmosphere, breaking up before falling in the ocean.

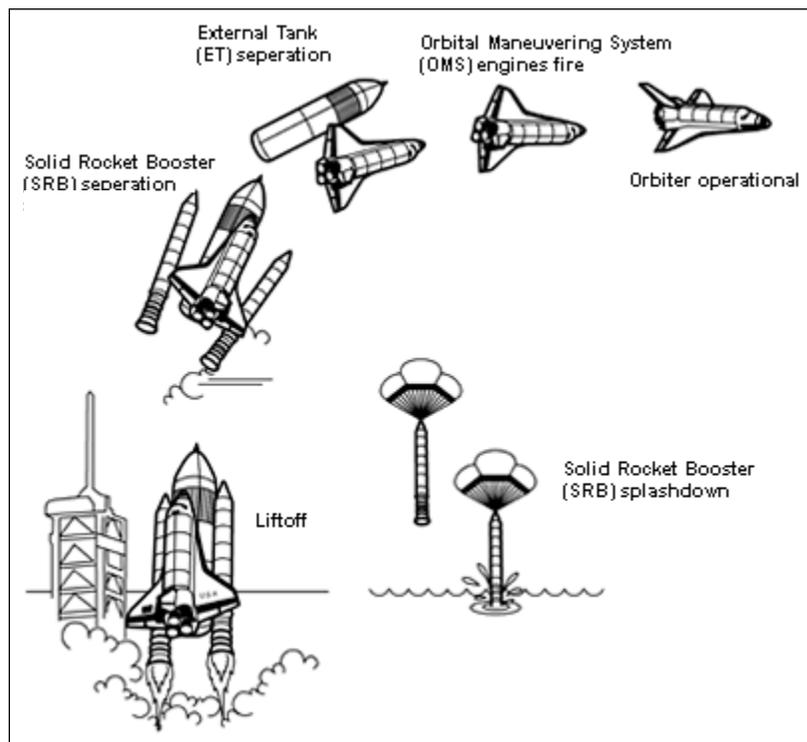


Figure 3: Space shuttle ascent process

NCTM Principles and Standards

Number and Operations

- Develop a deeper understanding of very large and very small numbers and of various representations of them.

Algebra

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Use symbolic algebra to represent and explain mathematical relationships.



- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Approximate and interpret rates of change from graphical and numerical data.

Data Analysis and Probability

- For bivariate measurement data, be able to display a scatter plot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools.
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

Problem Solving

- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

Communication

- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and apply mathematics in contexts outside of mathematics.

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (20 minutes)

- Either assign or let students choose small groups of 3-4 in order to work through this activity.
- Each group should review and discuss the main points of the Background section for several minutes to be sure that they understand the material. Circulate to help facilitate discussion in small groups. Ask if any group needs clarification.
- Read through *Instructions for Google Earth Space Shuttle Ascent Tour* (Appendix A) prior to completing this with students. Open the Google Earth Tour and in the left frame click on “Play me” to show students the actual trajectory of the space shuttle ascent of STS-119. After viewing the Google Earth Tour click on “Help and Vocabulary” in the left frame to discuss NASA terms and acronyms.
- Encourage student discussion and ask if there are any questions.
- Optional: You may choose to view the video, *STS-121 Shuttle Launch* (3.5 min), which is featured in a previous problem in this series, *Space Shuttle Ascent: Mass vs. Time*.
<http://humanresearch.jsc.nasa.gov/education/algebra>.

2 – Explore (20 minutes)

- Distribute graphing calculators. If necessary, review some of the steps to enter lists and finding regression equations, including types of data correlation. Stress the difference between linear and quadratic regression. As an alternative to the calculator, a spreadsheet application may be used to analyze the data.
- Distribute the worksheet, *Regression Equations*.
- Have students read the problem statement.
- Ask students to work through questions 1-3 as a group.



- Call on students to give their answers and discuss. If available, students can use presentation technology to demonstrate the graphing calculator aspects of the questions.

3 – Explain (15 minutes)

- Have the students work with their group to answer question 4 on the Regression Equations worksheet.
- Call on students to give their answers and discuss. If available, students can use presentation technology to demonstrate the graphing calculator aspects of the questions.

4 – Extend (15 minutes)

- Use the Google Earth Tour in a computer lab. Ask students to work with a partner to investigate the Google Earth Tour.
- Point out that the distance that the space shuttle travels in the first 8.5 minutes is approximately the same as the distance from Orlando, Florida, to Boston, Massachusetts.
- Ask students to use the Internet to research how much time it would take for a commercial jet to fly nonstop from Orlando to Boston and the time it would take to drive a car nonstop between the two cities.
- Have students compare the times for the space shuttle, the jet, and the car and discuss. (If you only have one computer, you could call on a different student to demonstrate on the computer finding the approximate time it would take for the space shuttle, a commercial jet, and a car. If you prefer, assign the research for students to complete at home and discuss the next day.

5 – Evaluate (20 minutes)

- Continuing in the computer lab with the Google Earth Tour, have students work with a partner to record the required time and altitude data for Table 2 on the Regression Equations worksheet. (If you only have one computer, let them record it together as a class. Call on a different student to go to the display and point out the values to record for time and altitude for each placemark.)
- When the data is recorded, have students complete questions 5-8 with a partner.

ENGAGE

Google Earth Tour

- View the Google Earth Tour. Use Appendix A.
- Discuss the NASA terms and acronyms on the “Help and Vocabulary” menu.
- Encourage student discussion using the following questions:
 - What are the major events that occur during the space shuttle ascent? (liftoff, SRB separation, MECO, ET separation, and orbit). Click on Ascent Events to demonstrate this feature.
 - How much time has elapsed in the mission when the space shuttle reaches 100,000 ft? (1:39 min). Click on the Altitude Placemarks.
 - What is the approximate altitude of the space shuttle 2:03 minutes after liftoff? (150,000 ft).
 - Encourage students to share their impressions of the ascent.



EXPLORE

Regression Equations

Solution Key

Problem

On July 4, 2006 Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121 to begin a rendezvous with the International Space Station (ISS). Before each mission, projected trajectory data is compiled to assist in the launch of the space shuttle and to ensure safety and success during the ascent. To complete this data, flight design specialists take into consideration a multitude of factors such as space shuttle mass, propellant used, mass of payload being carried to space, and the mass of any payload returning. They must also factor in atmospheric density, which changes throughout the year. After running multiple tests and simulations, information is compiled into a table showing exactly what should happen each second of the ascent. A “state vector” is a set of data describing exactly where an object is located in space, and how it is moving. From a state vector, the object’s current and future position can be determined. One of the responsibilities of the Flight Dynamics Officer (FDO) is to compute state vectors for position and velocity, as the space shuttle gains altitude above Earth in order to achieve orbit.

Table 1 shows the altitude of Discovery for mission STS-121 every 10 seconds from liftoff to SRB separation.

Table 1: STS-121 Discovery Ascent data (altitude)

Time (s)	Altitude (ft)
0	7
10	938
20	4,160
30	9,872
40	17,635
50	26,969
60	37,746
70	50,548
80	66,033
90	83,966
100	103,911
110	125,512
120	147,411

Directions: Answer questions 1 – 3 with your group. Discuss answers to be sure everyone understands and agrees on the solutions. Share your answers with the class and discuss.

Use the graphing calculator to analyze the data from flight STS-121. To enter the data press the **STAT** button and select the option **1: Edit**. Enter the times in seconds into **L1** and enter the altitude values in



feet in **L2**. Rounding to the nearest whole number in these activities will simplify the equations without losing essential elements of the problem. (Directions are for a TI-84 series calculator. Consult user manual for other models.)

1. Use Table 1 to help you predict the shape of the graph of Altitude versus Time and determine the appropriate ranges and scales for the viewing window.
 - a. Study the data in Table 1 and predict the shape of the graph.
Answers will vary. The predicted graph should rise from left to right.
 - b. Sketch your prediction.
Answers will vary and should match the prediction. The graph should rise from left to right.
 - c. Look at the range of values in the Time column. What are reasonable numbers for **Xmin** and **Xmax**?
Xmin = 0 and **Xmax** = 120.
 - d. Considering the difference between any two consecutive times, what is a reasonable **Xscl** value?
Xscl = 10.
 - e. Look at the range of values in the Altitude column. What are reasonable numbers for **Ymin** and **Ymax**?
Ymin = 0 and **Ymax** = 148,000.
 - f. Since these numbers are quite large and in order to have visible space between the tick marks on the y-axis, what is a reasonable number for **Yscl**?
Yscl = 20,000.
2. Using graphing technology, create a scatter plot of altitude vs. time. Use the scatter plot to explain why a quadratic function is a better fit than a linear function.

Go to **STAT PLOT** (**2ND**, **Y=**). Select **Plot 1** and press **ENTER**. Select **ON** by pressing **ENTER**, and select scatterplot for Type. For Mark, choose the dot, and press **GRAPH**. Use the scatter plot to explain why a quadratic function is a better fit than a linear function.

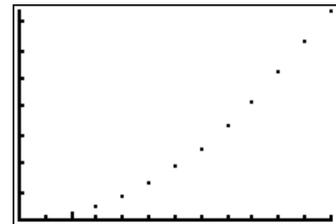
The points appear to lie along a parabola which is rising from left to right. The data are not rising at a constant rate and is therefore not linear. A linear function would show many outliers.

```

WINDOW
Xmin=0
Xmax=120
Xscl=10
Ymin=0
Ymax=148000
Yscl=20000
Xres=1
  
```

```

Plot1 Plot2 Plot3
On Off
Type: [Scatter] [Line] [Bar]
Mark: [Dot] [Cross] [Square]
Xlist:L1
Ylist:L2
  
```



3. Find the regression equation of the line that best fits the data. Using function notation, write the equation as a function of altitude vs. time. Use the variable t to represent time and round coefficients and constants to the nearest whole number.



On a TI graphing calculator you will find the equation by pressing the **STAT** key and selecting the **CALC** menu. Since the data is quadratic, select option **5: QuadReg** and press **ENTER**.

```
QuadReg
y=ax2+bx+c
a=10.03123377
b=31.94150849
c=-172.043956
```

$$\longrightarrow a(t) = 10t^2 + 32t - 172$$

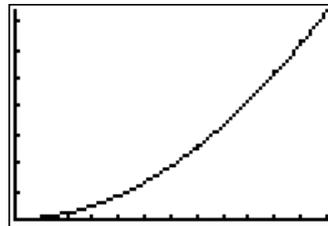
EXPLAIN

Solution Key

Directions: Answer question 4 with your group. Discuss answers to be sure everyone understands and agrees on the solutions. Share your answers with the class and discuss.

4. Graph the rounded function found in question 3 and compare to the scatter plot as you answer the following questions:

```
Plot2 Plot3
Y1=10X2+32X-172
Y2=
Y3=
Y4=
Y5=
Y6=
```



- a. Does the line fit the data? How can you tell?

Yes, because the points of the scatter plot appear to lie along the curve or are very close to it on either side.

- b. What is the correlation of the data (positive, negative, constant, or no correlation)? Explain what this represents with regard to the space shuttle.

The graph shows a positive correlation. As time passes, the altitude is increasing.

- c. What is the time in seconds for the first stage of ascent? Use this value and the function in question 3 to find the value of the function at the end of the first stage of ascent. Explain what this represents with regard to the space shuttle.

This is the altitude of the space shuttle at the end of the first stage of ascent.

- d. In Table 1 what is the altitude of the space shuttle at $t = 120$ seconds? Why do you think it is different from the value of the altitude which you found in part c above?

147,411 ft. This was the actual altitude of the space shuttle at $t = 120$ seconds.

The 147,668 ft is the estimated value from the line of best fit, which is simply a model of the data set and contains some error. The coefficients of the function were also rounded.



EXTEND

Solution Key

- Take students to a computer lab.
- Once students are paired at a computer, ask them to click on “Play me” in the Google Earth Tour.
- Have them play the tour again until the time shows 29 seconds and hit Pause.
- Ask them to notice the states that are labeled on the east coast of the United States. Point out that the red/black circle labeled ET Separation (which occurs at about 8.5 minutes) appears to be roughly opposite Rhode Island. In order to do the research on this question, a major airport will need to be used. Choose Boston, Massachusetts since it is relatively close and will yield fairly accurate results.
- Ask students to use the Internet to research the time it would take for a commercial jet to fly nonstop from Orlando, Florida, to Boston, Massachusetts.
- Ask them to also research the time it would take to drive a car nonstop between the two cities.
- Have students compare the travel times for the space shuttle, the commercial jet, and the car.

EVALUATE

Solution Key

Directions: Work with a partner on the Google Earth Tour of STS-119 on the computer. In the frame on the left, click on the Altitude Pacemarks Folder to expand it. Record in Table 2 on your worksheet the Mission Elapsed Time (MET) in seconds and the altitude in feet for each Altitude Placemark. Answer questions 5-8 with your partner.

Table 2: STS-119 Google Earth Tour ascent data (altitude)

Time (s)	Google Earth Tour Altitude (ft)
0	0
31	10,000
45	20,000
54	30,000
62	40,000
71	50,000
77	60,000
83	70,000
89	80,000
95	90,000
99	100,000
123	150,000



5. Enter the data from Table 2 in your calculator. Adjust your viewing window to accommodate the range of entries in the table. Create a scatter plot of the altitude vs. time and graph it. What type of function would best fit the data? Describe the scatter plot.

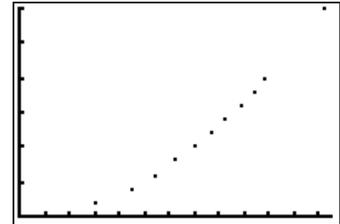
It is a quadratic function. The points lie along a parabolic curve that is rising from left to right.

```

WINDOW
Xmin=0
Xmax=125
Xscl=10
Ymin=0
Ymax=150000
Yscl=25000
Xres=1
    
```

```

Plot1 Plot2 Plot3
On Off
Type: [ ] [ ] [ ]
Xlist:L1
Ylist:L2
Mark: [ ] [ ] [ ]
    
```



6. Find the equation that best fits the data. Using function notation, write the equation as a function of altitude vs. time. Use the variable t to represent time and round coefficients and constants to the nearest whole number.

```

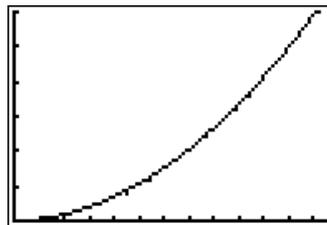
QuadReg
y=ax^2+bx+c
a=9.678283984
b=38.35959684
c=-354.7462699
    
```

→ $a(t) = 10t^2 + 38t - 355$

7. Graph the rounded function found in question 6 on the scatter plot. Does the line fit the data? How can you tell?

```

Plot1 Plot2 Plot3
Y1: 10X^2+38X-355
Y2=
Y3=
Y4=
Y5=
Y6=
    
```



The line fits the data because the points of the scatter plot appear to be on the parabolic curve or are very close to it on either side.

8. Use the function from question 6 to find the approximate altitude of Space Shuttle Discovery at 92 seconds after liftoff.

Let $t = 92$

$$a(92) = 3(92)^2 + 38(92) - 355$$

$$a(92) = 87,781 \text{ ft}$$

The altitude at 92 seconds after liftoff is 87,781 ft.



Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

NASA Experts

Dr. Greg Holt, Flight Controller, Flight Dynamics Division, Space Shuttle Mission Control Center
NASA Johnson Space Center

Kelly Smith, Co-op Student, Flight Dynamics Division, Space Shuttle Mission Control Center
NASA Johnson Space Center

Frank Hughes, V.P. for Education & Training Products, Tietronix Software,
Retired from NASA as Chief of Space Flight Training in 1999

Michael R. Sterling, Manager, Training Leads, Space Flight Training, NASA Johnson Space Center

Henry Lampazzi, Simulation Supervisor, Ascent Procedures Specialist, Spaceflight Training Division,
NASA Johnson Space Center

Chris Giersch - Education and Public Outreach Lead, Exploration and Space Operations Directorate,
Langley Research Center

Mathematics Educator

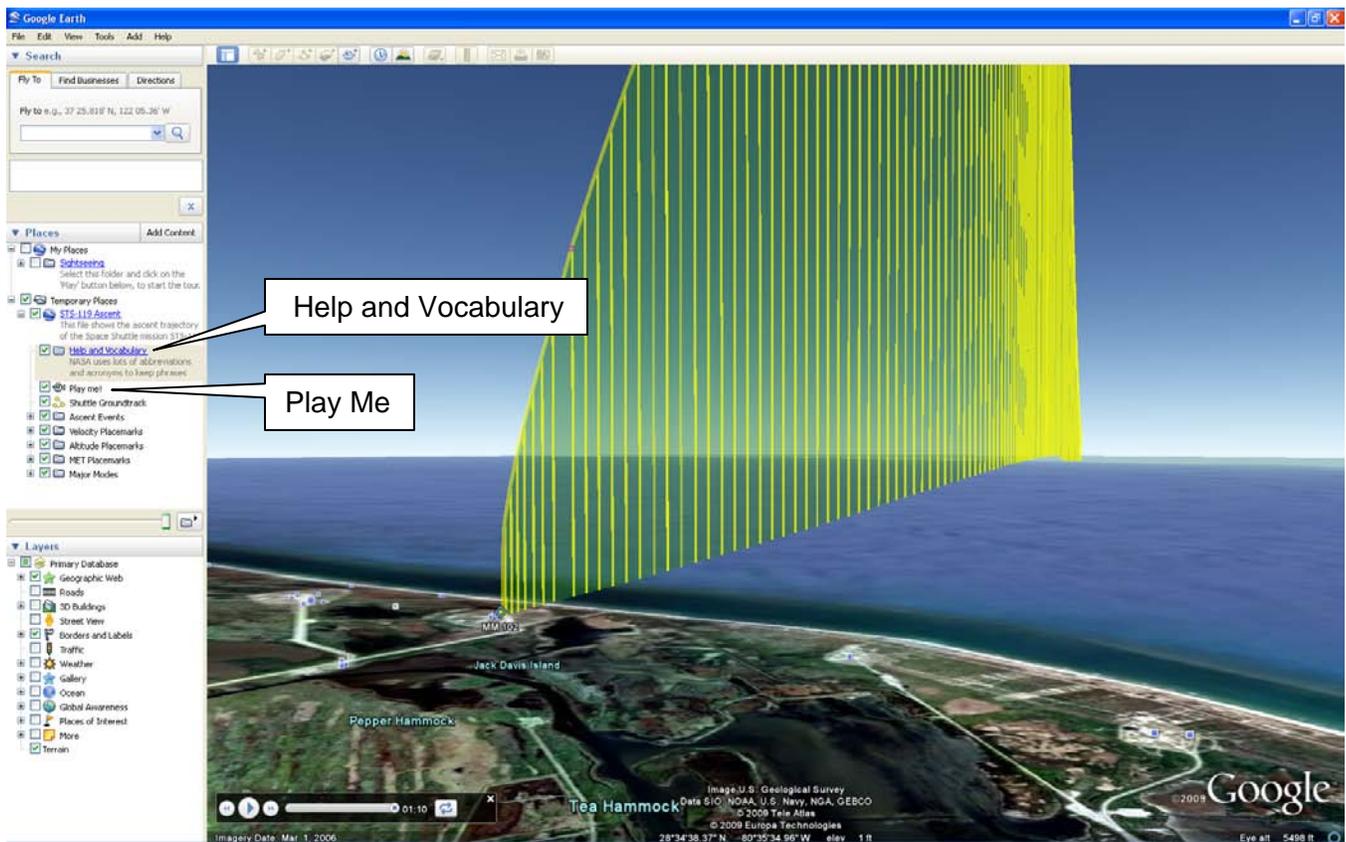
Natalee Lloyd, North Shore High School, Galena Park Independent School District



Appendix A

Instructions for Google Earth Space Shuttle Ascent Tour

1. Prior to presenting this tour, you should become familiar with the features of this file. This tour is 1:10 minutes long.
2. To access the tour, you must first have Google Earth installed on the computer. To download a free version of Google Earth, follow the link below and choose "Download Google Earth 5".
<http://earth.google.com>
3. Download the provided file SpaceShuttleAscentTour.kmz.
4. Open the file (SpaceShuttleAscentTour.kmz) on your computer by double clicking on it. Maximize the window.
5. To learn more about this file, click on **Help and Vocabulary**, located on the left frame under **Temporary Places**.
6. To start the animation, double click on "Play Me!"





7. The tour begins with an overview of the entire space shuttle ascent from several different angles. This overview is 20 seconds long.
8. After the overview, the sequence will begin again. There are three 5 second pauses for inserting *explanations of the ascent events: SRB Separation, MECO, and ET Separation*. Each event is marked with a black and red circle. At each pause, it is recommended that you click on the Pause button on the slider bar on the lower left of the screen and call on a student to describe the event and its consequences. When the student has finished, press Play to move to the next event.
9. Continue to play the animation and pause where appropriate until the tour is complete.
10. There are additional features that you may want to explore with students. You may also want to use the following subtopics to unclutter the animation, (for example, uncheck the altitude and velocity folders).

To expand a folder in the left frame, click on the “+” next to the folder. To collapse the folder, click on the “-” next to it. For each subtopic there is a pop-up information box that can be opened by clicking on the subtopic. To close the pop-up information box, click on the “x” in the upper right corner of the box (or uncheck the small checked box to the left of the subtopic by clicking on it).

- Ascent Events Folder (expanded)
 - Click on an event and a pop-up information box will appear on the graph.
 - Notice the bullets for the events are the same as the markers (red and black circles) that appear on the graph for these events.
- Velocity Placemarks Folder (expanded)
 - Click on a velocity and a pop-up information box will appear on the graph.
 - Notice the bullets for the velocities are the same as the markers (green circles) that appear on the graph for these velocities.
- Altitude Placemarks Folder (expanded)
 - Click on an altitude and a pop-up will information box will appear on the graph.
 - Notice the bullets for the altitudes are the same as the markers (pink circles) that appear on the graph for these altitudes.
- MET (Mission Elapsed Time) Placemarks Folder (expanded)
 - Click on an MET and a pop-up information box will appear on the graph.
 - Notice the bullets for the METs are the same as the markers (orange circles) that appear on the graph for these METs.
- Major Modes (MM) Folder (expanded)
 - Click on a major mode and a pop-up information box will appear on the graph.
 - Notice the bullets for the major modes are the same as the markers (blue diamonds) that appear on the graph for these major modes.