It All Comes Full Circle

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will
- use geometric definitions to describe the properties of circles;
- find the circumference of circles;
- calculate arc lengths;
- find the measure of central angles; and
- use the properties of similarity, such as scale factor.

Prerequisites

Students should have prior knowledge of the geometric principles of circles, circumference, central angles, arc length, and scale factor.

Background

This problem is part of a series that applies mathematical principles in NASA's human spaceflight.

The International Space Station (ISS) is an internationally developed research facility and is the largest human-built satellite in Earth's orbit. It travels around Earth at an altitude of approximately 250 miles (400 km). It is an incredible and complex engineering endeavor that has been assembled while in orbit.

Construction of the ISS began in November 1998, when Russia placed the Zarya module in orbit. With the exception of the Russian Zvezda module and Zarya module, all other modules were delivered by a NASA space shuttle. Each module was installed by ISS and space shuttle crewmembers during spacewalks and using a robotic arm. The space shuttle is the only space vehicle capable of carrying the large modules that make up the ISS.

When the space shuttle launches from NASA Kennedy Space Center, it must launch within a certain time frame (called a launch window) in order to successfully dock with the ISS. Launch windows are calculated so that the space shuttle will reach an orbit that is slightly lower than the ISS, but in the same orbital plane. The space shuttle travels slightly faster in its lower orbit, and thus "catches up" to the ISS, while making small orbit corrections to raise its orbit and align the vehicles. The space shuttle then docks with the

Key Concepts
Properties of circles, similarity

Problem Duration
50 minutes

Technology
Computer with projector, movie player

Materials
- *It All Comes Full Circle* Student Edition
- *Real World: Calculating Shuttle Launch Windows* video

Skills
Identify circumference, central angles, arc length, scale factor

NCTM Standards
- Number Operations
- Algebra
- Geometry

ISS to resupply, exchange crew members, and deliver hardware—such as a new ISS module, solar panels, or hardware for experiments.

During the rendezvous and docking maneuver sequence, teams from the ISS and Space Shuttle Mission Control Centers work closely together to ensure a safe and successful docking. The ISS Trajectory Operations Officer (TOPO) and space shuttle Flight Dynamics Officer (FDO) flight controllers work together throughout the entire maneuver sequence. The TOPO flight controller works hand in hand with other ISS flight controllers to determine and maintain the orbital position of the ISS. The FDO flight controller is responsible for matching that orbit by determining maneuvers for the space shuttle. Docking two fast-moving vehicles in space is a very delicate and potentially dangerous task, so the shuttle will perform up to ten maneuvers during the rendezvous sequence to align the orbits and maintain a safe trajectory while approaching the ISS.

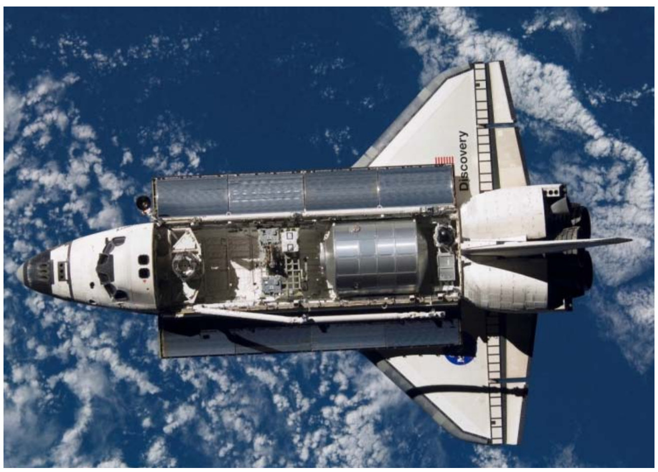

Figure 1: Space Shuttle Discovery, with payload doors open, is viewed from the ISS during rendezvous.

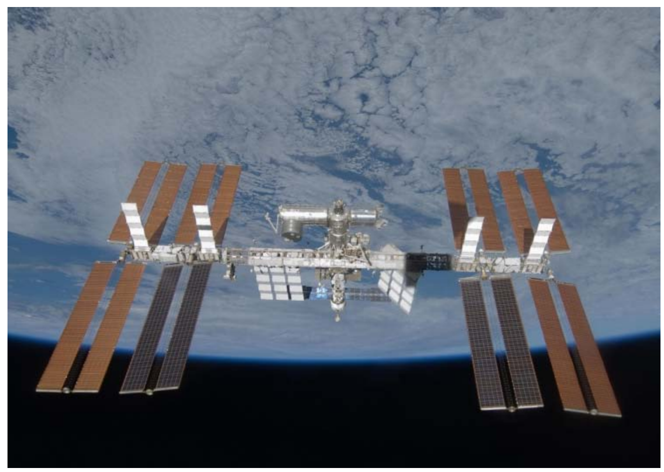

Figure 2: The ISS orbiting Earth as observed by Space Shuttle Discovery on March 26, 2009

NCTM Principles and Standards

Number and Operations

- Develop a deeper understanding of very large and very small numbers and of various representations of them
- Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases
- Judge the reasonableness of numerical computations and their results

Algebra

- Use symbolic algebra to represent and explain mathematical relationships
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology

Geometry

- Analyze properties and determine attributes of two- and three-dimensional objects;
- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them

Lesson Development

Following are the phases of the 5-E's Instructional model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (15 minutes)

- Play the video, *Real World: Calculating Shuttle Launch Windows*. (*approximately 7 minutes*) The video can be played or downloaded from this link: http://www.nasa.gov/audience/foreducators/nasaeclips/search.html?terms=&category=0100 To find the video after clicking on the link, type the video title in the search box or scroll down the page to find the title.
- Have students read the Background section aloud to the class. Stop occasionally to check for understanding.

2 – Explore (5 minutes)

- Distribute the worksheet, *It All Comes Full Circle* (Student Edition).
- With students in groups of two to four, answer questions 1-3.

3 – Explain (10 minutes)

- Allow students to remain in groups to answer questions 4-8.
- Call on students to give their answers and discuss.

4 – Extend (10 minutes)

- Allow students to remain in groups to answer questions 9-11.
- Encourage student discussion and ask if there are any questions.

5 – Evaluate (10 minutes)

- Have students work independently to complete questions 12-14. This may be done *in class or assigned* as homework.

It All Comes Full Circle

Solution Key

Problem

The space shuttle and ISS are in circular orbits around the Earth at similar speeds. A FDO flight controller is responsible for the trajectory of the space shuttle, and a TOPO flight controller is responsible for the trajectory of the ISS. Orbital mechanics can be complicated, and there is a possibility that instruments that are used to predict and monitor the vehicles' positions could go offline. To address these potential issues, the FDO and TOPO must have basic math knowledge and be able to apply it.

Through understanding the mathematical definitions of a circle, the FDO can estimate parameters and compare those estimates to the actual data to check for accuracy. Figure 3 is a simplified drawing of the space shuttle and ISS, each in its circular orbit. In the diagram, the path labeled 2 (red) is the path the space shuttle takes when it needs to rendezvous and dock with the ISS.

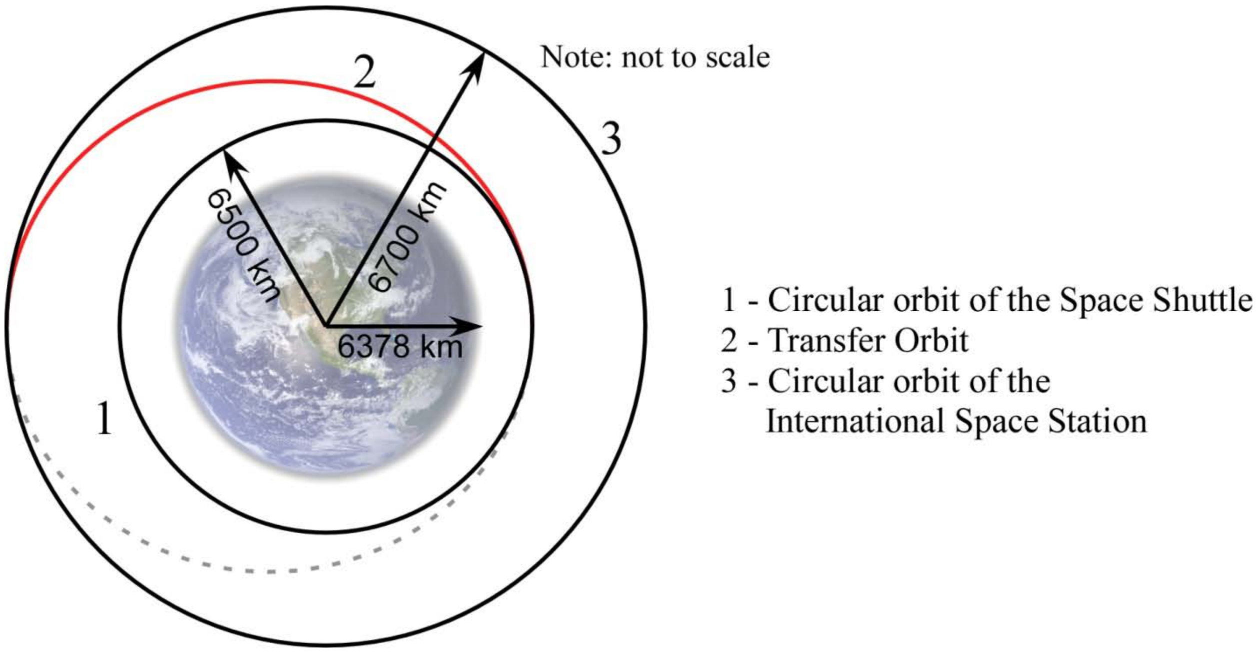

Figure 3: Simplified illustration of the space shuttle and ISS circular orbits and the elliptical transfer orbit

Directions: Show all work and justify your answers to questions 1-3. Label With the appropriate units.

1. The space shuttle orbits the Earth at a distance of 6,500 km from the center of the Earth. The ISS orbits the Earth at a distance of 6,700 km from the center of the Earth. In terms of a circle, what are these distances called? Explain.

   The distances are radii. Radius is the distance from the center of the circle to the edge of the circle. The center of the Earth is the center of each circle.

2. Use a geometric definition to describe the two circles and explain why this definition can be used.

   The two circles are concentric circles. They share the same center, but have different radii; or the two circles are coplanar (or are in the same plane).

Directions: Answer questions 4-8 in your group. Discuss answers to be sure everyone understands and agrees on the solutions. Use the π key on your calculator. Round all answers to the nearest tenth and then label with the appropriate units.

3. How many kilometers does the space shuttle have to travel to complete one orbit? In terms of a circle, what is this distance called? Explain.

   $C_{SS} = 2\pi r$

   $C_{SS} = 2\pi(6{,}500\ km)$

   $C_{SS} = 40{,}840.7\ km$

   One orbit would be the distance around the circle and is called the circumference of the circle.

4. The space shuttle is traveling around the Earth at a rate *v*, of 28,191 km/hr in order to remain in orbit. How many minutes would it take the space shuttle to complete one orbit around the Earth?

$$d = t_{SS} v$$
$$t_{SS} = \frac{d}{v}$$
$$t_{SS} = \frac{40{,}840.7\ km}{28{,}191\ \frac{km}{hr}}$$
$$t_{SS} = 1.447\ hr$$
$$t_{SS} = (1.448\ hr)(60\ \tfrac{min}{hr})$$
$$t_{SS} = 86.9\ minutes$$

5. How many kilometers does the ISS have to travel to complete one orbit? In terms of a circle, what is this distance called? Explain.

$$C_{ISS} = 2\pi r$$
$$C_{ISS} = 2\pi(6{,}700\ km)$$
$$C_{ISS} = 42{,}097.3\ km$$

One orbit would be the distance around the circle and is called the circumference of the circle.

6. The ISS is traveling around the Earth at the rate *v*, of 27,767 km/hr in order to remain in orbit. How many minutes would it take the ISS to complete one orbit around the Earth?

$$d = t_{ISS} v$$
$$t_{ISS} = \frac{d}{v}$$
$$t_{ISS} = \frac{42{,}097.2\ km}{27{,}767\ \frac{km}{hr}}$$
$$t_{ISS} = 1.516\ hr$$
$$t_{ISS} = (1.516\ hr)(60\ \tfrac{min}{hr})$$
$$t_{ISS} = 90.9\ minutes$$

7. The ISS is orbiting the Earth every day, experiencing one daylight and one darkness with each orbit. Approximately how many times in one day do the crewmembers experience daylight and darkness?

$$\frac{24\ hr}{1.516\ hr} = 15.8\ \textit{orbits in one day}$$

Since there are approximately 16 orbits per day, the crewmembers experience the daylight/darkness cycle 16 times throughout a 24-hour period of time.

Directions: Answer questions 9-11 in your group. Discuss answers to be sure everyone understands and agrees on the solutions. Round all answers to the nearest tenth and then label with the appropriate units.

8. The space shuttle has been in orbit for 30 minutes. How many kilometers has it traveled since leaving its initial point of orbit? What geometry term describes this distance?

$$d_{SS} = vt_s$$
$$d_{SS} = (0.5\ hr)\left(28{,}191\ \frac{km}{hr}\right)$$
$$d_{SS} = 14{,}095.5\ km$$

Since the space shuttle is traveling a circular path and has traveled part of the circumference of the circle, the distance is an example of arc length.

9. A portion of the space shuttle's orbit could be defined by a central angle of 127°. How long would the space shuttle have to travel on one orbit to create this angle?

$$\frac{t\ min}{86.9\ min} = \frac{127^\circ}{360^\circ}$$
$$t = 30.7\ min$$

10. How many kilometers does the ISS travel during a 30-minute time period?

$$d_{ISS} = vt_s$$
$$d_{ISS} = (0.5\ hr)\left(27{,}767\ \frac{km}{hr}\right)$$
$$d_{ISS} = 13{,}883.5\ km$$

Directions: Complete questions 12-14 independently. Label with the appropriate units.

11. The circles that are created by the orbits of the two vehicles are concentric. What common properties do they have and how are they different?

Answers may vary. They have the same center, but slightly different radii and they are similar figures.

12. Assume that the vehicles are in rendezvous position (right before docking) and each has traveled in this position for 30 minutes. What is the measure of the central angle that the ISS creates?

$$\frac{13{,}883.5\ km}{42{,}076\ km} = \frac{\theta}{360^\circ}$$
$$\theta = 118.8^\circ$$

13. What is the scale factor of the ISS and space shuttle orbits? What does the scale factor tell you about the orbits? Round to the nearest hundredth.

$$\frac{C_{ISS}}{C_{SS}} = \frac{42{,}076\ km}{40{,}820\ km} = 1.03 \quad or \quad \frac{r_{ISS}}{r_{SS}} = \frac{6{,}700\ km}{6{,}500\ km} = 1.03$$

The scale factor shows how similar the orbits are and how close they are in size.

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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