Exploring Space Through MATH

Applications in Precalculus

EDUCATOR EDITION

Earth, Can You Hear Me Now?

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will

- use trigonometric functions to model periodic behavior of real-life data;
- make predictions based on the trigonometric properties of a function;
- graph variations of sine curves;
- determine the domain and range of functions using graphs and tables.

Prerequisites

Students should have prior knowledge of plotting data on a coordinate plane, of solving equations, and of key characteristics of a sinusoidal function and the parent function, $f(x) = \sin x$.

Background

This problem applies mathematical principles in NASA's human spaceflight program.

Human spaceflight is an important part of NASA's mission. From lunar exploration to the completion of the International Space Station (ISS), NASA has been preparing humans to explore the unknown. The research and innovation required to explore space has led to technological advancements on Earth. Space exploration has brought benefits to medicine, medical care, transportation, public safety, computer technology, and to many other areas that enrich our everyday lives.

Exploring space is a complex endeavor. Missions to explore beyond Low-Earth Orbit (LEO) require extensive research and precise planning. Future destinations for exploration include Near-Earth Objects (NEO), the Moon, Mars, and the moons of Mars.

For over 40 years, NASA has been exploring Mars. The Mars Exploration Program began in the 1960's and has made and documented some remarkable findings about the "Red Planet". The first phase of the Mars Exploration Program consisted of activities known as "flybys", which involved satellites flying by the planet and taking as many pictures as

Key Concepts
Graphs of trigonometric functions, trigonometric equations

Problem Duration
50 minutes

Technology
Projector with movie player, graphing technology

Materials
- *Earth, Can You Hear Me Now?* Student Edition
- Graph paper

Skills
Graphing, equation manipulation

NCTM Standards
- Numbers and Operations
- Algebra

possible. The satellite series was called Mariner. The Mariners' cameras have captured some surprisingly great images of giant volcanoes and a grand canyon on the planet's surface. Still in orbit around Mars today, the Mars Odyssey spacecraft, with its onboard camera, has captured images that have been used to create an accurate Martian map.

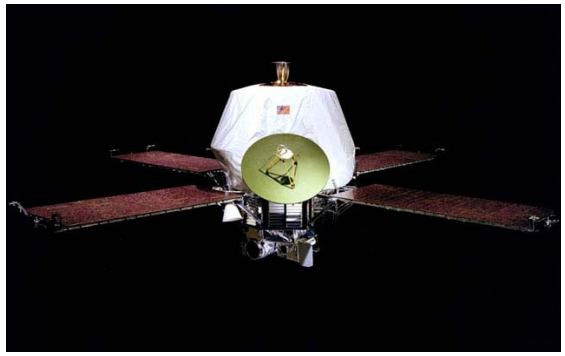


Figure 1: Mariner Satellites

NASA landed on the surface of Mars with the Mars Exploration Rovers, Spirit and Opportunity, in January 2004. These rovers moved about the surface of the planet collecting full color images of the terrain and microscopic image of the rock and soil. In January 2010, they found evidence of subsurface water on Mars.

Figure 2: Artist concept of rover on Mars

Humans have not yet flown to Mars. In an April 2010 speech at NASA Kennedy Space Center, President Obama predicted that humans would orbit the planet by the mid 2030's, and that a human landing would soon follow. Human spaceflight to Mars may seem straightforward since NASA has been successful with satellites and rovers. However, at its closest distance from Earth, Mars is 55 million kilometers away, and at its farthest distance, is over 400 million kilometers. Studies have shown that traveling to Mars with the current rocket technology would take about nine months. A round trip mission would take nine months to get there, a two to six month stay on Mars to study the planet, and then nine months to get back (or approximately two years total).

Imagine what it would take to plan a two-year piloted mission to Mars. Fuel, communication, food and medical care are only some of the things to consider. The needs of the crew would be extremely important. Being in communication with Mission Control becomes more difficult and crucial for mission success. Since the distance between the two planets is so enormous, there is a communication time delay. Transmitting a message from Earth to Mars could take up to 20 minutes. A successful mission would have very few communication issues and would require detailed and in-depth planning.

Any future space exploration beyond Low-Earth Orbit would be an ambitious undertaking and would require new strategies, new technology, research and development, and careful planning. Completion of these missions would increase humanity's reach far beyond Earth's boundaries.

NCTM Principles and Standards

Numbers and Operations

- Judge the reasonableness of numerical computations and their results.

Algebra

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions.
- Interpret representations of functions of two variables.
- Use symbolic algebra to represent and explain mathematical relationships.
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Draw reasonable conclusions about a situation being modeled.

Lesson Development

Following are the phases of the 5-E's Instructional Model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (10 minutes)

- With students in teams of three or four, have them read and discuss the main points of the Background section for several minutes to be sure that they understand the material. Circulate to facilitate discussion and provide clarification where needed.
- Play the video, *Spirit: Six Years of Roving Mars* (approximately 5 minutes), by following this link: http://www.jpl.nasa.gov/video/index.cfm?id=886
- Encourage student discussion of the video and ask if there are any questions.

2 – Explore (15 minutes)

- Distribute the student worksheets *Earth, Can You Hear Me Now?*
- Distribute graphing calculators. (*This is optional since they are not required until question 7.)*
- Call on one student to read the problem aloud.
- Using presentation technology, show the Mars and Earth orbit animation: http://www-mgcm.arc.nasa.gov/MarsToday/season.QT
- Ask students to work as teams on questions 1-4.

3 – Explain (10 minutes)

- Have students work as teams to answer questions 5-6.
- Call on students to give their answers and discuss.

4 – Extend (10 minutes)

- Distribute graphing calculators. *Note: The instructions and solution given in this section reference the use of a TI-84 graphing calculator.*
- Have students work as teams to answer questions 7-9.
- Encourage student discussion and ask if there are any questions.

5 – Evaluate (5 minutes)

- Have students work as teams to answer question 10.

Earth, Can You Hear Me Now?

Solution Key

Problem

In preparation for human exploration missions to Mars, NASA has been sending spacecrafts to orbit the planet and rovers to explore the surface. Vital information has been gathered about the flight to Mars and about the planet itself. Since Mars and the Earth orbit the sun at different rates, the distance between Mars and the Earth is constantly changing. In one proposed Mars exploration plan, astronauts will remain on the surface of Mars for approximately 18 months. The ability to communicate between Mars and Earth will be extremely critical.

Information sent from Mars to Earth is carried on radio waves that travel at the speed of light in space. Because the distance between the two planets is, on average, over 100 million kilometers, there still is a significant communication time delay. NASA's engineers and scientists are working to understand this delay by collecting data and modeling the distance between Mars and Earth in 26-month cycles. This is the time required for Mars to make its closest approach to Earth.

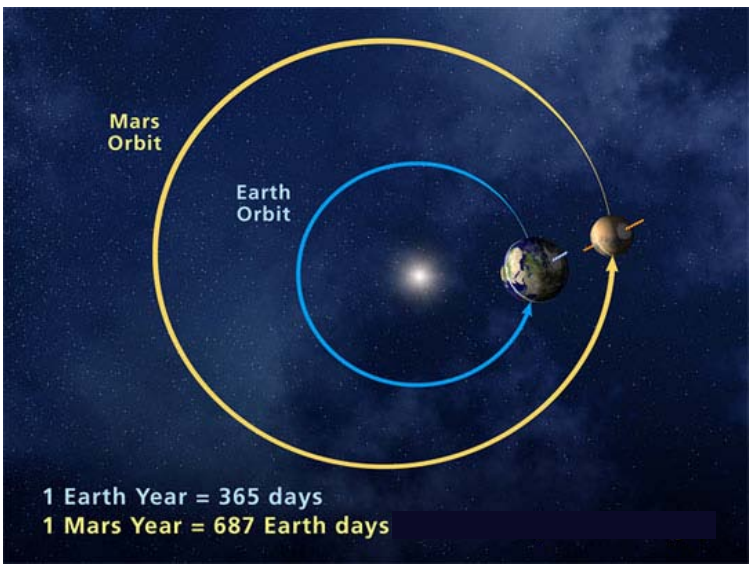


Figure 3: Mars and Earth orbits

Table 1: Mars-Earth Distance & Communication Delay 2002 – 2004 (approximately 26 months), where day zero is the start of the distance and communication delay tracking.

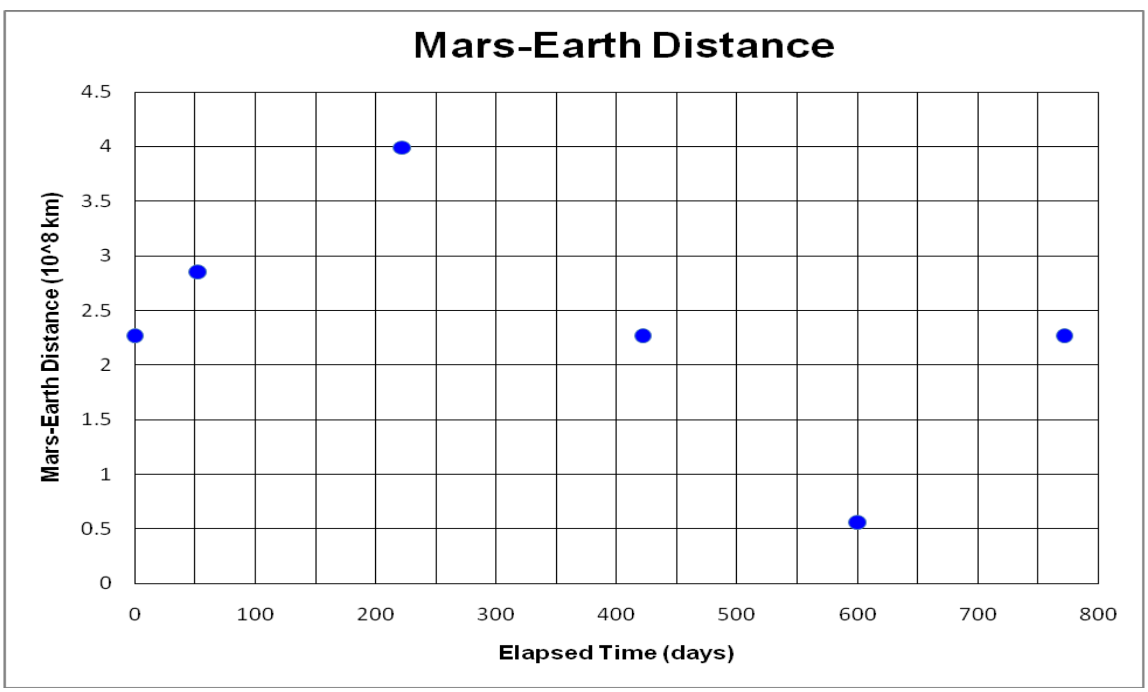
Elapsed Time (days)	Mars-Earth Distance (km)	One Way Communication Time (min)
0	2.27×10^8	12.6
52	2.85×10^8	15.9
222	3.99×10^8	22.2
422	2.27×10^8	12.6
600	5.57×10^7	3.1
772	2.27×10^8	12.6

Directions: Answer questions 1-6 in your teams. Discuss answers to be sure everyone understands and agrees on the solutions.

1. As Mars and Earth travel along their respective orbits, what happens to the one-way communication time of radio waves carrying information between the two planets?

   The time delay increases as the distance between Mars and Earth increases, and decreases as the distance decreases. This pattern repeats as the two planets travel along their orbits.

2. Plot the number of days and distance in km x 10^8 data from Table 1 on graph paper. Assume the number of days is your independent variable.



3. What type of function would you choose to model the graph of number of days and distance from Earth to Mars?

   A sinusoidal function would best model this data.

4. Analyze the characteristics of the data and determine the parameters of the model.

   a. A periodic function repeats itself. The time taken to complete one full cycle is called the period. Use Table 1 to determine the period (T) of your model.

      The period of the model is 772 days.

   b. Determine the phase shift (or horizontal shift) of the function.

      Based on the graph, a cycle starts on day 0, so there is no phase shift.

   c. The amplitude of your model is one-half the difference between the maximum and minimum distance; therefore the amplitude of your model is given by the equation $a=\frac{1}{2}(f(x)_{max}-f(x)_{min})$. Determine the amplitude of your model in terms of km x 10^8.

      $$a=\frac{1}{2}(f(x)_{max}-f(x)_{min})$$
      $$a=\frac{1}{2}(3.99-.56)$$
      $$a=1.72$$

      The amplitude of the model is 1.72 x 10^8 kilometers.

   d. The vertical shift is the difference between the position of the equilibrium of the model and its parent function. Determine the vertical shift of the model when compared to the parent function, $f(x)=\sin x$, in terms of km x 10^8.

      The function, $f(x)=\sin x$, oscillates about the equilibrium y = 0. The model's equilibrium is 2.27 units above y = 0. The vertical shift is 2.27x10^8 km.

5. Define the parameters *a*, *b*, *c,* and *d* needed to write the equation of the function in the form, $f(x)=a\sin(bx+c)+d$.

   a. Using your calculation from question 4c, what is the value of *a*, or the amplitude of the function, in terms of km x 10^8?

      The value of a is 1.72.x10^8 km.

b. Use the formula for the period $T = \frac{2\pi}{b}$ to determine the value of *b*. Write your answer in terms of pi.

$$772 = \frac{2\pi}{b}$$

$$b = \frac{\pi}{386}$$

c. Use the formula phase shift $= -\frac{c}{b}$ to determine the value of *c*. Write your answer in terms of pi.

$$0 = \frac{-c}{\frac{\pi}{386}}$$

d. Using your answer from question 4d, what is the value of *d*, or the vertical shift, of the graph of the function?

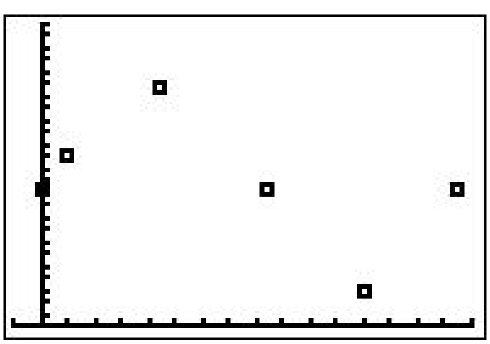
The value of d is 2.27x10^8 km.

6. Write the equation of the function in the form, $f(x) = a\sin(bx + c) + d$.

$$f(x) = 1.72x10^8 \sin\left(\frac{\pi}{386}x\right) + 2.27x10^8$$

Directions: Answer questions 7-10 in your teams. Discuss answers to be sure everyone understands and agrees on the solutions. Round all answers to the nearest thousandth and label with the appropriate units.

7. Compare the function to the data for your graph by creating a scatterplot. To enter the data, first clear all list by pressing **MEM (2 nd**, **+**). Select option 4 (**ClrAll Lists)** and press **Enter.** Press the **STAT** button and select the option **1: EDIT**. Enter the elapsed time in days into **L1** and enter the distance in km into **L2**. To create a scatter plot of elapsed time vs. distance, go to **STAT PLOT** (**2nd**, **Y=**). Select **PLOT 1** and press **ENTER**. Select On by pressing **ENTER**, and select scatterplot for Type.



8. Determine the appropriate viewing window setting based on the data and graph.

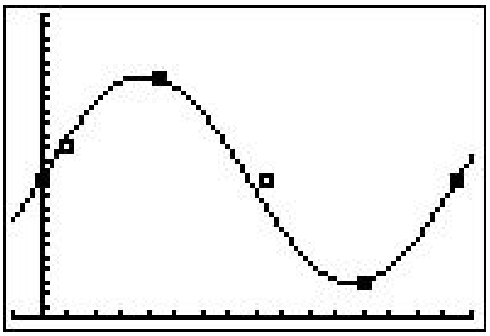
   a. Look at the domain of values in the number of days column. What are reasonable values for **Xmin** and **Xmax**?

      ***Xmin** = 0 and **Xmax** = 800 (Answers may vary.)*

   b. Look at the range of values in the Mars-Earth Distance column. What are reasonable values for **Ymin** and **Ymax**?

      ***Ymin** = 0 and **Ymax** = 5 x 10^8 (Answers may vary.)*

9. Graph your function in km by entering it into **Y=**. Press **GRAPH**. (Be sure the calculator is in RADIAN mode). Does your function fit the data well? Explain.



The graph of the equation passes through five of the six data points. (Answers may vary.)

10. In its latest quest to prepare for manned flights to Mars, NASA is planning to land Curiosity, a roving science laboratory, on Mars in the year 2012. Engineers have determined that the landing window must be between August 6, 2012 (day 3867) and August 20, 2012 (day 3881). Determine which of the two days would be most favorable for information relay to Earth from Curiosity as it lands on Mars.

$$f(x) = 1.72x10^8 \sin\left(\frac{\pi}{386}x\right) + 2.27 \text{ x } 10^8 km$$

$$f(3867) = 1.72x10^8 \sin\left(\frac{\pi}{386}3867\right) + 2.27 \text{ x } 10^8 km$$

$$f(3867) = 2.368 \text{ x } 10^8 km$$

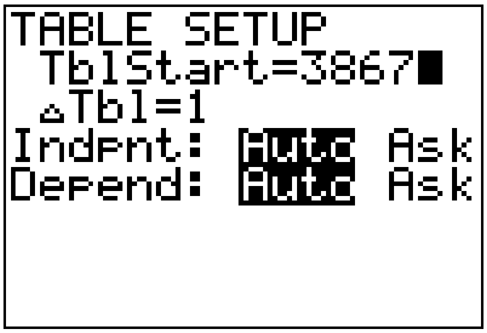
$$f(x) = 1.72x10^8 \sin\left(\frac{\pi}{386}x\right) + 2.27 \text{ x } 10^8 km$$

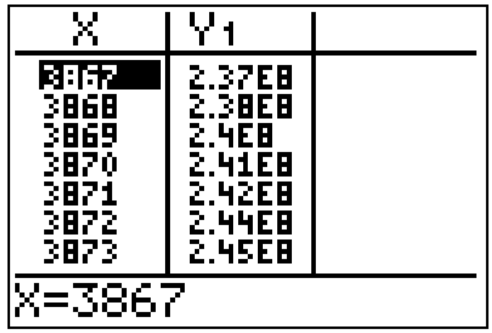
$$f(3881) = 1.72x10^8 \sin\left(\frac{\pi}{386}3881\right) + 2.27 \text{ x } 10^8 km$$

$$f(3881) = 2.563 \text{ x } 10^8 km$$

The distance between Earth and Mars on August 6, 2012 is expected to be approximately 2.368 x 10^8 kilometers. The distance between Earth and Mars on August 20, 2012 is expected to be approximately 2.563 x 10^8 kilometers. Therefore, August 6, 2012 would be more favorable. The shorter distance would have a shorter communication delay.

*Optional calculator method: Go to **TBLSET** (**2nd**, **WINDOW**) and enter 3867. Go to **TABLE** (**2nd**, **Graph**) and locate the corresponding y value.*





Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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