Flight Testing Newton's Laws

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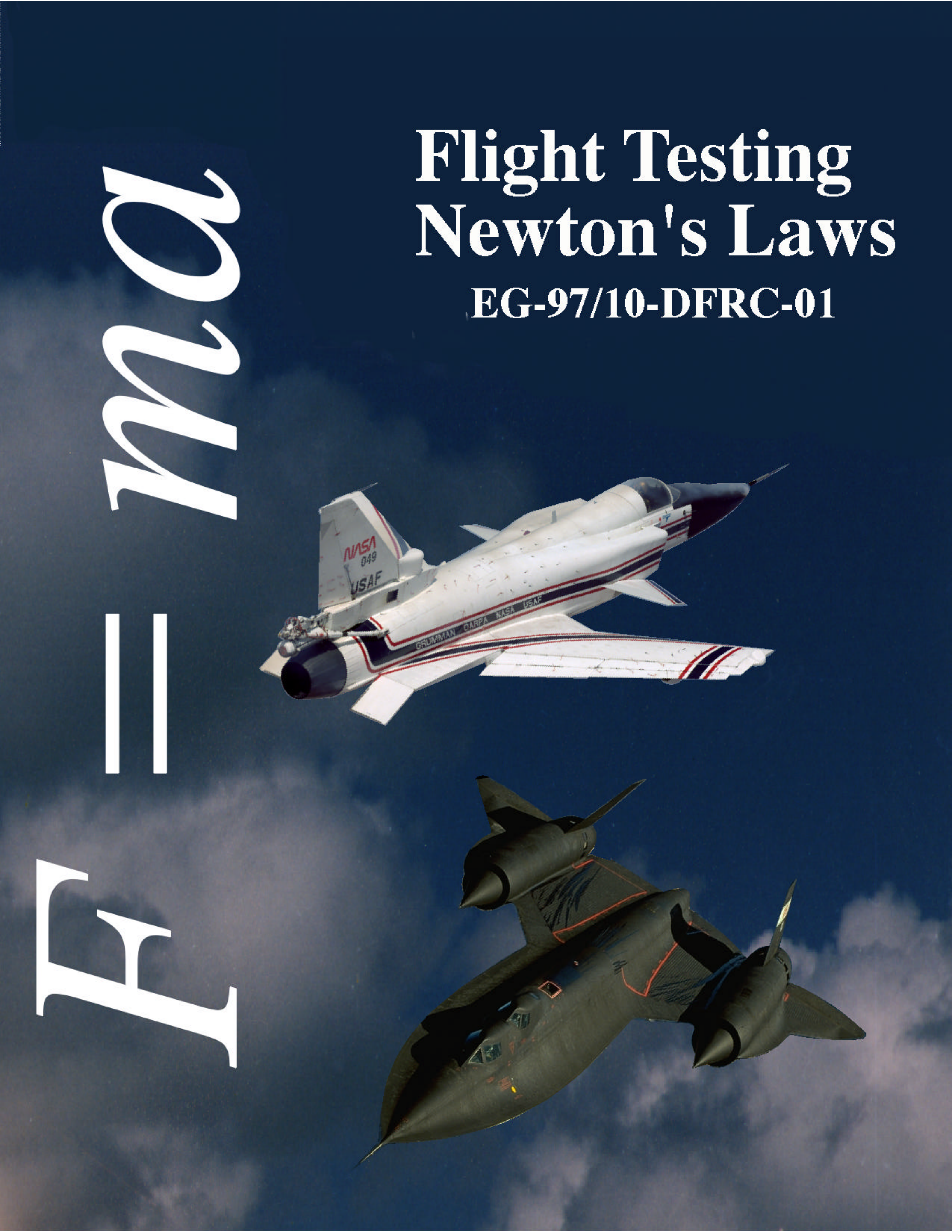

Flight Testing Newton's Laws

Instructor's Flight Manual

NASA / DFRC - X41 - 1

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Instructor's Flight Manual

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Preface

The "Flight Testing Newton's Laws" NASA Education Series uses aircraft to stimulate the student's interest in the physical sciences and mathematics. The main emphasis lies in showing how Newton's three laws of motion apply to flight testing an aircraft. However, complementary areas of trigonometry, vector addition, weight and balance, along with resolution of forces are also employed. Following a brief review in the first video of Newton's Three Laws and the four basic forces of flight, the presentation follows the typical sequence employed by test pilots and engineers preparing for a test flight. Aircraft weight and balance, determining takeoff distance, cruise performance, and landing distance are addressed in turn.

Each lesson guide is presented in the format of a Flight Instructor's Manual used by aircraft manufactures and pilots. This Manual contains certain areas where the teacher should direct the student's attention. Each of these areas are identified by their relative importance according to the following criteria:

NOTE: Sidelight information which may add to ensuing discussions but which is not considered essential to the material content.

Caution: Should the student fail to consider a particular aspect of the topic of discussion, the result may be the wrong answer to the example problem.

Warning: This block will identify background information the student should already possess. Knowledge of identified concepts is essential to understanding the material being presented. The material is not given during this session but is identified to the instructor in order to permit discussion of the material prior to undertaking the current lesson.

By way of example, consider the following:

NOTE: In an actual Pilot's Flight Manual, the Notes, Cautions, and Warnings are defined as follows:

> **NOTE:** An operating procedures, techniques, etc., which is considered essential to emphasize.
>
> Caution: Operating procedures, techniques, etc., which could *result in damage to equipment* if not carefully followed.
>
> Warning: Operating procedures, techniques, etc., which could *result in personal injury or loss of life* if not carefully followed.

Often information that is not critical to flight safety, but which enhances the pilot's understanding, is provided in the form of an Operational Supplement. Throughout this manual, Operational Supplements are provided at the end of the session to enhance the understanding of the material. When appropriate, a note is added to direct the reader's attention to the end of session Operational Supplement.

All units in the Flight Instructor's Manual are presented in the English system. The rationale behind this is twofold. First, engineers and pilots in the United States still use the English system exclusively. All cockpits have instrumentation measured in feet, statue or nautical miles per hour, pounds per square inch, and foot-pounds. Second, it is felt that if so desired, by converting the example problems into the metric system, the student will develop a feel for the relative magnitudes of units between the two systems. The accompanying text often presents both sets of units in its examples and explanations.

Occasionally, the teacher may want to stop the video to reinforce or clarify subjects being presented. Throughout this guide, there will be areas annotated by ****STOP VIDEO**** where clarification may be appropriate. In addition, where definitions are presented at the beginning of the session, it may be advantageous to review the definitions **before** showing the video. The recommended areas to start the video are annotated with ****START VIDEO****. All material presented prior to the ****START VIDEO**** symbol should be covered before hand.

The National Aeronautics and Space Administration's Education Division supports the National Education Standards. The activities in Flight Testing Newton's Laws were developed in accordance with the National Education Standards and satisfy the content requirements for science and mathematics specified below:

National Science Education Standards

Physical Science

<u>Motions and Forces:</u>

- Objects change their motion only when a net force is applied. Laws of motion are used to calculate precisely the effects of forces on the motion of objects. The magnitude of the change in motion can be calculated using the relationship $F = ma$, which is independent of the nature of the force. Whenever one object exerts force on another, a force equal in magnitude and opposite in direction is exerted on the first object.
- Gravitation is a universal force that each mass exerts on any other mass. The strength of the gravitation attractive force between two masses is proportional to the masses and inversely proportional to the square of the distance between them.

Curriculum Standards for School Mathematics

<u>Mathematics as Problem Solving</u>

- Use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content.
- Apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics.

<u>Mathematics as Communication</u>

- Reflect upon and clarify their thinking about mathematical ideas and relationships
- Read written presentations of mathematics with understanding

<u>Mathematics as Reasoning</u>

- Make and test conjectures

<u>Mathematical Connections</u>

- Relate procedures in one representation to procedures in an equivalent representation

- Use and value the connections between mathematical and other disciplines

Algebra

- Represent situations that involve variable quantities with expressions, equations, inequalities, and matrices;
- Operate on expressions and matrices, and solve equations and inequalities;
- Appreciate the power of mathematical abstraction and symbolism;
- Demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations.

Functions

- Model real-world phenomena with a variety of functions.

Trigonometry

- Explore periodic real-world phenomena using the sine and cosine functions;
- Solve trigonometric equations and verify trigonometric identities

References

NRC (National Research Council). 1996. *National Science Education Standards*. Washington, DC: National Academy Press.

National Council of Teachers of Mathematics. Commission on Standards for School Mathematics. 1989. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Virginia: National Council of Teachers of Mathematics.

It is our intent that through the use of videos and Flight Manuals, the thrill of aviation can be enjoyed by both the student and the teacher.

Acknowledgments

Endeavors such as "Flight Testing Newton's Laws" require the efforts of numerous people. The NASA/Dyden Flight Research Center's Education Division and the National Test Pilot School staff would like to extend a special thanks to the following individuals:

Ms. Lauern Holen of the Tehachapi (CA) Unified School District
Mr. Jim Nickel of the Antelope Valley (CA) Union High School District
Ms. LeAnn B. Tichenor of Desert High, Edwards AFB, CA

The tireless support of these teachers in the review and trial presentations of the "Flight Testing Newton's Laws" series is greatly appreciated.

Session 1
Review of Newton's Laws of Motion

1.0 Importance of Physics

Testing airplanes requires pilots to know a lot more than just how to fly the plane; they must also know why an airplane flies. The science of flight is totally dependent upon physics. In fact, without a good understanding of physics, Orville and Wilbur Wright would never have gotten their Flyer off the ground.

Wright Flyer

Many "would-be" aircraft designers never took the time to study Newton's Laws, and as a result, built contraptions that flew worse than they looked.

"Would-Be" Design

Newton's Laws of Physics are still applied by aircraft designers every day for every type of aircraft. Using these laws, designers are able to determine such things as the overall shape of the aircraft, how many engines are required, how far it can go, and how much runway is needed to takeoff and land. All these areas must be addressed for the design to be successful.

NOTE:

The following types of aircraft are shown in the first video:

SAAB Draken
(supersonic fighter)

Aeromacchi Impala
(jet trainer)

Sikorsky S-55
(transport helicopter)

In order to make effective use of Newton's Laws, a brief review of each is in order.

Warning:

The assumption is made that the students have already been taught the development of Newton's Laws. The following presentations are meant to serve only as a refresher. Sections 5.1 through 5.4 of the accompanying text should be reviewed prior to starting the video.

****START VIDEO****

2.0 Newton's First Law

The First Law of Motion is often referred to as the Law of Inertia. The formal definition states:

> **"A body in motion at a uniform speed will remain in motion at that speed unless acted upon by an external force, and a body at rest will remain at rest unless acted upon by an external force."**

NOTE:

See the Operational Supplement at the end of this session for a description of speed and velocity.

A body in motion is exactly what seatbelts are designed to restrain. Seatbelts are known to save lives by preventing the vehicle occupant from continuing forward when the vehicle stops suddenly. The tendency for the occupant to continue forward is a classic case of inertia at work. The "external force" which acts upon the body comes in the form of the seatbelt.

Seatbelts at Work

For the speeds experienced in a car, the seatbelts/shoulder strap combination should provide sufficient stopping force for the occupant. However, since aircraft travel at much faster speeds and are free to move in three dimensions, a "five point" harness is often used; 2 shoulder straps, a right and left seatbelt, and a "negative-g" strap between the legs. The effectiveness of this arrangement can be seen on the sled track occupant.

Sled Track Test Subject

Each belt must be made of the proper material and to the correct size to provide enough external force to limit the pilot's movement. Determining the size of forces is the topic of Newton's Second Law.

NOTE:

The rapid deceleration rate caused Col. Stapp's eyes to hemorrhage, giving him two completely red eyes.

3.0 Newton's Second Law

Newton's Second Law of Motion relates force to acceleration. The formal definition is:

> **"Force is equal to mass times acceleration, or $F = ma$."**

NOTE:

See the Operational Supplement at the end of this session for discussion of acceleration.

An everyday example of this law occurs when we step on the scale to weigh ourselves.

Session 1
Review of Newton's Laws of Motion

Second Law at Work

The force can be measured directly as our weight, refered to as F_w. Additionally, on Earth, the acceleration of gravity is found to be 32.2 ft/sec^2. Substitution into the

$$F = ma,$$

equation and rearranging where $a = g$

$$m = \frac{F_w}{g}$$

this equation yields our mass.

NOTE:

The development of the universal gravitational constant is contained in the Operational Supplement.

The significance of knowing our mass comes to light when we are not subjected to the Earth's gravity. A man standing on the Earth has the same mass as he would standing on the moon. However on the moon he would weigh 1/6th of what he would weigh on the Earth. The difference in his weight comes from the differences in the gravitational acceleration constants between the Earth and the moon. The moon's gravitational acceleration is only 1/6th that of the Earth's. As a result, his weight is only 1/6th of his Earthly weight. In aviation, the Earth's gravitational acceleration is referred to as a "*g*." Often times a pilot may feel the effect of more (or less) than 1 "*g*."

As an aircraft maneuvers, the pilot experiences a change in the "g-factor." This factor is multiplied times the standard gravitational acceleration of 32.2 ft/sec^2. A mathematical expression of this would look like

$$F = ma$$

$$F_w = mg \ \ (\text{"}g\text{" factor})$$

Caution:

The "*g*" factor is actually a result of centripetal acceleration. The equation for this is

$$F = \frac{mV^2}{R}$$

However, for this application the "g" factor can be envisioned as simply a multiplication factor.

Example: The pilot in the video weighs 155 pounds. To determine his mass,

NOTE:

Descriptions of this and other acceleration factors are contained in the Operational Supplement.

$$F_w = mg$$

$$155 \text{ lbs} = m \ (32.2 \text{ ft/sec}^2)$$

therefore

$$m = \frac{F_w}{g} = 155 \text{ lbs}/32.2 \text{ ft/sec}^2$$

or

$$m = 4.8 \text{ slugs}$$

****STOP VIDEO after pilot talks about slugs****

NOTE:

The use of slugs as a unit of measurement may be foreign to some students. See page 4.2 of the text for a complete definition of a slug and its equivalent in the metric system.

During a "2-g" turn, the pilot's weight can be found by:

$$F_w = mg(\text{"}g\text{" factor})$$

$$F_w = (4.8 \text{ slugs}) \ (32.2 \text{ feet/sec}^2) \ (2)$$

F_w = 309 pounds

NOTE:

The mass actually comes out to 4.8137 slugs. Due to round-off error, the 2*g* turn results in 309 lbs instead of the more exact 310 lbs.

****START VIDEO****

From this example, it can be seen that a "*g*" factor is purely a multiplication factor used to determine an increase in weight. Since this increase in weight acts towards the pilot's feet, the force may cause the blood to leave his upper body causing him to black out. As a result, he wears an "anti-*g*" suit to provide an opposing force on his legs keeping the blood in his head and chest. Opposing forces is the subject of Newton' third law.

4.0 Newton's Third Law

The third law of motion is often thought of as the law of action and reaction. Specifically it states

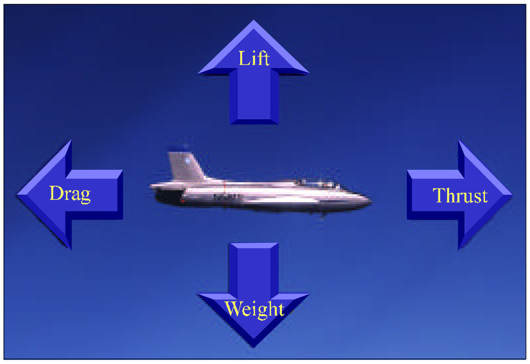
> **"When one object exerts a force on another, the second object must exert an equal and opposite force on the first."**

The simplest example of this is when we stand from sitting in a chair. We place our feet on the floor, use our legs to push against the floor, and push ourselves up. If the earth, or floor, didn't push back with an equal amount of force, we would fall into the earth (earth pushing back with less force) or we would be propelled into the air (earth pushing back with more force). The same principle applies to a jet engine. Thrust is the force produced by the hot gas coming out the back of the engine. Since Newton's third law must also be obeyed here, the air exerts a force equal to the thrust but in the opposite direction, propelling the jet forward in the same manner as the floor exerting a force on your legs allows you to stand.

Thrust is just one of four primary forces which act upon an aircraft in flight. The plane can't violate the third law, therefore thrust must be opposed by an equal and opposite force. This second force is called drag. Drag is the resistance of the atmosphere to the aircraft, like you feel when you put your hand out the window of a moving car. When drag and thrust are equal, the aircraft is no longer accelerating, but remains at the same speed since these forces are equal. If thrust is increased by adding more power, the aircraft will initially accelerate to a new speed. However as the plane's speed increases, so does the drag and eventually, thrust and drag will again be equal, but at a faster speed.

The remaining two forces on the aircraft highlight how a plane stays in the air. Lift is the force provided by the wings as the plane moves forward through the air. If lift is the force which causes a plane to rise, then it seems logical the opposing force would act in a downward direction. Not surprisingly, this force is the plane's weight. Now it may seem strange, but lift is always equal to weight, otherwise the aircraft couldn't stay in the air.

The aircraft is controlled by changing the lift forces over the wings and tail. Moving the control stick forward or backward causes more or less lift on the tail causing the nose to move up or down. Likewise moving the stick from side to side causes more lift on one wing, which results in a roll.



Forces in Flight

All four of these forces are actually dependent upon each other and in a future session, their interrelationships will be explored.

5.0 Summary

Newton's three laws were highlighted here to provide the student with an exposure into only a few areas where the laws come into play in aviation. The pilot's restraint system reviewed the principle of inertia, the "g" factors emphasized the change in force with a change in acceleration, while the jet engine and the forces of flight showed the concept of action and reaction. All of these laws can be demonstrated in the following example.

6.0 Measures of Performance

1 What does Newton's first law state?

ANSWER

"A body in motion at a uniform speed will remain in motion at that speed unless acted upon by an external force, and a body at rest will remain at rest unless acted upon by an external force."

2 What does Newton's second law state?

ANSWER

"Force is equal to mass times acceleration, or $F = ma$."

3 What does Newton's third law state?

ANSWER

"When one object exerts a force on another, the second object must exert an equal and opposite force on the first."

4 What is a "*g*"?

ANSWER

In aviation, the earth's gravational acceleration is referred to as a "*g*".

5 What is a "*g*" factor?

ANSWER

The "*g*" factor is actually a result of centripetal acceleration.

7.0 Example

Problem:

The pilot in the video said he weighs 155 pounds. The restraint system in the aircraft consists of five seatbelts (two shoulder belts, two lap belts and one negative "*g*" belt). How much force does each belt have to withstand to keep him from hitting the instrument panel if he experiences a positive "*g*" factor of +12 when the plane comes to a rapid stop during a crash landing?

Solution:

1. The pilot's mass is found by use of the second law:

$$F = ma$$

On Earth, one "*g*" is the acceleration of gravity (32.2 ft/sec^2) and the force is equal to his weight. Therefore, this mass is

$$w = mg$$

$$155 \text{ pounds} = m\,(32.2 \text{ feet/sec}^2)$$

$$m = 4.8 \text{ slugs}$$

2. Again using the second law, at a "*g*" factor of +12, the force is now

$$w = mg\,(\text{"}g\text{" factor})$$

$$w = (4.8 \text{ slugs})(32.2 \text{ feet/sec}^2)(12)$$

$$w = 1855.7 \text{ pounds}$$

3. This total force can be divided among the seat belts.

Caution:

One of the seatbelts is a "negative g" belt. Since the question states a "positive g-factor of 12" this belt should not be included in the calculations.

1855.7 pounds / 4 belts = 463.7 pounds per belt

4. Therefore each belt must be designed to be able to provide an opposing force of 463.7 pounds to contain the pilot's inertia.

NOTE:

The "negative *g"* belt only provides an anchor point for the lap belt and shoulder belt. This anchor prevents the belts form slackening during negative *g* maneuvers.

8.0 Suggested Activities

1 Have each student weigh themselves and determine their mass from the relationship

$$F_w = mg$$

$$m = \frac{F_w}{g}$$

2 Have each student determine how much they would weigh during a 2*g*, 4*g* and 9*g* turn.

Operational Supplement

Speed and Velocity

The simplest kind of motion that an object can have is a uniform motion in a straight line. This means an object moving in this manner is moving with a constant *velocity*. Constant velocity implies not only constant *speed*, but, unchanging *direction* as well. For this reason velocity is a vector quantity.

The speed of a moving body is the distance it moves per unit time in any arbitrary direction. If the speed is uniform, the object moves an equal distance in each successive unit of time. Speed is a scalar measurement since the direction of motion is immaterial. Whether or not the speed is constant, the *average speed* is the distance the body moves divided by the time required for the motion:

$$V_{avg} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t} \qquad (1)$$

where Δs is the distance traveled, V_{avg} is the average speed, and Δt is the elasped time. The British system unit of speed is the foot per second (ft/sec); the SI unit is the meter per second (m/sec); many other units are common, such as the mile per hour (mi/hr), centimeter per second (cm/sec), knot (kts), etc.

The terminology used above is very important. The concept of *speed* does not involve the idea of direction. A body moving with constant speed may move in a straight line or in a circle or in any one of an infinite variety of paths so long as the distance moved in any unit of time is the same as that moved in any other equal unit of time. The concept of *velocity* includes the idea of direction as well as magnitude. Hence we must consider the *displacement* of a body and not merely the *distance* traveled. The definition of average velocity, then, is given by:

$$\bar{V}_{avg} = \frac{\bar{s}_2 - \bar{s}_1}{t_2 - t_1} = \frac{\Delta \bar{s}}{\Delta t} \qquad (2)$$

The defining equation for average velocity (Equation 2) is different from the equation for average speed (Equation 1) in that $\bar{v}$ and $\bar{s}$ are vector quantities. The bar over the symbol is used to emphasize this fact. Constant velocity is a particular case of constant speed. Not only does the distance traveled in unit time remain the same, but, the direction is unchanged as well.

Accelerated Motion

Objects seldom move with constant velocity. In almost all cases, the velocity of an object is continually changing in magnitude or in direction or both. Motion in which the velocity is changing is called *accelerated motion*, and the rate at which the velocity changes is called the *acceleration*. The velocity of a body may be changed by changing the speed, by changing the direction, or by changing both speed and direction. If the direction of the acceleration is parallel to the direction of motion, only the speed changes, while, if the acceleration is at right angles to the direction of motion, only the direction changes. Acceleration in any other direction produces changes in both speed and direction. For the present, we will confine our attention to the simplest type of accelerated motion, called *uniformly accelerated motion*. In this case the direction is always the same and only the speed changes at a constant rate in the direction of the original motion. The acceleration in this case is equal to the rate of change of speed, since there is no change in direction. The acceleration is *positive* if the speed is increasing, *negative* if the speed is decreasing. Negative acceleration is sometimes called *deceleration*.

The acceleration of a body is defined as the time rate of change of velocity. Using algebraic symbols to represent average acceleration, the defining equation is written:

$$a_{avg} = \frac{V_f - V_i}{t} = \frac{\Delta V}{\Delta t} \qquad (3)$$

where a_{avg} represents the average acceleration, V_f the final velocity, V_i the initial velocity, and t the elapsed time. Since units of acceleration are obtained by dividing a unit of velocity by a unit of time, it may be seen that the British unit of acceleration is the foot per second per second (ft/sec^2) and the SI unit is the meter per second per second (m/sec^2).

Uniformly Accelerated Motion

Because they are often encountered, it is convenient to remember and list the equations for the special cases which apply to a body moving with constant acceleration in a straight line. If both sides of Equation 3 are multiplied by t, we obtain:

$$V_f - V_i = at \qquad (4)$$

which expresses the fact that the change in speed is equal to the rate of change in speed multiplied by the time during which it is changing. The distance traveled during any time is gotten by multiplying Equation 1 by t:

$$s = V_{avg}t \qquad (5)$$

But, the average speed V_{avg} must be obtained from the initial and final speeds V_i and V_f. Since the speed changes at a uniform rate, the average speed is equal to the average of the initial and final speeds:

$$V_{avg} = \frac{V_i + V_f}{2} \qquad (6)$$

By combining these equations, two other useful equations can be obtained. Eliminating V_f and V_{avg}, we obtain:

$$s = V_i t + \tfrac{1}{2}at^2 \qquad (7)$$

If we eliminate V_{avg} and t, we obtain:

$$V_f^2 = V_i^2 + 2as \qquad (8)$$

Of these five equations, Equation 5 is true for all types of motion; the remaining four equations hold only for *uniformly accelerated linear motion.*

Universal Gravitation

In addition to the three laws of motion, Newton formulated a law of great importance in mechanics, the law of *universal gravitation. Every particle in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the two particles and inversely proportional to the square of the distance between them.* This relation may be expressed symbolically by the equation:

$$F = \frac{Gm_1m_2}{s^2} \qquad (9)$$

where F is the force of attraction, m_1 and m_2 are the respective masses of the two particles, s is the distance between them, and G is a constant called the *gravitational constant.* The value of G depends on the system of units used in Equation 4. If the force is expressed in newtons, mass in kilograms, and distance in meters, G has the value $6.67 \times 10^{-11} m^3/kg\text{–sec}^2$. If the force is expressed in pounds, mass in slugs, and distance in feet, G has the value $3.42 \times 10^{-8} ft^4/lb\text{–sec}^4$.

Newton checked his law of gravitation by calculation and observation of the orbit of the moon. With the approximate data at his disposal, he still found reasonable agreement between his computations and his observations. Careful subsequent experimentation and measurement of the force of attraction between small bodies in the laboratory has further established the validity of the law of universal gravitation and led to the determination of the value of G given above.

Uniform Circular Motion

In *uniform circular motion*, the velocity vector remains constant in magnitude while the direction continually changes. Just as a force is required to change the speed of an object, so a force must also act to cause a change in the direction of the motion. Whenever the net force on a body acts in a direction other than the original direction of motion, it changes the direction of the motion. Such acceleration is very common, for it is present whenever a car turns a corner, an airplane changes its direction, or in any other similar motions.

Central Acceleration

When an object is moving in a circular path with constant speed, its *velocity* is continually changing. The acceleration produces a change in direction but no change in speed. Therefore, the acceleration must always be at right angles to the motion, since any component in the direction of the motion would produce a change in speed. The acceleration is always directed toward the center of the circle in which the body moves. It is constant in magnitude but contiually changing direction. In Figure 1 a body is moving with uniform speed, v, and constant angular speed, ω, in a circular path. The linear speed and angular speed are related by the equation:

$$V = \omega r \tag{10}$$

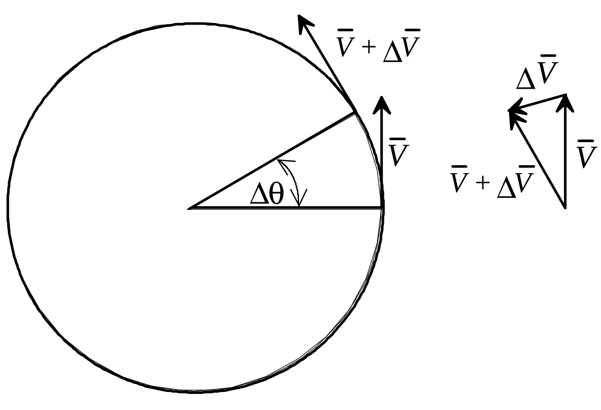


Figure 1 Uniform Circular Motion

where r is the radius of the circular path. The velocities of the object at points A and B are, respectively, $\bar{V}$ and $\bar{V} + \Delta\bar{V}$, equal in magnitude, but, differing in direction by a small angle $\Delta\theta$. In the vector triangle, $\Delta\bar{V}$

represents the change in velocity in the time Δt required for the object to to move from A to B. If the angle $\Delta\theta$ is small, the chord is approximately equal to the arc, and thus:

$$\Delta V = V\Delta\theta \tag{11}$$

But, since:

$$\Delta\theta = \omega\Delta t \tag{12}$$

Hence:

$$\Delta V = V\omega\Delta t = \omega^2 r\Delta t = \frac{V^2}{r}\Delta t \tag{13}$$

and:

$$\frac{\Delta V}{\Delta t} = \omega^2 r = \frac{V^2}{r} \tag{14}$$

As Δt is made smaller, the approximation in this equation becomes less and less, and the direction of $\Delta\bar{V}$ becomes more nearly perpendicular to that of $\bar{V}$. As Δt approaches zero, the instantaneous acceleration is found to be directed toward the center of the circle and is given by:

$$a_c = \frac{dV}{dt} = \omega^2 r = \frac{V^2}{r} \tag{15}$$

This equation states that the acceleration increases as the speed is increased and, for a given speed, is greater for a shorter radius. The acceleration is at right angles to the velocity and hence is directed toward the center of the circle. If the angular speed in Equation 15 is expressed in radians per second, then the units of a_c then depend upon the units in which r and V are expressed in. If the units of r are in feet and V in feet per second, then the units of a_c are in ft/sec^2 . If the units of r are in meters and V in meters per second, then the units of a_c are in m/sec^2.

<u>Centripetal Force</u>

According to Newton's laws of motion, any object that experiences an acceleration is acted upon by an unbalanced force, a force that is proportional to the acceleration and in the direction of the acceleration. The net force that produces the central acceleration is called the *centripetal force* and is directed toward the center of the circular path. Every body that moves in a circular path does so under the action of a centripetal force. A body moving with uniform speed in a circle is not in equilibrium. From Newton's second law, the magnitude of the centripetal force is given by:

$$F_c = ma_c = m\frac{V^2}{r} = m\,\omega^2 r \tag{16}$$

where m is the mass of the moving object, V is its linear speed, r is the radius of the circular path, and ω is the angular speed. If m is in slugs, V in ft/sec, and r in ft, then F_c is in lb. If m is in m/sec, V in m/sec, and r in m, then F_c is in newtons.

An inspection of Equation 16 discloses that the centripetal force necessary to keep a body in a circular path, as shown in Figure 2, is directly proportional to the square of the speed at which the body moves and inversely proportional to the radius of the circular path. If the speed is doubled, keeping the radius constant, the centripetal force becomes four times as great. If instead, the radius is cut in half, with the speed remaining constant, the centripetal force increases to twice as great. If at any instant the cord in

Figure 2 breaks, eliminating the centripetal force, the rock will retain the velocity it has at the instant the cord breaks and travel at constant speed along a line tangent to the circular path at that point. The act of throwing a baseball follows the exact same principle.

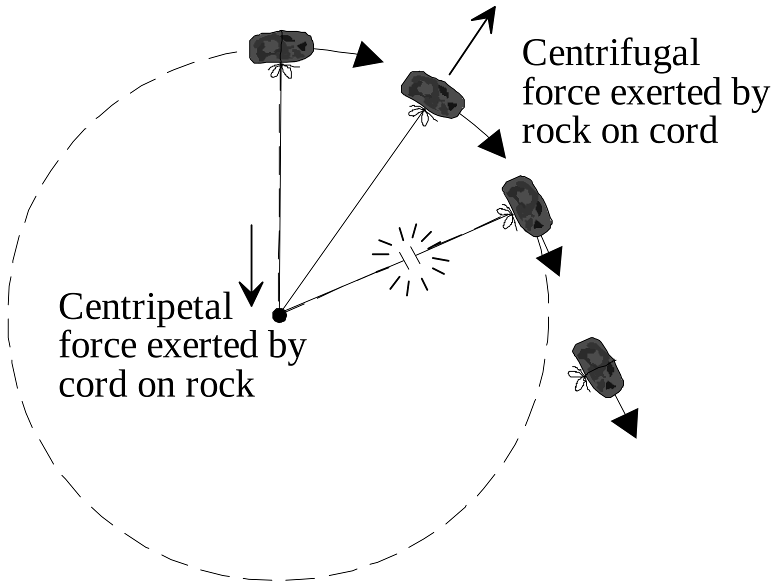


Figure 2 Centripetal and Centrifugal Forces

Work has been defined as the product of a force and a displacement in the direction of the force. Since centripetal force acts at right angles to the direction of motion, there is no displacement in the direction of the centripetal force, and it accomplishes no work. No energy is expended on or by an object while it is moving at constant speed in a horizontal circular path. This conclusion is consistent with the observation that, if the speed is constant, the kinetic energy of the body is also constant.

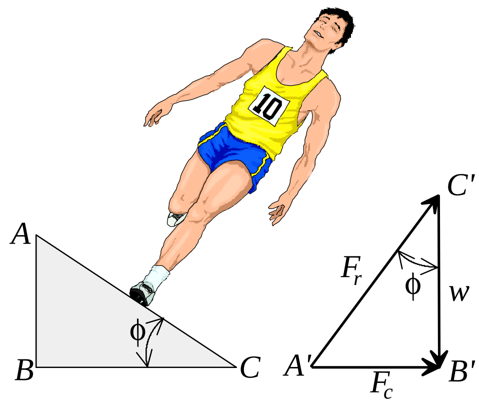
As the speed of a flywheel increases, the force needed to hold the parts of the wheel in circular motion increases with the square of the angular speed, as indicated by Equation 8.7. If the speed becomes high enough, the cohesive forces between the molecules of the material that the flywheel is made of are no longer sufficient and the wheel disintegrates, the parts flying off along tangent lines like mud from an automobile tire. Whenever news reports of an aircraft engine failure during flight, it is often due to rotating fan blades in the engine coming apart from the stresses created by the combination of heat and rotational forces.

When a container of liquid is being whirled in a horizontal circular motion, the container exerts an inward force on the liquid sufficient to keep it from spilling out. The bottom of the container presses on the layer of liquid next to it; that layer in turn exerts a force on the next; and so on. In each layer, the pressure must be the same all over the layer or the liquid will not remain in the layer. If the liquid is of uniform density, each element of volume with a mass m in a given layer will experience an inward force (mV^2/r) just great enough to maintain it in that layer and there will be no motion of the liquid from one layer to another. If, however, the layer is made up of a mixture of particles of different densities, the force required to maintain a given element of volume in that layer will depend upon the density of liquid in that element. Since the inward force is the same on all the elements in a single layer, there will be motion between the layers. For those elements which are less dense than the average, the central force is greater than that necessary to hold them in the layer; hence they are forced inward. For the elements more dense than the average, the central force is insufficient to hold them in the layer and they will move to a layer farther out. As rotation continues, the elements of the mixture become separated, with the least dense nearest the axis of rotation and the most dense farthest from the axis. This behavior is utilized to our advantage in the *centrifuge*, a device for separating liquids of different densities. Very high speed centrifuges may be used to separate gases of different densities.

Airplane pilots, who put their aircraft into a very tight turn or pull out of a steep dive at high speed, often experience centripetal accelerations several times as large as the acceleration due to gravity. Under these circumstances, the flow of blood to the pilot's brain is decreased unless other measures are taken to counteract these forces. Without a "*g*-suit" strapped to his torso, these high g-forces can cause the pilot to lose consiousness ("black out") during such periods of maximum acceleration.

Turns

A runner, in going around a curve, leans inward to obtain the centripetal force that causes him to turn as shown in Figure 3. The track must exert an upward force sufficient to sustain his weight, while at the same time it must provide a horizontal centripetal force. If the track is flat, the horizontal force must be entirely frictional. In that case, the frictional force may not be large enough to enable a sharp turn if the surface of the track were smooth. If the track is tilted from the horizontal, a portion of the horizontal force can be sustained by the horizontal component of the reaction force provided by the track surface while the remainder is still supplied by friction. If the angle of banking is properly selected, the force the track exerts, which is perpendicular to its surface, will be sufficient to provide the necessary horizontal force without friction.



(a) (b)

Figure 3 A Banked Turn

For this ideal case, as shown in Figure 3, the reaction force F_r of the track is perpendicular to the surface AC. The force due to the weight of the runner w is directed vertically downward. The resultant force F_c is the horizontal centripetal force. In the force triangle in Figure 3, the angle ϕ is the angle of bank of the track:

$$\tan v = F_c/w = \frac{mV^2/r}{mg} = \frac{V^2}{rg} \tag{17}$$

Equation 17 indicates that, since the angle of bank depends upon the speed, the curve can be ideally banked for only one speed. At any other speed, the force of friction must be depended upon to prevent slipping.

Let us now consider the turning flight of an airplane. In particular, we will only examine three specialized cases: (1) a level turn, (2) a pullup, and (3) an inverted pulldown (split-s). A study of the generalized motion of an airplane along a three-dimensional flight path is beyond the scope of this series.

A level turn is illustrated in Figure 4. Here the wings of the airplane are banked through the angle ϕ; hence the lift vector is inclined at the angle ϕ to the vertical. The bank angle ϕ and the lift L are such that the component of lift in the vertical direction exactly equals the force due to weight of the aircraft:

$$w = L \cos \phi \tag{18}$$

and therefore the airplane maintains a constant altitude, moving in the horizontal plane. The resultant of L and F_w leads to a resultant centripetal force F_c which acts in the horizontal plane causing the airplane to turn in a circular path with a radius of curvature equal to R and a turn rate of ω.

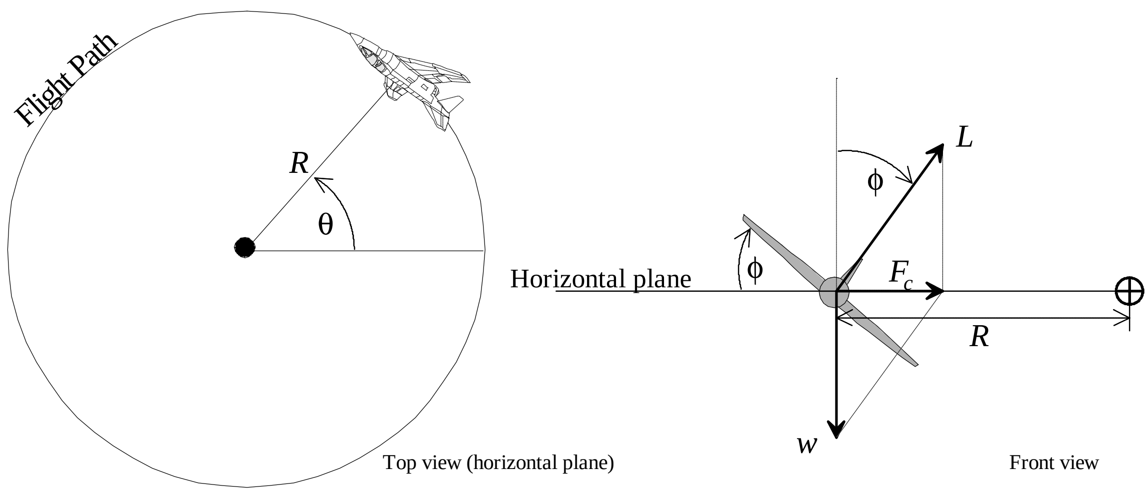


Figure 4 An Airplane in a Level Turn

From the force diagram in Figure 4, the magnitude of the resultant force is:

$$F_c = \sqrt{L^2 - w^2}$$

If we introduce a new term, the *load factor n*, defined as:

$$n \equiv L/w$$

and combine the above equation with Equation 18, we can show that load factor can be expressed as a function of bank angle only:

$$n = \frac{L}{L \cos v} = 1/\cos v \tag{19}$$

Load factor is usually quoted in terms of "g's"; for example, an airplane with lift equal to five times the weight is said to be experiencing a load factor of 5 g's. Hence, the centripetal force can be written as:

$$F_c = w\sqrt{n^2 - 1} \tag{20}$$

The airplane is moving in a circular path at the velocity V; therefore, the centripetal force can also be expressed from Equation 16 as:

$$F_c = m\frac{V^2}{R} = \frac{wV^2}{gR} \tag{21}$$

Combining Equations 20 and 21 and solving for R, we have:

$$R = \frac{V^2}{g}\sqrt{n^2 - 1} \tag{22}$$

And, the turn rate ω = V/R. Thus, from Equation 21, we have:

$$z = \frac{g}{V}\sqrt{n^2 - 1} \tag{23}$$

For the maneuvering performance of an aircraft, both military and civilian, it is frequently advantageous to have the smallest possible R and the largest posssible ω. Equations 22 and 23 show that, to obtain both a small turn radius and a large turn rate, we must have:

1. The highest possible load factor ($n = L/w$)
2. The lowest possible velocity

Consider the second case of a pullup maneuver where the airplane, initially in straight and level flight, suddenly experiences an increase in lift. Since the lift is greater than the weight of the airplane in this case, the airplane will begin to accelerate upward in a "vertical turn" or circular path in the vertical plane as shown in Figure 5. From the force diagram in Figure 5, the centripetal force F_c is vertical and is given by:

$$F_c = L - w = w(n - 1) \tag{24}$$

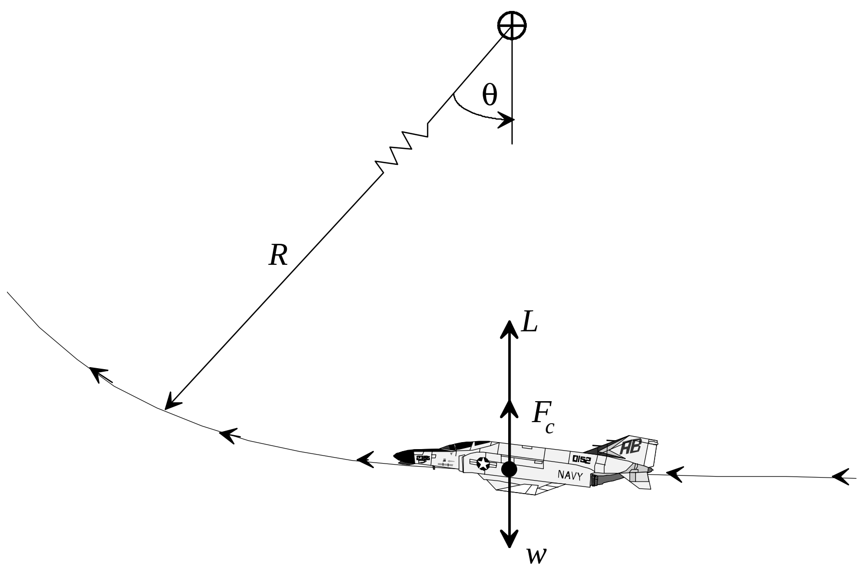


Figure 5 The Pullup Maneuver

We have from Equation 21:

$$F_c = m\frac{V^2}{R} = \frac{wV^2}{gR} \tag{21}$$

Combining Equations 21 and 24 and solving for R we get:

$$R = V^2/g(n-1) \tag{25}$$

And, the turn rate $\omega = V/R$. Thus, from Equation 25, we have:

$$\omega = g(n-1)/V \tag{26}$$

A related case is case 3, the inverted pulldown maneuver, illustrated in Figure 6. Here, an airplane, initially in straight and level flight, suddenly rolls to an inverted position, such that both L and F_w are pointing downward. The airplane will begin to turn, in the vertical plane, downward in a circular flight path with turn radius R and turn rate ω. By an analysis similar to the pullup above, the following results are easily obtained:

$$F_c = L + w = w(n+1) = \frac{wV^2}{gR} \tag{27}$$

$$R = V^2/g(n+1) \tag{28}$$

$$\omega = g(n+1)/V \tag{29}$$

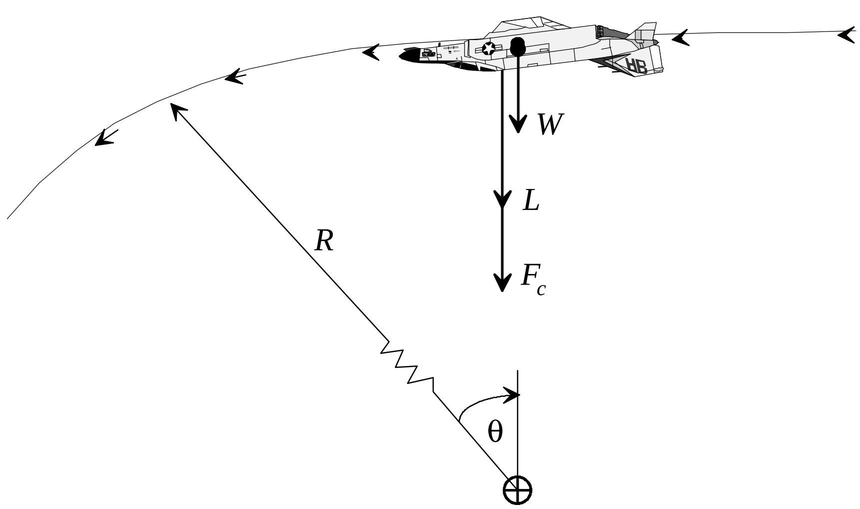


Figure 6 The Inverted Pulldown Maneuver

Considerations of turn radius and turn rate are particularly important to military fighter aircraft; everything else being equal, those airplanes with the smallest R and the largest ω will have definite advantages in air combat. High performance fighter aircraft are designed to operate at high load factors, typically from 5 to 9 g's; and if the turn is accomplished at the exact speed where the aerodynamic lift generated by the wing is sufficient to produce the maximum g at the minimum speed, the tightest turn will result with the aircraft possessing its highest energy level. This speed is often referred to as the "corner velocity" of the aircraft.

Curvilinear Motion

Frequently the net force acting on a body is neither parallel to the direction of its motion nor at right angles to that direction. In this case, neither the speed nor the direction remains constant. Such motion may be readily studied by considering two components of the acceleration, one parallel to the original direction of motion, the other perpendicular to that direction.

One of the most common of such motions is planetary motion, in which the force on the moving body is inversely proportional to the square of the radius and always directed toward a fixed point. The body travels in an ellipse, the fixed point being at one focus. The speed of the moving body is greatest when the body is nearest the focus, less when it is further away. This motion is called planetary motion because the planets move in this manner in their journeys around the sun. Comets have much more elongated elliptical paths that carry them outside the solar system at their furthest distance from our sun. Since electrified particles show a similar law of attraction, we should expect them to behave in the same manner as those moving under the action of gravitational forces.

Another simpler example of curvilinear motion that is closer to home is projectile motion. The science of the motion of projectiles is called *ballistics*. The simplest type of ballistic motion is that in which the projectile is given an initial velocity and then allowed to move under the influence of gravity alone. True projectile motion is that in which an object is given an initial velocity and then allowed to proceed under the action of gravity and also air resistance. Other objects which are self-propelled, such as rockets and missiles, move in the same manner as projectiles except that they do not depend upon an initial impulse alone, but also upon a sustained force throughout most of its flightpath. The initial speed of the rocket or missile may be quite low since it is continually gaining speed along its path.

All of these examples of curvilinear motion are outside the scope of this series. They are mentioned here to provide a knowledge of their existence.

Summary

In uniform circular motion: (a) the speed V is constant; (b) the direction of the motion is continually and uniformly changing; and (c) the acceleration a_c constant in magnitude and is directed toward the center of the circular path. The magnitude of the *central acceleration* is given by:

$$a_c = \frac{V^2}{r} = \mathrm{z}^2 r \tag{15}$$

where V is the linear speed, r is the radius, and ω is the angular speed.

The *centripetal force*, the inward force that causes the central acceleration, is given by:

$$F_c = m\frac{V^2}{r} = m\,\mathrm{z}^2 r \tag{16}$$

The proper banking of a curve to eliminate the necessity for a horizontal frictional force is given by the relation:

$$\tan \mathrm{h} = \frac{V^2}{gr} \tag{17}$$

The *load factor* being pulled by an airplane in *level turning flight* is defined as:

$$n = L/w = 1/\cos\phi \tag{19}$$

The turn radius is given by:

$$R = \frac{V^2}{g}\sqrt{n^2 - 1} \tag{22}$$

and the turn rate is:

$$z = g\sqrt{n^2 - 1}\,/V \tag{23}$$

The turn radius and turn rate for a pullup is given by:

$$R = V^2/g(n-1) \tag{25}$$

$$\omega = g(n-1)/V \tag{26}$$

And, the turn radius and turn rate for an inverted pulldown is given by:

$$R = V^2/g(n+1) \tag{28}$$

$$\omega = g(n+1)/V \tag{29}$$

Often in curvilinear motion, the accelerating force is neither parallel nor perpendicular to the direction of motion. In this case, the acceleration produces change in both speed and direction of motion.

Session 2
Weight and Balance

1.0 Definitions

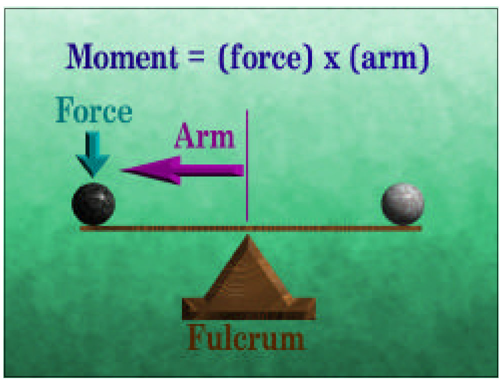
Center of gravity (*cg*) - The point about which the plane would balance if it were possible to suspend the plane at that point; the mass center of the plane at which the entire weight of the plane is assumed to be concentrated.

Center of gravity limits - The specified forward and aft points within which the *cg* must be located during flight.

Reference datum line (RDL) - An imaginary vertical line from which all arm measurements are taken.

Arm - The horizontal distance from the Reference Datum Line to the *cg of any particular item.*

Moment - The product of a force (or weight of an item) multiplied by its arm. The total moment of an object is the weight of the object multiplied by the length of the arm from the RDL to the *cg*.



Moments and Arms

Control - The ability to generate desired movements through the use of forces.

Fulcrum - The pivot point of a lever; balance point of a beam.

Longitudinal axis - An axis of rotation through the *cg* which runs from nose to tail of the aircraft. (Figure 2.1)

Lateral axis - An axis of rotation through the *cg* which runs from wingtip to wingtip of an aircraft. (Figure 2.1)

Directional axis - An axis of rotation perpendicular to the longitudinal and lateral axis which runs vertically through the center of gravity. (Figure 2.1)

NOTE:

All axes pass through the center of gravity and are perpendicular to each other at that point.

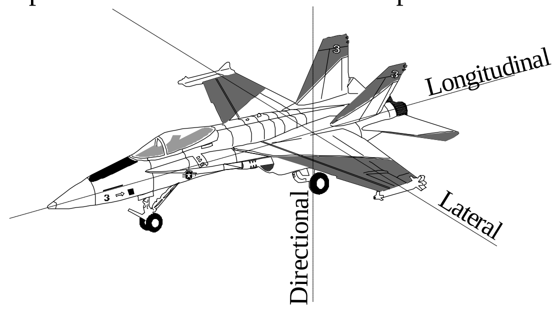


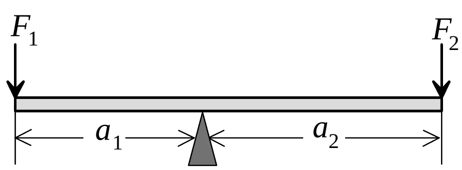
Figure 2.1 Aircraft Axis

NOTE:

Session 3 of the text should be reviewed prior to starting the video.

2.0 Balancing Forces and Moments

During the first session we determined that an object's weight is a measure of the force it exerts on the Earth. We also saw how according to Newton's third law, forces exist in "equal and opposite" pairs. Any time an out of balance force exists, there is an acceleration in the direction of the greater force. Many times when we use a see-saw we are faced with two forces (or weights) which are not equal. Then to level the board over the fulcrum, we used the concept of balanced moments, as shown in Figure 2.2



$F_1a_1 = F_2a_2$

Figure 2.2 Balanced Moments of a See-Saw

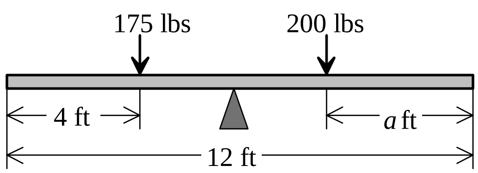
Recall that a moment is the product of a force multiplied by a distance, or arm. Therefore it stands to reason that a smaller force acting at a greater distance could quite easily balance a larger force acting at a smaller distance.

NOTE:

Mechanics often refer to a moment as a "torque" and will list automobile performance in the form of "foot-pounds of torque." Additionally, the "torque wrench" used in auto repair is simply the mechanic's arm strength applied over the known length of the wrench. A gauge indicates "foot-pounds or inch-pounds" of torque based on the amount of force applied by the mechanic.

The following example highlights how moments are balanced.

***Example 1*:** A person weighing 175 pounds sits 4 feet from the end of a bench seat. A second person weighing 200 pounds wants to sit at the opposite end of the bench. If the fulcrum is in the middle of the 12 foot bench, how far from the end should the 200 pound person sit for the bench to remain level?



***Solution*:**

1. Calculate the moments on the left side of the fulcrum:

   force × *arm* = *moment*

   (175 lbs) × (6 ft – 4 ft) = 350 ft-lbs

Caution:

The question is asking how far from the END of the board should the person sit. Since in this case we are balancing the bench with reference to the fulcrum, and the fulcrum is in the middle of the board, the arm is subtracted from half the board length.

2. To balance the bench, the moments on the left must equal the moments on the right, therefore:

   $350\ ft\text{-}lbs = (200\ lbs) \times (6\ ft - a\ ft)$

   $350\ ft\text{-}lbs = 1200\ ft\text{-}lbs - 200\ lbs \times (a)ft$

   $(350\ ft\text{-}lbs) - (1200\ ft\text{-}lbs) = -200\ lbs \times (a)ft$

   $$\frac{-850ft - lbs}{-200lbs} = 4.25ft = (a)ft$$

3. Therefore, the 200 pound person should sit 4.25 feet from the right end of the bench.

4. If the fulcrum is in the middle, 6 ft are on each side. Therefore the distance from the fulcrum is 6 - 4.25 ft. = 1.75 ft.

Proof:

175 lbs. × 2 ft. = 200 lbs. × 1.75 ft

300 ft-lbs = 300 ft-lbs

In the above example, the arms were measured with respect to the pivot point. However when dealing with aircraft we're trying to FIND the balance point, or more specifically, the center of gravity. Therefore the arms are measured with respect to the reference datum line (RDL). This imaginary line is usually located at the nose of the aircraft and is used solely as a reference point for calculating the center of gravity.

3.0 Significance of Weight & Balance

Determining an aircraft's total weight and the location of the center of gravity is crucial to predicting the aircraft's performance and controllability. As we will see in future sessions, an increase in an aircraft's weight has a direct impact on the following areas of that plane's performance:

higher takeoff speed
longer takeoff run
longer landing roll

How the weight is distributed aboard an aircraft is in part determined by the pilot and in part determined by the designer. The pilot can affect how much fuel, people, and cargo is put onboard but the designer decides where the fuel, people, and cargo are placed. The designer's decisions are based upon being able to balance the airplane and control the aircraft's movements. Uppermost in the designer's concern for balancing the aircraft is the fore and aft location of the center of gravity along the longitudinal axis of the aircraft. Balancing the aircraft results when the sum of the moments around the center of gravity equals zero, often written as $\sum M_{cg} = 0$. However, balancing the aircraft on the lateral axis left and right of the longitudinal axis is also very important. Each item in an aircraft has weight and subsequently exerts a force at a specific location on the plane. Resolving these forces into one resultant force acting at a specific location will yield the center of gravity of the airplane.

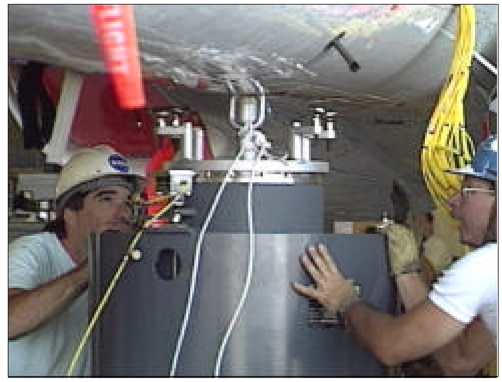
Caution:

The student should be familiar with the method involved in resolving parallel forces into a single resultant force. See Example 2.1 and 2.2 of the Operational Supplement for an explanation of resolution of forces.

The *cg* is not necessarily a fixed point for every loading condition; its location depends on the weight distribution of the aircraft. As fuel is burned throughout a flight or passengers change seats, the *cg* shifts accordingly. The designer has accounted for this movement to a certain degree by providing the pilot with a range of acceptable *cg* locations where aircraft control may be retained. The amount of control a pilot has is a function of the size of the control surfaces and how large of a moment these surfaces can generate. Control surfaces include the elevator on the tail which creates a force to rotate the aircraft about the lateral axis; ailerons on the wing which rotate the aircraft about the longitudinal axis; and the rudder on the tail which rotates the aircraft about the directional axis. In order to generate a moment, the control surface must create a force located at a distance from the center of gravity. When an aircraft is in flight, any force exerted by a control surface tends to rotate the aircraft around the center of gravity making knowledge of the *cg* location critical. Determining the location of the *cg* begins with weighing the aircraft.

4.0 Weighing An Aircraft

Determining the weight of an aircraft is simply a matter of summing forces. A scale is placed under each point where the aircraft touches the ground, and the readings of all the scales are then added together.



Weighing the Aircraft

Caution:

This may seem very basic however it is very important to place a scale under each point of ground contact. Failure to do so will result in an erroneous total.

Some aircraft have very unusual landing gear arrangements. For example, the U.S. Air Force B-52 bomber has an “outrigger” landing gear under each wingtip. These support the weight of the wings when they are full of fuel. Therefore, in order to get the total weight of a B-52, a scale would also have to be placed under each “outrigger” gear. The video depicts the procedures involved with weighing NASA’s F-18 High Angle of Attack Research Vehicle (HARV). The aircraft was lifted by a crane and large scales were placed under the landing gear.

Lifting Aircraft

Once the total weight of the aircraft is determined, determining the center of gravity location is accomplished through resolution of the forces into a resultant force acting at the *cg* of the aircraft.

****STOP VIDEO after two lines marked on metal****

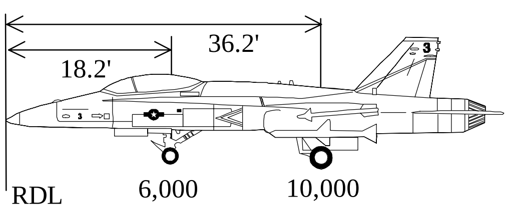
cg of irregular shape

NOTE:

The video also demonstrated an experimental way of determining the *cg* of an irregular shaped object. Further explanation may be found in the Operational Supplement for this session.

***Example 2*:** Determine the center of gravity location of the F-18 HARV given the following weights and arms:

item	weight (lbs)	arm (ft)	moment (ft-lbs)
Nose Wheel	6,000	18.2	
Main Wheels (ea.)	10,000	36.2	
Total	**26,000**		



F-18 HARV Weights

Solution:

Using the relationship that the $cg = \frac{\Sigma moment}{\Sigma weight}$ we:

First, find the total weight and moment for the entire aircraft.

item	weight (lbs)	arm (ft)	moment (ft-lbs)
Nose Wheel	6,000	18.2	109,200
Main Wheels (ea.)	10,000	36.2	362,000
Total	**26,000**		**833,200**

Second, by dividing the total moment by the total weight, the location of the *cg* is found.

$$\frac{833,200 ft-lbs}{26,000 lbs}$$

= 32.04*ft* from the Reference Datum Line

NOTE:

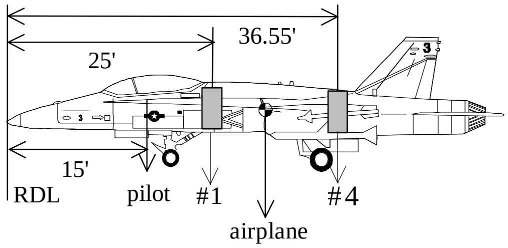
For the purposes of this example, the Reference Datum Line is assumed to be at the nose of the aircraft. The arm to the nose wheel is 18.2 feet and the arm to the main wheel is 36.2 feet. Therefore, the *cg* is 32.04 feet from the nose.

****START VIDEO****

As stated previously, the designer accounts for movement of the *cg* in flight by providing an acceptable *cg* range where control of the aircraft can be maintained. When fuel is burned, weight is removed, so there is less force acting at a given point on the aircraft. The *cg* location will therefore change. For the aircraft to be balanced in flight, the moments forward of the *cg* must be equally opposed by the moments aft of the *cg*. The following example will highlight how center of gravity moves in flight.

***Example 3*:** Given the following items and associated arm lengths, calculate how much the center of gravity moves when all of the fuel is burned from the # 1 fuel tank.

item	weight (lbs)	arm (ft)	moment (ft-lbs)
Empty airplane	26,000	32.04	833,200
pilot	155	15.00	
#1 fuel tank	2,150	25.00	
#4 fuel tank	3,620	36.55	



F-18 HARV Loading Diagram

Solution:

Again using the relation that the

$$arm_{cg} = \frac{total\ moment}{total\ weight}$$

1. Calculate the moment for each item:

pilot	155 lbs	× 15.0 ft	=	2,325	ft-lbs
#1	2,150 lbs	× 25.0 ft	=	53,750	ft-lbs
#4	3,620 lbs	× 36.5 ft	=	132,311	ft-lbs

2. Determine the current *cg* location

item	weight (lbs)	moment (ft-lbs)
Empty airplane	26,000	833,200
pilot	155	2,325
#1 fuel tank	2,150	53,750
#4 fuel tank	3,620	132,311
Total	**31,925**	**1,021,586**

$$\frac{1,021,586 ft - lbs}{31,925 lbs}$$

= 32 feet from the Reference Datum Line

NOTE:

As you can see even adding fuel and pilot, the *cg* only moved 0.4 feet (4.8 inches) compared to the over all length of the airplane. This is negligible.

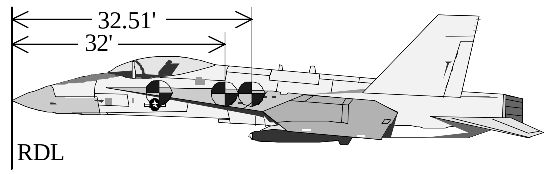
3. Determine how much the *cg* moves when the fuel in # 1 tank is burned off. To do this, simply subtract the weight of the fuel in # 1 tank from the total weight, and subtract the moment from the total moment. Then calculate the new *cg* by dividing the new moment by the new weight.

$$31,925\ lbs - 2150\ lbs = 29,775\ lbs$$

$$1,021,586\ ft\text{-}lbs - 53,750\ ft\text{-}lbs = 967,836\ ft\text{-}lbs$$

New *cg* location is

$$\frac{967,836 ft - lbs}{29,775 ft} = 32.51 ft$$



The movement of the center of gravity may not appear to be very significant, however if the airplane is to stay balanced (remain in level flight) the force that has been lost due to removal of the fuel weight, must be replaced with a force created by the tail. If the amount of weight removed cannot be replaced by a force generated by the tail, the aircraft experiences what is termed “loss of control authority.” What this really means is the *cg* has moved to a location where the force created by the tail is no longer sufficient to keep the plane level.

Downforce Created by Tail

Consider the following:

Example 4: Assuming the fuel has burned out of Tank # 1, as shown in Example 3, how much force must be generated by the tail to keep the *cg* in the same location (32 feet) if the tail is located 51 feet from the Reference Datum Line?

***Solution*:**

Using the basic relationship $arm_{cg} = \frac{total\ moment}{total\ weight}$

1. Determine the total moment lost when the fuel in # 1 tank burned.

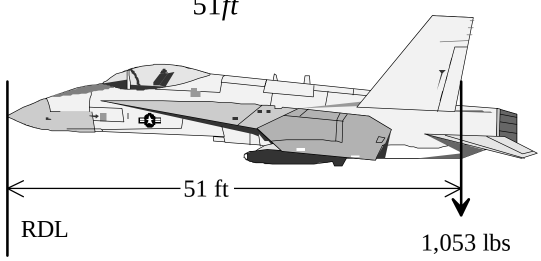
item	weight (lbs)	arm (ft)	moment (ft-lbs)
#1 fuel tank	2,150	25.00	53,750

2. Determine the tail force required.

The moment which must be replaced to keep the *cg* at 32 feet is found by step 1. Since the tail has an arm of 51 feet, the force is found by dividing the moment needed by the arm length.

$$\frac{moment}{arm} = force$$

$$\frac{53,750ft - lbs}{51ft} = 1053lbs$$



Then to keep the aircraft in level flight, the tail must generate 1053 pounds of force. The total amount of force a tail can generate is based on a number of factors including the speed, distance of the tail from the reference line, and the size of the tail. Based on these factors the designer sets the amount the *cg* can move in flight since he has calculated the maximum amount of up (or down) force the tail can generate. A force generated by an aerodynamic surface, such as a wing or tail, is termed lift. How lift is generated is the subject of the next session.

5.0 Measures of Performance

1 What is a moment?

ANSWER

Moment - The product of a force (or weight of an item) multiplied by its arm. The total moment of an object is the weight of the object multiplied by the length of the arm from the RDL to the *cg*.

2 What is the relationship between the flight control surfaces and the *cg*?

ANSWER

The designer has accounted for this movement to a certain degree by providing the pilot with a range of acceptable *cg* locations where aircraft control may be retained. The amount of control a pilot has is a function of the size of the control surfaces and how large of a moment these surfaces can generate. In order to generate a moment, the control surface must create a force located at a distance from the center of gravity. When an aircraft is in flight, any force exerted by a control surface tends to rotate the aircraft around the center of gravity making knowledge of the *cg* location critical.

6.0 Suggested Activity

1 A suggested activity is to have each student determine the center of gravity of an object found in the classroom by both the experimental and the analytical methods.

2 Calculate the *cg* of a model car or plane.

- **a** Place a postal scale under each wheel. Add the readings of each scale to get the model total weight.
- **b** Now measure the location of the wheel with respect to the nose of the model, i.e., find the arm of each wheel.
- **c** Generate a chart similar to that of Example 3. Divide the total moment by the total weight and get the *cg* location.
- **d** Suspend the model by a string located at the calculated *cg* location and determine if the model is level. If not, remeasure and try again.

Session 2
Weight and Balance

Operational Supplement

Center of Gravity

We often represent the weight of a body by a single force $\overline{w}$, acting downward. Actually, the earth exerts a force of attraction on each particle of a body; the weight of the body results from adding all the forces that act on all of the particles of the body. The weight $\overline{w}$ not only has magnitude and direction, but, it has a line of action which passes through a special point in the body known as the *center of gravity*.

A single force $\overline{\boldsymbol{F}}$ acting vertically upward can be used to support a body of weight $\overline{w}$. The first condition of equilibrium states the vector sum of all the external forces acting on the body must be zero, so the magnitude of the force $\overline{F}$equals the weight $\overline{w}$. This condition, however, is not sufficient to ensure equilibrium. The second condition of equilibrium states the vector sum of all the moments which result from these forces, must equal zero. To accomplish this the forces must be equal in magnitude and opposite in direction. When only two forces act on a body, this second condition of equilibrium can be fulfilled only if $\overline{F}$ and $\overline{w}$ act along the same straight line. If the force $\overline{F}$ is applied at any arbitrary point A in the body shown in Figure 1(a), the body will, in general, rotate about point A as an axis and then ultimately come to rest in an orientation which places $\overline{F}$ and $\overline{w}$ along the same line of action as in Figure 1(b). If the body is now supported at some other point B, the body will rotate about point B as an axis and ultimately come to rest in an orientation which again places $\overline{F}$ and $\overline{w}$ along the same line of action as in Figure 1(c). The lines which pass through A and B intersect at a point C which is the center of gravity (*cg*) of the body. If a single force $\overline{F} = -\overline{w}$ could be applied at the center of gravity C, the body will be in equilibrium no matter how it is oriented as shown in Figure 1(d).

In many cases of practical interest, the position of the center of gravity of a body can be calculated with the aid of a simple theorem that states: *The moment about any axis produced by the weight of the body acting through the center of gravity must equal the sum of the moments about the same axis produced by the weights of the individual particles of the body.*

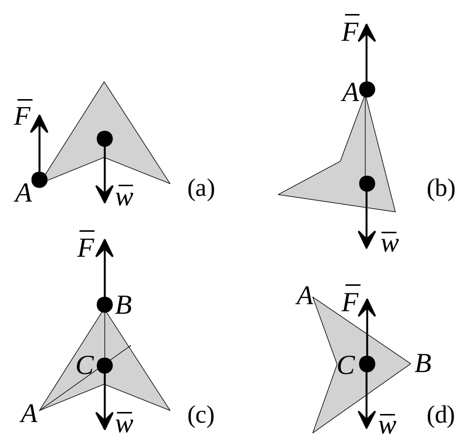


Figure 1 Determining the Center of Gravity

Example 1. Let's assume that we want to know the weight and center of gravity of an empty passenger aircraft sitting on the ground at Kennedy International Airport in New York as depicted in Figure 2. Since the aircraft is at rest, we know that weight of the aircraft is supported by the forces exerted by the pavement beneath each landing gear. We also know from the above theorem that the sum of the clockwise moments produced by each gear about some axis of rotation, say, the tip of the nose of the aircraft, is exactly balanced by the counterclockwise moment produced by the weight of the aircraft acting through the center of gravity. If we pulled the aircraft onto a set of platform scales and measured the gear reaction forces as 100,000 lb for each of the two main gear and 25,000 lb for the nose gear and the distances of the main and nose gear aft of the nose of the aircraft were measured to be 50 ft and 10 ft, respectively, we would get the following results:

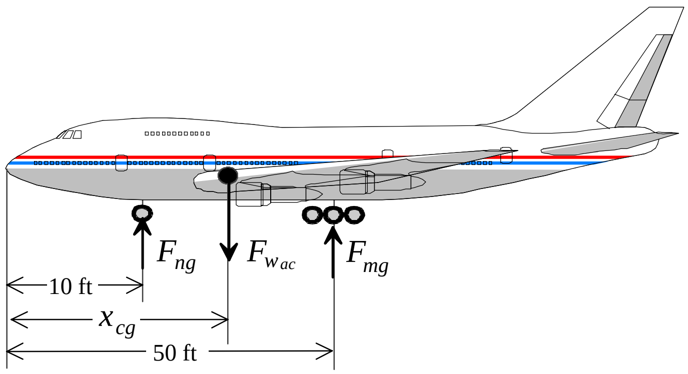


Figure 2 Measuring Weight and Center of Gravity

$$\Sigma\overline{F} = \overline{F}_{w_{ac}} + \overline{F}_{ng} + \overline{F}_{mg} = 0$$

$$w_{ac} = -F_{w_{ac}} = F_{mg} + F_{ng} = 2 \times 100,000lb + 25,000lb = 225,000lb$$

$$\Sigma\overline{M} = \overline{M}_{w_{ac}} + \overline{M}_{ng} + \overline{M}_{mg} = 0$$

$$-M_{w_{ac}} = -F_{w_{ac}} \times x_{cg} = M_{ng} + M_{mg}$$

$$225,000lb \times x_{cg} = 2 \times 100,000lb \times 50ft + 25,000lb \times 10ft = 10.25 \times 10^6 ft - lb$$

$$x_{cg} = 10.25 \times 10^6 ft - lb/225,000lb = 45.56ft$$

We now know that the aircraft, empty of fuel, passengers, and baggage weighs 225,000 lb and has a *cg* 45.56 ft aft of the nose. If we then fuel the aircraft with 40,000 gal of jet fuel weighing 6.25 lb/gal in fuel tanks that have a centroid (center of the volume or mass) location of 40 ft aft of the nose and load the aircraft with 200 passengers weighing an estimated total of 40,000 lb with a centroid of 55 ft aft of the nose and 10,000 lb of baggage in a baggage hold with a centroid of 50 ft aft of the nose, what would be the engine-start gross weight and *cg*? The results are:

$$\overline{F}_{w_{tot}} = \overline{F}_{w_{ac}} + \overline{F}_{w_f} + \overline{F}_{w_p} + \overline{F}_{w_b}$$

$$F_{w_{tot}} = 225,000lb + 40,000gal \times 6.25\tfrac{lb}{gal} + 40,000lb + 10,000lb = 525,000lb$$

$$\overline{M}_{w_{tot}} = \overline{F}_{w_{ac}} \times x_{cg} + \overline{F}_{w_f} \times x_f + \overline{F}_{w_p} \times x_p + \overline{F}_{w_b} \times x_b$$

$$M_{w_{tot}} = (225,000lb \times 45.56lb) + (40,000gal \times 6.25\tfrac{lb}{gal} \times 40ft) + (40,000lb \times 55ft) + (10,000lb \times 50ft)$$
$$= 22.95 \times 10^6 ft - lb$$

$$x_{cg} = M_{w_{tot}}/F_{w_{tot}} = 22.95 \times 10^6 ft - lb/525,000lb = 43.71ft$$

So, the fully loaded aircraft has a weight of 525,000 lb and a *cg* 43.71 ft aft of the nose. Notice that the c.g. is at a location forward of the main gear. What would happen if the *cg* was aft of the main gear?

Example 2. Let's take the same fully loaded aircraft in the above example and look at forces acting on the aircraft after takeoff and after it has levelled off at cruise altitude with 4,000 lb of fuel having been burned to get there as shown in Figure 3 . The aircraft is in stabilized, level flight, such that the weight of the aircraft is supported by the lift forces being generated by the wing and the tail. If the center of pressure on the wing (where the resultant wing lift force acts) is located at 45.0 ft aft of the nose of the aircraft and the center of pressure of the tail is 105 ft aft of the nose, what are the magnitude and direction of the forces acting on the wing and tail? Again, the sum of the lift forces on the wing and tail are equal and opposite to the weight of the airplane and the sum of the clockwise moments produced by the lift forces about the nose is exactly balanced by the counterclockwise moments produced by the weight of the aircraft acting through the center of gravity. We must first recalculate the weight and center of gravity of the aircraft after 4,000 lb of fuel have been consumed:

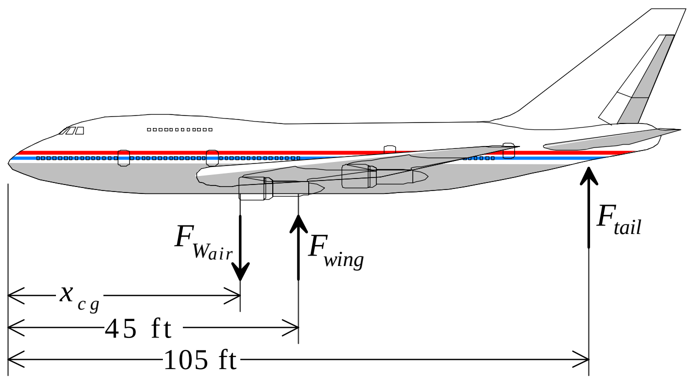


Figure 3 Level Flight Forces and Moments

$$F_{w_{air}} = F_{w_{gnd}} - F_{w_{fu}} = 525,000lb - 4,000gal \cdot 6.25\tfrac{lb}{gal} = 500,000lb$$

$$M_{w_{air}} = F_{w_{air}} \cdot x_{cg} = F_{w_{ac}} \cdot x_{ac} + F_{w_f} \cdot x_f + F_{w_p} \cdot x_p + F_{w_b} \cdot x_b$$

$$M_{w_{air}} = 500,000lb \cdot x_{cg} = 225,000lb \cdot 45.56ft + 36,000gal \cdot 6.25\tfrac{lb}{gal} \cdot 40ft + 40,000lb \cdot 55ft$$
$$+10,000lb \cdot 50ft = 21.95 \cdot 10^6 ft - lb$$

$$x_{cg} = M_{w_{air}}/F_{w_{air}} = 21.95 \cdot 10^6 ft - lb/500,000lb = 43.90ft$$

We must now express the equilibrium conditions of level flight in terms of the force and moment equations:

$$\Sigma\overline{F} = \overline{F}_{w_{air}} + \overline{F}_{wing} + \overline{F}_{tail} = 0$$

$$\Sigma\overline{M} = \overline{F}_{w_{air}} \cdot \boldsymbol{x}_{cg} + \overline{F}_{wing} \cdot \boldsymbol{x}_{wing} + \overline{F}_{tail} \cdot \boldsymbol{x}_{tail} = \mathbf{0}$$

Solving these equations simultaneously for the lift forces on the wing and tail, we get:

$$500,000lb = F_{wing} + F_{tail}$$

$$500,000lb \cdot 43.90ft = F_{wing} \cdot 45ft + F_{tail} \cdot 105ft$$

And by substitution, we can solve for the lift forces on the wing and tail :

$$500,000lb \cdot 43.90ft = F_{wing} \cdot 45ft + (500,000lb - F_{wing}) \cdot 105ft$$

$$F_{wing} = 500,000lb \,\$ (105ft - 43.90ft)/(105ft - 45ft) = 509,167lb$$

$$F_{tail} = 500,000lb - 509,167lb = -9,167lb$$

So, the lift force on the wing is 509,167 and the lift force on the tail is - 9,167 (a downward force). This is so, because the center of gravity of the aircraft was forward of the center of pressure of the wing, requiring a counterclockwise moment by the tail to balance the moment equation.

Session 3
Developing Lift

1.0 Definitions

Acceleration (*a*) - The rate of change of velocity with respect to time, or the change in velocity over a given period of time.

Airfoil - The two-dimensional profile of a wing section.

Coefficient of lift (C_L) - A measure of how efficiently a wing transforms dynamic pressure into a lift force; a proportionality constant which measures how much the pressure changes between the top and bottom of the airfoil.

Density (r) (pronounced "row") - The mass of a substance divided by a unit of volume.

Dynamic pressure (*q*) - The force exerted by a gas in motion.

Momentum (*mV*) - The product of the mass of a substance multiplied by the velocity.

Nautical mile - 6,076 feet. Equal to a one minute arc of latitude at the Earth's equator.

Knot - a measure of speed given as nautical miles per hour.

Static pressure (P_S) - The force exerted by a gas at rest.

Total pressure (P_T) - The sum of the static and dynamic pressures.

Velocity (*V*) - The rate of change of distance with respect to time, or the change in distance over a given period of time.

Wing area - The amount of wing surface an aircraft possesses.

NOTE:

An indepth development of lift is included in sections 7.1 through 7.3 of the text.

2.0 Bernoulli's Equation

Development of the Pressure Relationship

In the mid-1800's, a scientist by the name of Daniel Bernoulli used Newton's second law to mathematically explain the pressure relationship between a moving fluid and a fluid at rest. Bernoulli phrased the relationship as;

"The pressure of a mass of moving fluid in an open area is a constant; and that constant is the sum of the static pressure plus the dynamic pressure."

Static pressure is presented to most people daily in the form of the barometer reading given by the local weather forecaster. This reading is in the form of inches (or millimeters) of mercury which can be directly converted to a pressure, normally, 14.7 lbs/sq. in. at sea level. Changes in static pressure can be felt when our ears feel like they have filled up, causing us to "pop" our ears. Dynamic pressure is perhaps a bit more obscure, but none the less common. When you put your hand out the window of a moving car, you appear to feel the "force of the wind" pushing your hand backward. What you are really experiencing is the dynamic pressure (*q*) which is the result of the velocity of the air mass (or in this case, the velocity of the car through the air mass). The only time dynamic pressure can be measured is when the flow of the air mass is brought to rest upon some type of measuring device. In fact, Bernoulli determined that dynamic pressure can be given the numerical value of:

$$q = 1/2\ (\rho)\ V^2$$

where ρ is the fluid density and *V* is the velocity of the fluid mass.

Then Bernoulli's relationship can be written mathematically as:

$$\text{constant} = P_S + q$$

or

$$\text{constant} = P_S + 1/2\ (\rho)\ V^2$$

where P_S is the static pressure of the fluid.

Since the constant is just the sum of the static and dynamic pressures, it is given the name Total Pressure or P_T. Therefore:

$$P_T = P_S + 1/2\ (\rho)\ V^2$$

From this relationship, you can see that if the left side of the equation is to remain constant, when one of the pressures on the right increases, the second pressure on the right must make a corresponding decrease. Consider the following example:

Example**:** A man is standing still in calm air on the beach holding a barometer which indicates 29.92 inches of mercury. If one inch of mercury equals 0.4912 pounds/square inch of pressure, what is the total pressure the man experiences?

Solution**:**
1. From Bernoulli's equation

$$P_T = P_S + 1/2\ (\rho)\ V^2$$

we can see

$$P_T = 29.92 \text{ inches of Hg} + 0$$

so

$$P_T = (29.92 \text{ inches of Hg})\ (0.4912 \text{ inches of Hg/Pounds per square inch})$$

$$P_T = 14.7 \text{ pounds/square inch}$$

Caution:

The man is standing still in calm air, therefore there is NO dynamic pressure. As a result, the total pressure is simply equal to the static pressure.

In the above example neither the man nor the air was moving. Therefore since the velocity was zero, there was no dynamic pressure to be considered. The early aviation pioneers built upon Bernoulli's equation and went back to Newton's second law to develop the origins of lift.

****START VIDEO****

3.0 Lift and the Rate of Change of Momentum

We have written Newton's second law as $F = ma$, however we know that acceleration, a, is the rate of change of velocity, V, with respect to time. In other words, acceleration is the measure of how the velocity changes written as "ΔV" or "dV" over a given period of time written as "Δt" or "dt." We also know that momentum is the product of an object's mass multiplied by its velocity, or (mV).

Incorporating these relationships into the equation for Newton's second law, we can rewrite the law as:

$$F = (mass) \times (\Delta Velocity/\Delta time)$$

or

$$F = m\ (\Delta V/\Delta t)$$

If we look at the change in velocity during any given period of time as being the difference between the beginning velocity (V_0) and the ending velocity (V_f) we can again rewrite Newton's second law as

$$F = \frac{m(V_f - V_0)}{dt}$$

Since we also know mV is momentum, then the Second Law can be termed;

F = Rate of Change of Momentum

****STOP VIDEO****

Early aerodynamicists used this theory to predict that if a downward rate of change of momentum could be achieved, the equal and opposite force would be in the upward, or "lifting" direction. The difficulty in putting this theory into practice came from determining how to get a rate of change of momentum in the downward direction. Here is where the use of an airfoil became invaluable for a number of reasons.

Looking at the profile of a wing, (Figure 3.1) we can see the shape looks like an elongated water drop laying on its side. This shape is referred to as an airfoil. Usually the top is curved more than the bottom making the upper surface slightly longer than the bottom. Since air passing over the top and bottom must reach the rear of the wing at the same time, the air passing over the top must not only travel faster, but also changes direction and is deflected downward. This actually results in lift being generated due to a rate of change of vertical momentum and a difference in static pressure between the top and bottom of the wing.

Session 3
Developing Lift

At this point it is important to explain several terms used by pilots and engineers. Looking at the airfoil in Figure 3.2 will help clarify these terms.

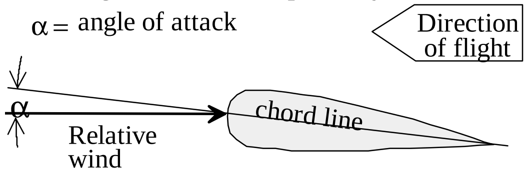


Figure 3.1 Terminology

To begin, the chord line is an imaginary line drawn from the leading edge to the trailing edge of an airfoil. Secondly, the relative wind is the airflow which acts on the airfoil and is parallel to but opposite the direction of flight. The angle between the chord line and the relative wind is called the angle of attack. This is called "alpha" and the symbol used is α. As the angle of attack increases, the change of vertical momentum increases. Additionally, as the angle of attack increases, the coefficient of lift (C_L) increases. The result is an increase in lift. However, there are limits to how much the angle of attack can be increased. Looking at a graph of how the lift coefficient changes with angle of attack, Figure 3.2 shows that at some higher angle of attack, the lift coefficient begins to decrease.

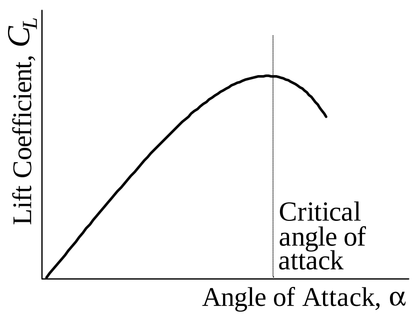


Figure 3.2 Plot of C_L vs α

The angle of attack where the lift coefficient begins to decrease is called the critical angle of attack. Once the critical angle is exceeded, the wing can no longer produce enough lift to support the weight of the aircraft and the wing is said to be "stalled." In other words, the aircraft will stall when the critical angle of attack is exceeded.

To investigate further, first go back to the second law and look at the vertical rate of change of momentum.

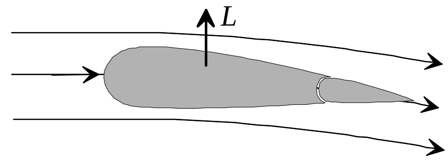


Figure 3.3 Airfoil

Caution:

Recall that momentum is the mass multiplied by the rate of change of velocity in a particular direction. Here we are referring to vertical momentum so we are only concerned with the rate of change of vertical velocity.

****START VIDEO****

The force, F, we are looking for is the lift and is equal to the mass of the air multiplied by the change in vertical velocity of the air over the wing. Whether a wing moves through stationary air, or air is blown over a stationary wing, the physics involved is the same.

Therefore, we can say that in flight there exists an initial velocity of the air in front of the wing (V_0) which has no vertical velocity. In Figure 3.4 we can also see that there is a downward deflection of the air at the rear of the wing (V_f).

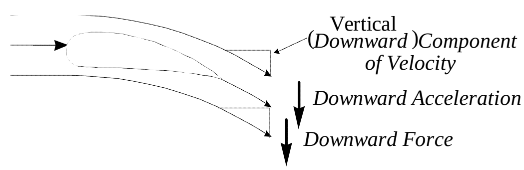


Figure 3.4 Airfoil In Flight

Change in Vertical Momentum

Employing the second law, the rate of change of vertical momentum over the wing the equation becomes:

$$F = \frac{m(V_f - V_0)}{dt}$$

$$F = \frac{m(V_f - 0)}{dt}$$

or

$$F = \frac{m(V_f\,)}{dt}$$

Caution:

Keep in mind these velocities are measured in the vertical direction.

Aerodynamicists knew that density is equal to mass divided by a unit of volume of the air.

$$(\rho) \;=\; m/v$$

Taking a unit volume of air then v equals 1, so the equation becomes

$$(\rho) \;=\; m/1 \;=\; m$$

In this case, the density equals the mass, then the force equaiton can be written.

$$F = \frac{q_{vert}}{Dt}$$

Then according to Newton's third law, the upward force, or lift, would be equal and opposite to the downward rate of change of momentum. Scientists found, however, that it was very difficult to measure the vertical velocity over an airfoil, but measuring the velocity of the air the airfoil was moving through was very simple. This is where they employed Bernoulli's work to help with their research.

Lift Force Opposite Downward
Change of Momentum

4.0 Lift and the Bernoulli Equation

Bernoulli equated the total pressure to the sum of the static and dynamic pressures. The dynamic pressure is a function of the air velocity and the air density.

Density is directly related to temperature, which can be directly measured, and since the air velocity can also be measured, researchers had the dynamic pressure part of Bernoulli's equation well in hand.

$$q = 1/2\ (\rho)\ V^2$$

However, this was only half of the equation. Recall that since the upper surface of the wing is longer, the air must move faster over the top of the wing. Measuring the air velocity would only get the dynamic pressure, not the change in vertical velocity over the wing. Remember the total pressure is the sum of the static and dynamic pressures; and the total pressure must remain constant. So as one increases the other decreases. Then what is needed is how much the static pressure changes over the top of the wing. Since the change in static pressure will be different for different wing shapes, scientists used wind tunnels to measure that static pressure changes between the top and bottom of different wing shapes, assigning each a value referred to as the "Coefficient of Lift" or C_L. Now they had a means of determining all the pressures necessary to find lift. A force, however, is a pressure multiplied by an area.

$$F = (P_T)(\,area)$$

Since researchers were dealing with airfoils, and a wing is just several airfoils side-by-side, the logical area to use was the wing area, given the symbol "*S*." At last they had all the ingredients necessary to define lift. The equation is:

$$L = \tfrac{1}{2}\rho V^2 S C_L$$

or

$$L = q\,S\,C_L$$

5.0 Summary

We have seen how using Newton's third law, scientists conceived how lift could be developed. Taking that conceptual notion, they employed Bernoulli's pressure relationships to determine how to predict the amount of lift generated by a wing. We know that lift and weight are equal and opposite forces so lets look at one final example to tie all of this together.

6.0 Measures of Performance

1 What is momentum?

ANSWER

Momentum - The product of the mass of a substance multiplied by the velocity written as ***mV***.

2 How does momentum relate to Newton's second law?

ANSWER

We can rewrite the law as:

$$F = (mass) \times (\Delta Velocity/\Delta time)$$

or

$$F = m\,(\Delta V/\Delta t)$$

If we look at the change in velocity during any given period of time as being the difference between the beginning velocity (V_0) and the ending velocity (V_f) we can again rewrite Newton's Second Law as:

$$F = \frac{m(V_f - V_0)}{dt}$$

Since we also know mV is momentum, then the Second Law can be termed:

F = Rate of Change of Momentum

3 What is lift?

ANSWER

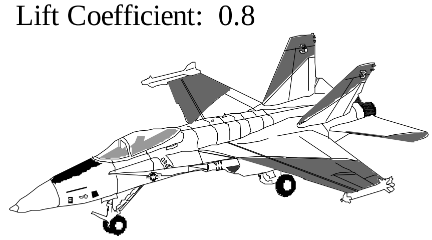
The force equal and opposite to the downward rate of change of momentum.

7.0 EXAMPLE

Problem:

To what speed must an aircraft be propelled before it can become airborne given the following information:

Aircraft weight: 26,000 pounds
Wing area: 600 square feet
Air density: 0.002378 slugs/ft3
Lift Coefficient: 0.8



SOLUTION:

1. The lift equation is: $L = \tfrac{1}{2}\rho V^2 S C_L$
2. We are asked to find the velocity, therefore we must rearrange the equation and solve for the velocity term:
$$V = \left(\frac{2L}{\rho S C_L}\right)^{\frac{1}{2}}$$
3. We are given the weight and since lift must equal weight we can simply put the weight directly into the equation.
4. Substituting the appropriate values into the equation:
$$V = \left(\frac{2(26{,}000\,lbs)}{\left(0.002378\frac{slugs}{ft^3}\right)(600\,ft^2)(0.8)}\right)^{\frac{1}{2}}$$

NOTE:

Recall that a slug/cubic foot is equivalent to a lb-sec^2/cubic foot.

$$V = \left(\frac{52,000 lbs}{1.14 \frac{lbs-\sec}{ft}} \right)^{\frac{1}{2}}$$

$$V = 213.4 \text{ ft/sec}$$

5. Now we usually don't speak of aircraft speeds in feet per second, so to convert to miles per hour, multiply feet per second by 0.6818. Then:

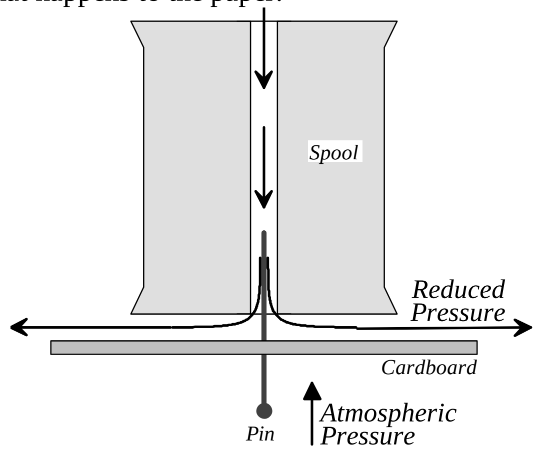
$$V = (213.4 \tfrac{ft}{\sec})(0.6818 \tfrac{miles/hour}{ft/\sec})$$

$$V = 145 \text{ mph}$$

Then assuming we keep the wing area, coefficient of lift, and air density constant, we can change the amount of weight we can lift by simply changing the aircraft's velocity. Changing the airplanes velocity requires changing the engine thrust, which is the subject of the next session.

8.0 Suggested Activity

Take an thread spool and hold a piece of cardboard (like from the back of a tablet) with a pin stuck through it in the hole at the bottom of the spool. Holding the spool vertically, blow air through the hole in the top of the spool and watch what happens to the paper.



A jet of air moves horizontally from the hole at the bottom and spreads out over the surface of the cardboard. If air is blown through with sufficient speed, the outward movement of the air at the bottom of the spool will create a low static pressure at the base of the spool. The higher pressure from the atmosphere under the cardboard will hold it close to the spool so you can now let go of the cardboard. This shows the pressure force overcoming the weight of the cardboard.

Session 4
Developing Drag

1.0 Definitions

Coefficient of drag (C_D) - A measure of how much of the dynamic pressure gets converted into drag.

****START VIDEO****

2.0 Introduction

The previous section started with a discussion of the change in momentum of a particle of air. As the air hit the wing, its new trajectory was split into two components; one parallel to the original direction (relative wind) and one perpendicular to it. The new perpendicular momentum was shown to be related to the lift. The change in horizontal momentum was mentioned only briefly. The emphasis of this session is to correlate this change to *profile* drag.

Caution:

An airplane must fight its way through *two* kinds of drag in order to maintain steady flight; profile drag is the same kind of drag experienced from all objects in a flow. Cars, rocks, and hockey pucks must all overcome profile drag. Objects that create lift must also overcome *induced* drag, also known as drag-due-to-lift. Discussions of induced drag are saved for later. The video footage uses the word "drag" instead of "profile drag."

The concepts from the previous session all apply to drag, so many of the calculations are repeated as well. As before, the aerodynamic force generated can be calculated as the rate of change of momentum. Since drag is defined to be along the direction of the relative wind, then we need only to look at this component of momentum.

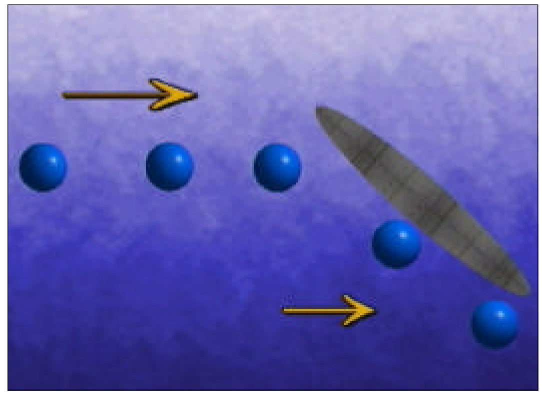


Figure 4.1 Change in Momentum

Figure 4.1 shows that the air particle's horizontal momentum decreases as it moves along. Since its mass isn't changing, we can conclude that only the speed is decreasing. The profile drag is the mass times the deceleration (of the air).

$$Profile\ Drag = m \cdot a_{horiz} = m\frac{\Delta V_{horiz}}{\Delta t}$$

The cause of this deceleration is the loss of energy from skin friction and from pressure.

3.0 Skin Friction

Skin friction is a function of the surface area wetted by the airstream. Any increase in surface area will increase skin friction drag. In addition to this area in contact with the flow, skin friction drag is also affected by what's happening at the contact point between the fluid and the surface. More specifically, it is affected by the fluid's speed and viscosity (stickiness) and by the roughness of the surface.

NOTE:

The Operational Supplement at the end of this session defines the various types of friction.

Some of these effects can be demonstrated with experiments. To eliminate the effect of pressure drag, we need to use an object with constant weight and aerodynamic properties. A puck from an air hockey table should work nicely.

To determine how much skin friction drag exists, we must measure the force needed to overcome it. Place the puck on a flat piece of sheet metal or a smooth board (Figure 4.2). If you tilt the board slowly at an increasing angle until it starts to move, the weight of the puck will overcome the **Breakout Friction**. This is of course greater than the running friction.

Since aircraft skin friction is more like running friction, it would be appropriate to show this measurement with the puck. To do so, tilt the board, hold it, slide the puck slowly and see if it continues at the same speed. If it slows down, tilt the board more and try again. If the puck accelerates after the push, the reduce the tilt of the board and try again.

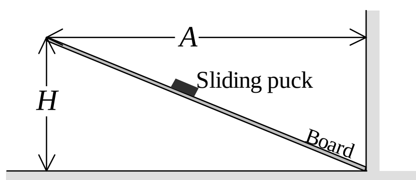


Figure 4.2 Coefficient of Friction, $C_f = \tan c = \frac{H}{A}$

Since the puck's weight increases the friction force **and** the propelling force, the weight effect essentially cancels out and the tangent of the angle of the board is used to define the friction coefficient.

With a measurement capability in place, we can show the effect of changing fluid viscosity. With the dry board as a baseline measurement, reduce the viscosity by adding a light oil or running water to the board. Once the puck starts moving, much less tilt is needed to keep it going. The test can be repeated with a thick, high viscosity fluid such as grease or molasses and will show a need for higher tilt. An extremely low viscosity fluid such as air requires very little tilt at all: If an air hockey table is turned on, the puck will barely slow down at all once set in motion. Only a very slight tilt is needed to keep the puck moving at constant velocity.

To illustrate the impact of speed on skin friction drag, this same series of experiments can be repeated with a higher initial velocity on the puck. Keeping the puck moving at a constant high speed requires only a little more tilt (compared to the low-speed tilt) on an air hockey table, but requires a lot more tilt when using a more viscous fluid. The increase in required tilt angle demonstrates the fact that the speed of the flow also affects the drag. Technically, some of the increase in tilt is due to the extra pressure drag at the higher speed, but this is such a small difference at the low speeds in this experiment that it is practically unmeasureable.

To demonstrate the effect of surface roughness, the experiment can be conducted with a highly polished board (or glass), a rough board, and a board with sandpaper. The above series of experiments can be conducted with a large combination of speeds, roughness, and fluids.

Since aircraft only fly in air, skin friction is due only to the speed and skin roughness. Many race pilots and ground crews spent time waxing their planes to get the smoothest possible surface.

NOTE:

There is a small change in the viscosity of air as it warms up. Unlike liquids, air actually gets more viscous as it heats up. The difference is not significant for general aviation aircraft like Cessnas and Beechcraft, but is more important for fast-movers like the Concorde and SR-71 because they fly so fast that they heat the air around them.

4.0 Pressure Drag

The other component of profile drag is pressure drag. Pressure drag is a function of the size of the wake behind an object in an airstream; it can be reduced by streamlining the object in order to delay separation of the flow. A side effect of streamlining is an increase in the wetted (exposed) area and hence the skin friction, so it is important to ensure that a net reduction in drag is actually achieved when adding streamlining. Figure 4.3 compares the drag coefficients of various shapes which are immersed in the same airstream.

The flat plate has almost no skin friction drag because the flow is attached to the plate only a short distance at the edge. The plate does, however, generate a strong, turbulent wake, so pressure drag

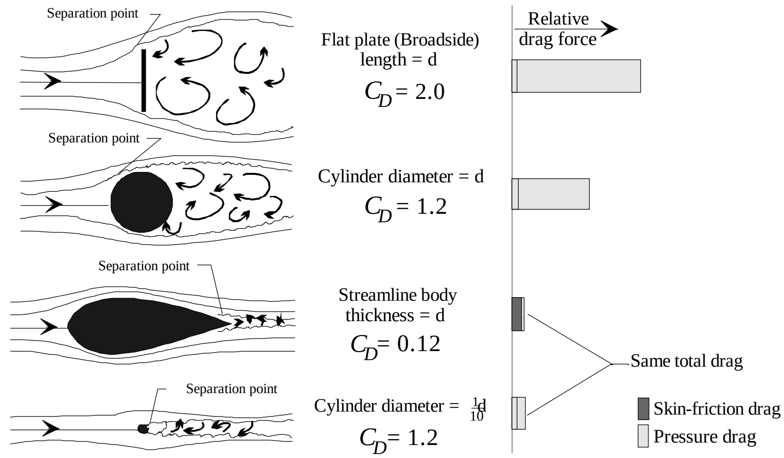


Figure 4.3 Drag Coefficients of Various Bodies

is very high. Because a flat plate normal to the airstream creates so much drag, aerodynamicists avoid such additions to aircraft or automobiles.

Figure 4.4 Large Flat Plate

The "blunt" motorhome is a good reminder that designers sometimes must make compromises to have an all-around good package. The C-23 Sherpa aircraft looks blunt from the front view, but is shaped enough in the side view to allow it to fly at 200 mph.

If a cylindrical cross-section is used instead of a flat plate, the airflow stays attached to the surface almost to the shoulder producing more skin friction drag. When the strength of the wake is reduced, so is pressure. The diagram shows that the total drag is 40% lower than that of the flat plate.

Proper streamlining of the same basic diameter reduces the total drag to 6% of the flat plate drag. The skin friction component is almost four times as large as in the flat plate's friction but, because the flow stays attached for almost all of the surface area of the streamlined shape, the wake and, therefore, the pressure drag, are minimized.

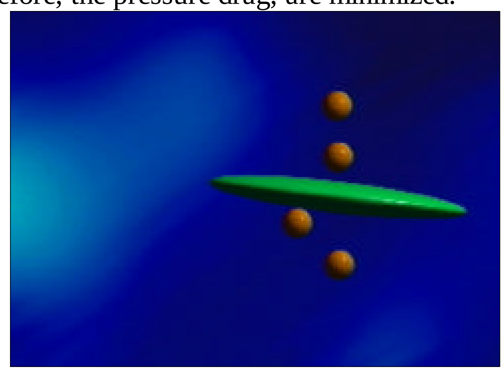
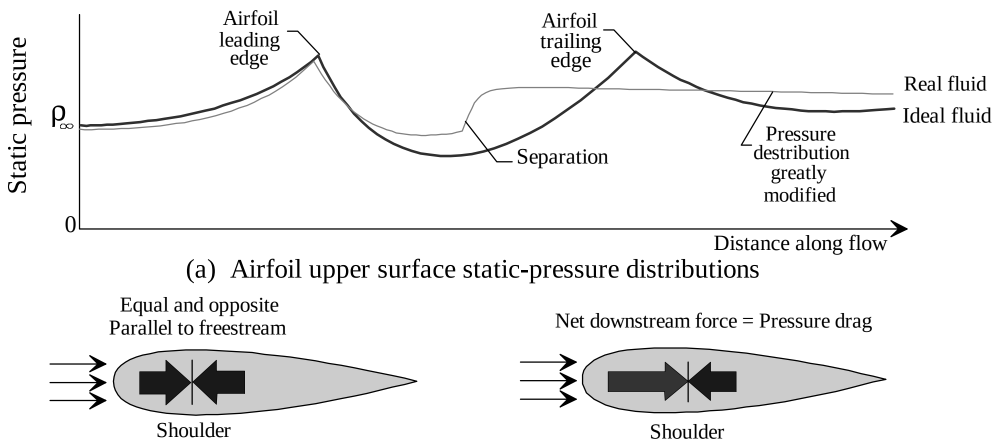


Figure 4.5 Streamlined Shape

4.1 Causes of Pressure Drag

If there was no such thing as friction, then the flow across a surface would retain its original energy and wouldn't separate from the surface. If this was true, then the pressure change across an airfoil would look like the ideal curve in Figure 4.6(a). This ideal situation is called "total pressure recovery" since the pressure at the trailing edge is the same as that at the leading edge. In this ideal situation, all the pressures acting in the drag direction are exactly offset by the pressures in the thrust direction (Figure 4.6(b)) and therefore, no drag exists. Our experience tells us this ideal case does not exist.

In reality, friction robs some of the energy of the flow (transforming it into heat and noise). When this happens, the flow will have insufficient energy and will separate from the airfoil surface.



(a) Airfoil upper surface static-pressure distributions

(b) Ideal fluid air foil (no pressure drag) Sum of horizontal pressures = 0

(c) Real fluid airfoil (net pressure drag) more drag pressure than thrust pressure

Figure 4.6

The actual pressure within the separated flow is typically random and changes quickly, but averages out to be the same as atmospheric pressure. This is illustrated as the line for the real fluid in Figure 4.6(a).

Since there is not total pressure recovery at the trailing edge, a pressure differential will exist between leading and trailing edges. This pressure differential will produce a retarding force called pressure drag (Figure 4.6(c)). For any given airspeed, the pressure drag is essentially proportional to the size of the wake behind the body. The force also increases with the square of velocity, (Figure 4.7).

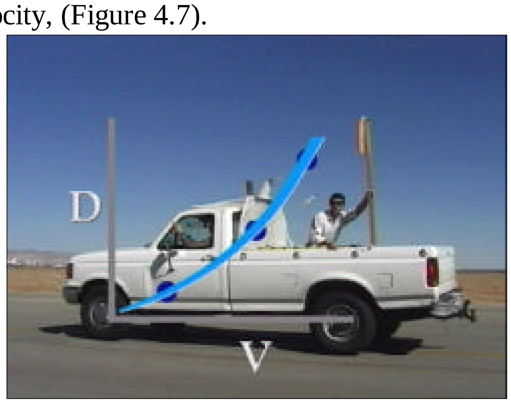


Figure 4.7 Drag Increase with Velocity

When the Wright Brothers were designing the first airplane, they needed to determine what shapes gave the lowest drag. Instead of trying to measure the actual drag force in pounds, they placed the test article on one end of a weathervane device and placed a flat plate on the opposite end at the same radial distance. The entire unit was placed inside a wind tunnel. The wind was forced through the tunnel by a fan after being straightened by a simple grid. The straightened flow then blew on the weathervane which pivoted about its vertical axis. For each shape tested, they increased or decreased the size of the flat plate until its drag force was the same as the shape. They knew the drag forces were equal when the weathervane didn't move when released. With this method they determined the "equivalent flat plate area" drag for a great many airfoil and propeller shapes, (Figure 4.8).

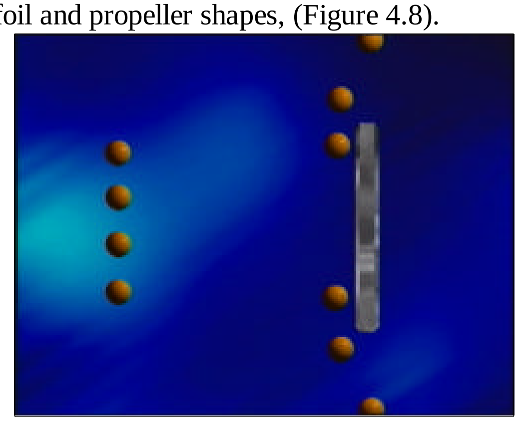


Figure 4.8 Equivalent Flat Plate Area

The Wright Brothers were very careful to eliminate unwanted effects. They made sure there were no other drafts in the room, and nothing upset the delicate test rig. Section 7 of this session describes an experiment similar to that performed by the Wright brothers.

5.0 Summary

The students should realize that the total change in momentum yields a total force called the resultant aerodynamic force (RAF). This is vectorally divided into the more common lift and drag forces (Figure 4.9). There is nothing special about the drag force, it is still measured as the rate of change of momentum - just in the drag direction. An aircraft designer tries to arrange the shapes so the RAF points in the lift direction as much as possible.

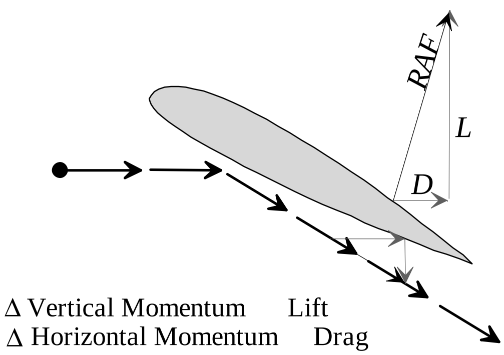


Figure 4.9 Resultant Aerodynamic Force

Figure 4.10 Overall Drag Equation

6.0 Measures of Performance

1. If an object's speed is tripled what happens to its drag?

$$D = C_D \tfrac{1}{2} \rho V^2 S$$

ANSWER

The V^2 effect will generate 9 times the drag.

2. If an airplane flies so high that the air density is only 1/10 of sea level density, then how does the drag compare?

ANSWER

The profile drag is also 1/10 of sea level drag.

3. What is streamlining?

ANSWER

Shaping a body so it changes the flow's horizontal momentum as little and as smoothly as possible.

4. Why are some vehicles not streamlined?

ANSWER

Other design goals outweigh the importance of low drag.

7.0 Suggested Activity

An experiment to demonstrate profile drag can be set up with the same idea the Wright Brothers used.

1) A leaf blower or one or two electric fans (in a row) can be used for power. (Figure 4.11(a))

2) The flow straightener can be made from boxes that are used for shipping wine or beer bottles or food jars. The cardboard dividers inside can be used side-by-side or in a row depending on their size. The airflow can be checked for straightness by taping a few 3" pieces of yarn to the cardboard dividers. (Figure 4.11(b) and 4.11(c))

3) Instead of a weathervane, you can use the front wheel of a small (motocross-type) bicycle. The wheel must have good quality, well-adjusted bearings that rotate freely. The wheel must be mounted so that its axle is perfectly horizontal. The wheel can be mounted separately or by just flipping the bike upside-down and leveling it. A bike wheel is used because it is readily available and has good bearings. Another device with a good axle will work also. (Figure 4.11(a))

4) Attach the test article to the wheel. This can be done at the spokes or the rim. Either way, it is

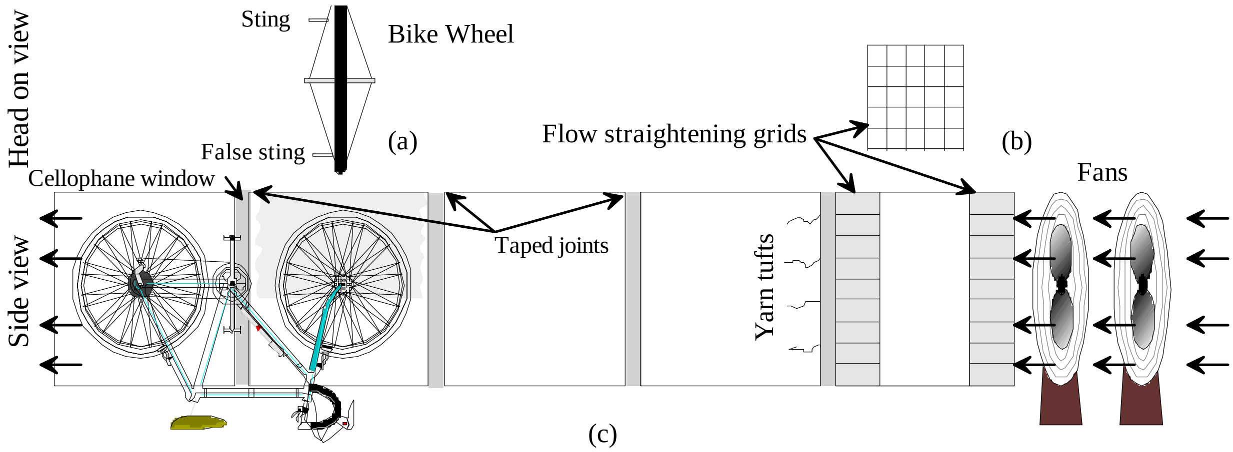


Figure 4.11

good practice to laterally separate the article from the wheel by a couple of inches to avoid airflow interference. The separating mount is called a sting. The wheel's aerodynamic interference can be further minimized by wrapping cellophane or mylar around it. (Figure 4.11(c))

5) Hook a small container (*w*) to the wheel (at the same radial distance as the sting for simple calculations). When the test article is at the top (directly above the axle), the weight should be on some point in front of and horizontal to the axle. (Figure 4.12)

6) Next, the wheel must be statically balanced so that it will stay in any angular position in which it is placed. If it isn't balanced, then it will always have a tendency to rest with the heavy side on the bottom. Balancing the wheel is easy: when the heavy end rotates to the bottom, simply tape some weights near the top of the wheel to offset the heavy part.

7) A useful rectangular wind tunnel can be built using cardboard boxes taped together end-to-end. The purpose of the tunnel is to constrain the air so the fan's energy isn't wasted by blowing around the test region. A powerful fan allows the use of a large refrigerator box tunnel, but a smaller fan requires a more narrow tunnel. Both ends must be open. Fans are typically placed so they blow into the tunnel, but some are built so the fan "sucks" air into it. A slot must be cut in the bottom for the wheel. It must be wide enough to allow for the test article as well as the wheel. A small flap may be employed to close especially wide openings. A cellophane window can be cut into the side of the tunnel at the test section. To prevent the wind from spinning only one side of the wheel, the tunnel should be large enough so that the entire wheel fits inside (Figure 4.11(a)). A false sting (aerodynamically similar to the actual sting) should be placed on the wheel opposite to the actual sting.

8) **If** the tunnel is not large enough for the entire wheel, **then** remove the test article only and perform step 6 without the test article. Next, submerge as much of the wheel as practical into the tunnel and turn on the fan. With the sting at the top, place additional "speed weights" on the wheel to prevent its rotation (due to wheel and sting drag). See Figure 4.12. Speed weight balancing must be accomplished for each fan speed setting, After balancing, turn off the fan and install the test article on the wheel and rotate the wheel so the test article is at the top. Be sure the article is fixed at the desired angle of attack.

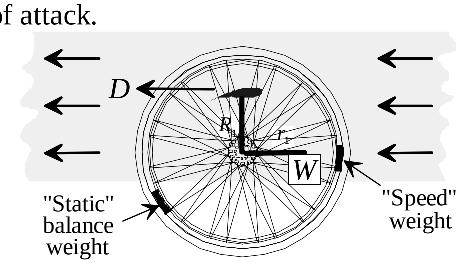


Figure 4.12

9) When the fan is turned on, the aerodynamic drag on the test article tries to force the wheel to turn. This can be prevented by adding some weight (lead shot, sand) to the container.

   If the container is placed at the same radial distance as the sting ($R_1 = r_1$), then the weight of the sand is exactly equal to the drag force. If the distances are different, then;

$$\text{Profile Drag} = \frac{\textit{sand weight}(\textit{HORIZONTAL}\text{ distance to weight})}{\textit{VERTICAL}\text{ distance to sting}}$$

   We can use this equation to simplify the test procedure: instead of adding and deleting mass from the container, slide the hook (moment arm) to compensate for profile drag.

NOTE:

The test article should be exactly at the top at all times, otherwise it's off-center weight and lift will tend to rotate the wheel as well the change in aerodynamic drag.

10) Repeat the test for different angles of attack and different shapes, i.e., balls, cylinders, model airplanes, flat plates, airfoils, molded clay shapes, etc.

11) If any of the test articles create lift, then they will also probably create their own pitching moment (M) that tends to rotate the wheel. If precise tests are to be done to eliminate this effect, then the test should be repeated with the sting at a different radius (R_2) and the same balancing weight (w) at whatever new radius (r_2) is required to maintain a vertical sting position. This twin test gives two equations with two unknowns, M and D:

$$w \times r_1 + M = D \times R_1$$

and

$$w \times r_2 + M = D \times R_2$$

solving simultaneously yields the profile drag and the pitching moment:

$$D = \frac{w[r_1 - r_2]}{[R_1 - R_2]}$$

$$M = w\left[\frac{(r_1 - r_2)R_1}{(R_1 - R_2)} - r_1\right]$$

With the wheel rotated so the sting is directly in front of the axle, weight can be tied to the sting to determine the lift of the airfoil section. Again, the weight required will be affected by the pitching moment. The nose-over moment (M) calculated previously can be added to the weight to get total lift:

$$L = w + \frac{M}{\textit{horizontal radius to weight}}$$

Once the experiment equipment is established, a wide variety of tests can be accomplished: lift, drag & pitching moment, measurements; effects of different shapes & surface roughness; the influence of test article frontal area and airspeed on drag.

Such a matrix of tests would be daunting for a single class, but a good local database can be established after only a few classes. Of course it would be important to be able to recreate the same test set-up and retain test article for future classes.

Alternate Approach:

To avoid the destabilizing effect of the test article moving off from the vertical, the test can be rigged with the wheel horizontal (vertical axle).

In this case, the balance weight is connected through a string and pulley. As shown in Figure 4.13.

The "speed weight" test is performed with all parts connected except the test article. When the test article is added, any **additional** weight compensates for the profile drag. This test apparatus is more elaborate, but is easier to work with.

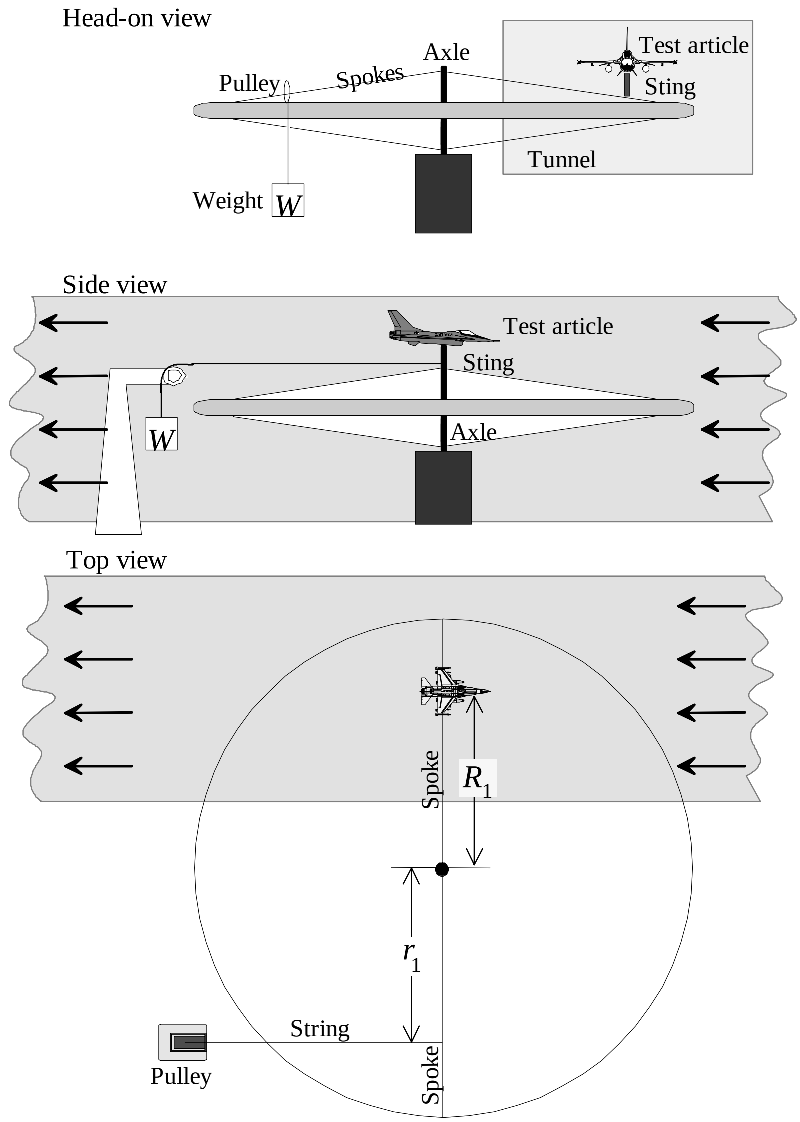

Head-on view
Axle
Test article
Pulley
Spokes
Sting
Tunnel
Weight
W
Side view
Test article
Sting
W
Axle
Top view
Spoke
R_1
r_1
String
Pulley
Spoke

Figure 4.13

Session 4
Developing Drag
Operational Supplement

Friction Forces

Friction forces always act to oppose the motion of one body over another when parts of their surfaces are in contact. These forces are caused by the adhesion of one surface to the other and by the interlocking of the irregularities of the rubbing surfaces. The magnitude of frictional force depends upon the properties of the surfaces and upon the normal force (force perpendicular to the surface). The effects of friction are often undesirable, because friction increases the work necessary to do a task, causes wear in machinery parts, and generates heat. To reduce this waste of energy, friction is minimized by the use of wheels, bearings, rollers, and lubricants. Automobiles and airplanes are streamlined in order to decrease air friction. On the other hand, friction is desirable in many cases. Nails and screws hold boards together by means of friction. Power may be transmitted from a motor to a machine drive-wheel by means of a clutch or a friction belt. In walking, driving a car, striking a match, tying our shoes, or sewing fabric together we find friction a useful tool. Cinders or sand are scattered on icy streets, grooves are cut into the tires of automobiles and aircraft, and special materials are developed for use in brakes - all for the purpose of increasing friction where it is desirable.

Sliding Friction. When we slide a box across a floor, we must continually apply a steady horizontal force to cause the box to slide uniformly over the horizontal surface. Newton's third law states there is a force, parallel to the surfaces in contact, opposing the motion. This opposing force is called *friction.* The frictional force is generally the result of the roughness of the two surfaces in contact, which causes interlocking between them. This interlocking gives rise to a force that resists motion. If the applied force is just equal to the opposing frictional force, the box will continue to move uniformly; if the applied force is greater than the frictional force, the body will accelerate.

The observations we can make regarding sliding frictional force are these:

1. It is parallel to the surfaces in contact.
2. It is proportional to the force which is normal (perpendicular) to the surfaces which presses them together.
3. It is generally independent of the area of the surface contact and independent of the speed of the sliding, provided that the resultant heat does not alter the condition of the surfaces or fluids are not introduced between the surfaces.
4. It depends upon the properties of the substances in contact and upon the condition of the surfaces, e.g., polish, roughness, grain, wetness, etc...

Sliding friction is sometimes called *kinetic friction.*

When one body is in uniform motion on another body, the ratio of the frictional force, F, to the perpendicular force pressing the two surfaces together, N, is called the *coefficient of kinetic friction*, μ. It can be expressed by the following equation:

$$\mu_k = F/N \qquad (1)$$

When the two surfaces are lubricated, the lubricant fills the surface irregularities, reducing the friction. The ratio F/N, however, is no longer a simple constant, but, depends upon the properties of the lubricant, the area, and relative speed of the moving surfaces.

Static Friction. When a body at rest on a horizontal surface is pushed gently by a horizontal force, it does not move because there is a frictional force just equal to the applied force. If the applied force is increased slowly, the frictional force increases to oppose motion until a *limiting force* is reached.

If the applied force exceeds the limiting friction force, the body "breaks out" into accelerated motion. The *coefficient of static friction* is the ratio of the "breakout" frictional force to the normal force.

$$\mu_s = F_{bo}/N \tag{2}$$

For any two surfaces the coefficient of static friction, μ_s, is somewhat greater than the coefficient of kinetic friction μ_k.

Rolling Friction. *Rolling friction* is the resistance to motion caused chiefly by the deformation produced where a wheel, bearing, or roller pushes against the surface on which it rolls. The deformation of an automobile tire in contact with the pavement is readily visible. Even in the case of a steel wheel rolling on a steel rail, there is some deformation of the two surfaces. The deformation of the two surfaces produce internal friction in the two bodies. The force of rolling friction varies inversely with the radius of the roller, and decreases as more rigid surfaces are used. Rolling friction is ordinarily much smaller than sliding friction.

Viscous Friction. The friction forces encountered by solid objects in passing through fluids and the frictional forces set up within liquids and gases in motion are examples of *viscous friction*. The laws of fluid friction differ greatly from those of sliding and rolling friction. The amount of frictional resistance encountered by an object moving through a fluid depends on the size, shape, and speed of the moving object, as well as on the properties of the fluid itself. The frictional resistance encountered by a man falling through the air increases with his speed until he reaches a terminal speed, about 120 mi/hr, at which time the retarding force of friction equals his weight. When he opens his parachute, the greater surface it presents increases the retarding force of friction and reduces the terminal speed to 14 ft/sec.

Viscosity is that property of a fluid, its internal friction, which causes it to resist flow. Viscosity is due fundamentally to cohesion and molecular momentum exchange between fluid layers, and, as flow occcurs, these effects appear as shearing forces (parallel to the layers) between the moving layers. Consider a layer of liquid in a shallow pan, onto which a flat plate, A, is placed, as shown in Figure 1. A force F is required to maintain the plate at a constant speed V with respect to the other surface B. On the surface of each solid, A and B, there will be a layer of liquid that adheres to the solid and has zero speed. The next layer of liquid moves slowly over the first, the third layer moves slowly over the second, and so on. This distribution of speeds results in a continual deformation of the liquid. This internal (or viscous) friction distorts the cube of fluid, C, into a new shape, R, as the force moves the upper plate.

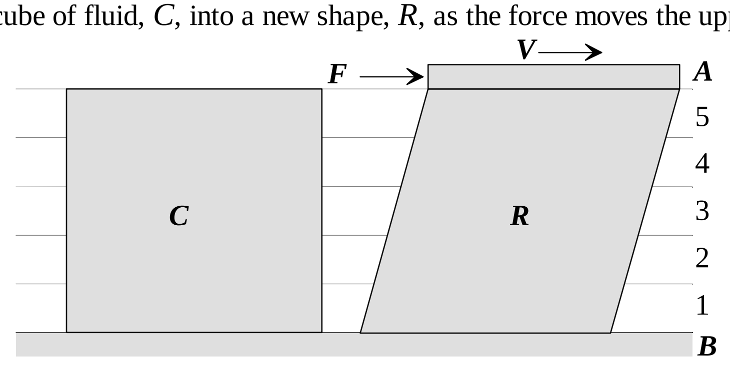


Figure 1 Viscous Friction

The viscosity of liquids decreases with increase in temperature. A liquid that flows as slowly as the proverbial molasses in January at low temperature may pour freely at higher temperature. Lubricating oil may fail to form a desired protective film at low temperatures; hence, when starting a car on a cold day, it is wise to allow the engine to idle for a time until the oil is warmed. The viscosities of gases, unlike those of liquids, increase with increase in temperature. The internal friction of liquids is attributed to the cohesive forces between closely packed molecules. In the case of gases, whose molecules have much larger separations, cohesive forces are much smaller and some other mechanism must be sought for internal friction. This other mechanism is in the form of a continual migration of molecules from one layer to another. Molecules diffuse from a fast-moving layer to a slower moving layer, and from the slower moving layer to the faster. Thus each layer exerts a drag on the other proportional to the mass of the molecules and their speeds. This description of gas viscosity accounts for the fact that an increase in temperature, which increases molecular speeds, results in an increase in the viscosity of a gas.

Session 5
Thrust

1.0 Definitions

Compressor - The part of an engine which forces the same amount of mass into a smaller volume, increasing the air density.

Combustion - The controlled explosion of a fuel-air mixture.

Disk area - The area described by a propeller as it turns through a full circle.

Turbojet - An engine which has a small intake area (for low drag purposes) that greatly compresses the air, adds fuel and generates rapid air velocities by combustion of the fuel/air mixture.

Turbofan - A engine which is essentially a large fan encased in a shroud mounted on the end of a turbojet shaft.

2.0 Introduction

The previous sessions have developed three of the forces of flight; weight, lift, and drag. Drag was shown as the force acting opposite the flight path of the aircraft, therefore the opposing force, thrust, must act in the same direction as the flight path. However, an engine produces a force which acts toward the rear of the aircraft. Through an application of Newton's third law, this force creates an equal and opposite reaction which "thrusts" the aircraft forward. For this reason, the force produced by the engine is called thrust. This session will describe the origins of thrust and highlight how various engines produce thrust. Thrust may be the most important force because regardless of the type of aircraft being studied (or tested) ALL need some type of thrust to propel them aloft. Even unpowered aircraft such as gliders need a tow plane to provide an external force to pull the aircraft into the air, where it can obtain airflow over the wings to provide the necessary lift to remain airborne. Hang gliders use foot power to initiate movement prior to "leaping" off a cliff. The most common means of developing thrust on powered airplanes comes from propellers or jets. Both of these types employ the same principle of operation involving Newton's second law.

****START VIDEO****

3.0 Principles of Thrust

The explanation of thrust is based entirely on Newton's second law. Recall that force equals the rate of change of momentum:

$$F = \frac{\Delta(mV)}{\Delta t} \tag{5.1}$$

Students will recognize the simplified version of this law that applies when the mass is constant:

$$F = ma$$

for thrust analysis, however, we use equation (5.1) in another form:

$$F = \frac{\Delta(mV)}{\Delta t} = F = \frac{\Delta m}{\Delta t}\Delta V \tag{5.2}$$

$\frac{\Delta m}{\Delta t}$ is known as the mass flow and is sometimes abbreviated as Q. ΔV is simply the total change in velocity of the airflow.

$$F = Q\,\Delta V \tag{5.3}$$

The amount of force, or thrust, generated is dependent upon two primary factors; 1) the amount of mass flow, and 2) the change of the air flow speed.

Each of the primary factors influencing thrust can be varied by different means. If more thrust is required, either the mass flow can be increased or the change in velocity of the air mass as it flows through the propeller can be increased. To create a given amount of thrust, a large amount of mass flow can be accelerated a little or a small amount of mass flow can be accelerated a lot. This concept was demonstrated in the video by the use of paper fans.

3.1 Propeller Aircraft

For a propeller powered aircraft, it can be proven through the use of the kinetic energy theory and the Bernoulli pressure relationship, that the total change of the air flow speed (ΔV) is a function of the aircraft's forward speed and the change in the speed of the air as it immediately passes through the propeller area. This is expressed as:

$$\Delta V = 2\left[\Delta v + \frac{\Delta v^2}{V}\right] \tag{5.4}$$

The Δυ is the change in velocity between the air in front of the propeller and the air immediately behind the propeller, as shown in the video.

Figure 5.1 Change in Air Velocity Directly behind Propeller

Caution:

The Δυ in this relationship should not be confused with the total change in velocity, Δ*V*, shown in equation 5.3.

The additional velocity imparted by the propeller was given in the video to be equivalent to a propeller constant, "*k*" times the engine RPM, giving the relationship:

$$\Delta\upsilon = k\,(\text{RPM}) \qquad (5.5)$$

NOTE:

The propeller constant "*k*," contains the conversion factor from "revolutions per minute" (RPM) to revolutions per second. "*k*" is a different constant for each given flight speed and propeller design.

Figure 5.2 Volume of Air

Mass flow (*Q*) is dependent upon the density speed and area of the air as shown by the relationship:

$$Q = \frac{\{\rho\gamma\}}{\Delta t} = \rho AV \qquad (5.6)$$

Recall the area of a circle or disk is:

$$\text{Propeller disk area, } A = \frac{\pi d^2}{4} \qquad (5.7)$$

If we assume the density of the air and the propeller diameter remain essentially constant, then equations 5.3, 5.6 and 5.7 can be combined:

$$F = T = Q\,\Delta V = \rho\left(\frac{\pi d^2}{4}\right)V(\Delta V)$$

Solely for the purpose of helping students relate with accelerations, the video replaced the *V*(Δ*V*) product with a "pseudo-acceleration," *a*:

$$F = T = \rho\left(\frac{\pi d^2}{4}\right)a \qquad (5.8)$$

If the air's acceleration (*a*) is replaced with *V*(Δ*V*) [Δ*V* shown in equation's 5.5 and 5.4] then $a = 2\left[Vk(RPM) + (k\,RPM)^2\right]$. The thrust equation identified in the video combine this and equation 5.8 to get:

$$F = T = \left[\rho\left(\frac{\pi d^2}{4}\right)\right]2\left[Vk(RPM) + (k\,RPM)^2\right] \qquad (5.9)$$

This equation can be quite "messy" therefore, an example may clarify the important points.

****STOP VIDEO****

***Example 1*:**

An aircraft has an engine that can turn 2750 rpm. How much thrust will be generated at 100 mph (147 feet per second) if the propeller diameter is 5 feet and has a "*k*" value of 0.0044 at an altitude where the density is 0.0022 slugs per cubic foot (which can also be written as $0.0022\frac{lb\sec^2}{ft^4}$)?

***Solution*:** By directly substituting into equation 5.8 the thrust can be determined.

$$F = T = \left\{\rho\left(\frac{\pi d^2}{4}\right)\right\}2\left\{Vk(RPM) + (k\,RPM)^2\right\} \qquad (5.9)$$

$$T = \left\{0.0022\tfrac{slugs}{ft^3}\left(\frac{\pi 5^2}{4}ft^2\right)\right\}$$
$$2\{147\tfrac{ft}{\sec}(0.0044)(2750\ RPM)$$
$$+(0.0044)(2750\ RPM)^2\}$$

$$T = \left\{0.0022\tfrac{slugs}{ft}\right\}2[147\tfrac{ft}{\sec}(12.1)+146.41]$$

$$T = \ 166 \text{ lbs}$$

Now to show the effect propeller size has on thrust, consider a forty percent, that is two feet, increase in the propeller diameter.

***Example 2*:**
Using the same aircraft in Example 1, how much thrust can be generated if the propeller diameter is increased to 7 feet?

***Solution*:** Again by direct substitution into equation 5.9, the thrust can be determined.

$$F = T = \left\{\rho\left(\tfrac{\pi d^2}{4}\right)\right\}2\{Vk(RPM)+(k\ RPM)^2\} \tag{5.9}$$

$$T = \left\{0.0022\tfrac{slugs}{ft^3}\left(\frac{\pi 7^2}{4}ft^2\right)\right\}$$
$$2\{147\tfrac{ft}{\sec}(0.0044)(2750\ RPM)$$
$$+(0.0044)(2750\ RPM)^2\}$$

$$T = \left\{0.08467\tfrac{slugs}{ft}\right\}2[147\tfrac{ft}{\sec}(12.1)+146.41]$$

$$T = \ 325 \text{ lbs}$$

This shows that a 40% increase in propeller diameter increased the thrust by 95% as a result of an increase in the mass flow. However, this increase in thrust creates an unbalanced force in the horizontal direction. Recall that in unaccelerated flight, thrust and drag must be equal, according to Newton's third law. The aircraft will therefore accelerate to a new speed where the drag and thrust are again equal. This concept will be covered in further detail in Session 8. For now let’s see what happens when the mass flow is kept the same, that is keep the same propeller size, but change the acceleration of the air by changing the RPM.

***Example 3*:**
Using the same aircraft as in example 1, what is the increase in thrust if the propeller RPM is increased to 3000?

***Solution*:** The answer can again be found by direct substitution into equation 5.9.

$$F = T = \left\{\rho\left(\tfrac{\pi d^2}{4}\right)\right\}2\{Vk(RPM)+(k\ RPM)^2\} \tag{5.9}$$

$$T = \left\{0.0022\tfrac{slugs}{ft^3}\left(\frac{\pi 5^2}{4}ft^2\right)\right\}$$
$$2\{147\tfrac{ft}{\sec}(0.0044)(3000\ RPM)$$
$$+(0.0044)(3000\ RPM)^2\}$$

$$T = \left\{0.0432\tfrac{slugs}{ft}\right\}2[147\tfrac{ft}{\sec}(13.2)+1174.241]$$

$$T = \ 182.7 \text{ lbs}$$

This example shows that a 9% increase in RPM (which is really a 9% increase in the change of the flow velocity through the propeller) over that in Example 1 yields a 10% increase in thrust. Highlighted here is the effect of increasing the acceleration of the airflow. Comparing this to Example 2, it would appear that the most effective way to increase thrust is to increase the size of the propeller, which really means increase the mass flow through the propeller. However, as shown in the video, there are practical limits on propeller size. These limits come from the fact that propellers mechanically accelerate the air. This type of acceleration also limits the amount of thrust that can be developed. Jet engines, on the other hand, use an increase in acceleration of the air to create much larger thrust values.

****START VIDEO****

3.2 Jet Engines

In a turbojet engine, the inlet area is small when compared to that of a propeller. As a result, there is a smaller amount of mass entering the engine. Recall previously we assumed the density remained constant. Now in the case of a turbojet, in order to allow for combustion the air density must be increased. This is done by the compressor section of the engine, as shown in the video. As the air progresses toward the rear of the engine, it is forced into the smaller and smaller spaces between the blades of each compressor ring. This compacting of the air results in an increase in the air pressure and density as well as an increase in the air temperature.

Figure 5.3 Jet Engine Compressor

As the air exits the compressor section of the engine, it enters the combustion chamber where fuel is added. This densely packed air/fuel mixture is ignited and the resultant "explosion" accelerates the gases out the rear of the engine at a very high rate of speed. This chemical acceleration of the air (combustion) adds to the thrust produced by the engine. Most jet fighters have a system called afterburners, which adds raw fuel into the hot jet exhaust generating even more thrust through higher accelerations of the air.

So the jet generates large amounts of thrust by chemically accelerating the air as the result of combustion. The fact that the jet compresses the air as much as 40 times (depending upon the number of compressor rings) allows the jet aircraft to fly at higher altitudes where the air is too thin for propeller driven aircraft to fly. These altitudes permit the jet aircraft to fly over most weather systems giving passengers a smoother ride. There is a price to pay for the ability to fly at higher speeds and altitudes. That price comes in the form of higher fuel consumption, or in more everyday terms, lower fuel mileage.

One type of engine is a combination of both the turbojet and a propeller called, appropriately, a turboprop. A turboprop is a small turbojet engine which turns a propeller. The turboprop uses the jet's ability to compress the thin air found at higher altitudes combined with the larger volume of air associated with a propeller to produce modest amounts of thrust at medium altitudes. Although it burns less fuel than a turbojet, it cannot fly as high, nor a fast. Both of these limitations are the result of propeller inefficiencies.

As stated earlier, air density decreases as altitude increases. Since propellers are simply airfoils, they have a tendency to become less effective as the air gets thinner. Additionally, although Examples 2 and 3 proved that increasing the prop size and speed increased thrust, as propellers get bigger and turn faster, the tips begin to reach supersonic speeds. At these tip speeds, shock waves begin to develop and destroy the effectiveness of the prop. It would seem, therefore that the most efficient engine would be a combination of the turbojet and a large, slow turning prop. In recent days, these engines have been developed and are called "high by-pass ratio turbofans."

The engines use a turbojet as a "core" to serve two purposes: 1) produce a portion of the total thrust, and 2) to turn a huge fan attached to the main shaft. The engine can operate at higher altitudes because the jet core can compress the thin air. The thrust produced by the core is supplemented by having a VERY large fan section attached to the main shaft of the core. The fan draws in huge amounts of air and therefore can turn slow enough to prevent the flow at the blade tips from becoming supersonic. The overall result is: 1) the fan mechanically generates a little acceleration to a large amount of air mass, and 2) the jet core compresses thin air and chemically generates large accelerations to relatively small amounts of air. Since the fan is mounted to the same shaft as the core, the by-pass ratio of these engines is determined by dividing the amount of air flowing through the fan blades by the amount of air passing through the engine core. This can be written as:

$$ratio = \frac{(\text{area of fan} - \text{area of core})}{\text{area of core}}$$

Consider the following example using the G.E. 90 engine shown in the video.

***Example 4*:**
If the G.E. 90 engine has a fan diameter of 10.25 feet, and a core diameter of 3.34 feet, what is the bypass ratio of the engine?

Figure 5.4 GE 90 Engine

***Solution*:** The area of the fan is:

$$A = \frac{\pi d^2}{4}$$

$$A = \frac{\pi 10.25^2 ft}{4}$$

$$A = 82.52\ ft^2$$

The area of the core is:

$$A = \frac{\pi 3.34^2 ft}{4}$$

$$A = 8.78\ ft^2$$

Then the bypass ratio is:

$$ratio = \frac{(82.52ft^2 - 8.78ft^2)}{8.78ft^2}$$

$$ratio = 8.4 \text{ to } 1$$

This means over eight times as much air moves around the outside of the engine as moves through the engine. Since this air is producing thrust but NOT using fuel directly, the efficiency of the engine is greatly increased.

4.0 Summary

As we have seen, whether an aircraft has a propeller, a turbojet, or a turbofan, all of these produce thrust by accelerating a mass of air to the rear of the aircraft. Let's finish this session by proving that the movement of this air to the rear creates an unbalanced force pushing the aircraft forward. In the video, a balloon was used to show how when the pressure is equal in all directions there is no net force.

Figure 5.5 Equal Forces in the Balloon

However, when the stem is released, the air escaping from the balloon causes an unbalanced force at the front of the balloon, propelling the balloon forward. The same principle applies to the thrust produced by an aircraft engine. The unbalanced force propels the aircraft forward, creating airflow over the wings which generate lift, causing the aircraft to become airborne. The first step to getting airborne is the takeoff, which it just so happens is the topic of the next session.

5.0 Measures of Performance

1 What is the basic principle of operation behind thrust?

ANSWER

Newton's second law; force equals the rate of change of momentum.

2 What are the two primary factors which determine the amount of thrust which can be generated?

ANSWER

1) The amount of mass flow
2) The change of the air velocity behind the rear of the engine.

3 What are the two ways the thrust can be increased on a propeller driven aircraft?

ANSWER

1) Increase the size of the propeller.
2) Increase the RPM of the propeller.

4 By what means does a propeller accelerate an air mass?

ANSWER
Mechanical

5 By what means does a jet accelerate an air mass?

ANSWER
Chemical or combustion

6 What makes a turbofan engine so efficient?

ANSWER
A smaller turbojet core permits flight at higher altitudes while burning a small amount of fuel. The large fan is connected to the same shaft as the jet and therefore turns while the jet compressors turn. These large fans turn slightly slower while accelerating large amounts of air. This increases the thrust of the engine while not burning any fuel.

6.0 Problems

1 A propeller driven aircraft requires 200 pounds of thrust to fly at 110 miles per hour (161.4 ft/sec). If the engine is capable of turning at 3000 RPM, and the propeller constant (k) is 0.005, how large does the propeller have to be to fly at an altitude where the density is 0.0021 slugs/ft^3?

ANSWER
By rearranging equation 5.9, all the information is given to solve directly for the propeller diameter.

$$F = T = \left\{ \rho\left(\frac{\pi d^2}{4}\right)\right\} 2\{Vk(RPM) + (k\ RPM)^2\} \tag{5.9}$$

rearranging to solve for "d" yields:

$$d = \sqrt{\frac{2T}{\rho\pi\{Vk(RPM) + (k\ RPM)^2\}}}$$

Now if we let $Vk(RPM) = x$ and $K\ RPM = y$, solving for x and y gives:

$$x = Vk(RPM)$$
$$x = (161.4\tfrac{ft}{\sec})(0.005)(3000RPM)$$
$$x = 2421\tfrac{ft}{\sec}$$

$$y = k\ RPM$$
$$y = (0.005)(3000RPM)$$
$$y = 15\tfrac{ft}{\sec}$$

$$d = \sqrt{\frac{2T}{\rho\pi\{x + y^2\}}}$$

$$d = \sqrt{\frac{2(200lbs)}{\left(0.0020\frac{slugs}{ft^3}\right)(3.14)\{2421\frac{ft}{\sec} + (15\frac{ft}{\sec})^2\}}}$$

$$d = 4.8 \text{ ft}$$

2 What is the size of the "fan" portion of a turbofan engine if the core has a diameter of 2.75 feet and the by-pass ratio is 6.3:1?

ANSWER
Rearranging the relationship

$$ratio = \frac{(\text{area of fan } - \text{ area of core})}{\text{area of core}}$$

to solve for the fan area gives:

$$A_{fan} = \{(\text{ratio}) \text{ x } (A_{core})\} + A_{core}$$

Then the area of the core is:

$$A_{core} = \frac{\pi d^2}{4} = \frac{(3.14)(2.75)^2}{4}$$

$$A_{core} = 5.94\ ft^2$$

By substitution into the relationship shown above:

$$A_{fan} = \{(6.3) \text{ x } (5.94 \text{ ft})\} + 5.94 \text{ ft}$$
$$A_{fan} = 43.4\ ft^2$$

Then substituting into the equation for the area of a circle, the fan diameter is determined:

$$A = \frac{\pi d^2}{4}$$

$$d = \frac{4A}{\pi}$$

$$d = \frac{4(43.4ft^2)}{3.14}$$

$$d = 7.4 \text{ ft}$$

Session 6
Takeoff Performance

1.0 Definitions

Coefficient of friction (m) - A measure of the resistance to movement of two surfaces which are in contact.

Takeoff distance - The distance required to accelerate an aircraft to takeoff speed

Takeoff speed - The speed where the wings of an aircraft generate enough lift to just equal the weight of the aircraft.

2.0 Introduction

Previous sessions have applied Newton's Laws to the forces of flight; weight, lift, drag and thrust. In this session, those same laws are applied to determine the takeoff performance of an aircraft. The primary consideration when analyzing takeoff performance is measuring the distance required to become airborne. In other words, determining the distance required to accelerate a stopped aircraft to an airspeed where the wings can generate enough lift to cause the aircraft to become airborne. This evaluation begins by considering the four forces associated with the test aircraft.

****START VIDEO****

3.0 Test Aircraft Description

Whenever a test report is written about an aircraft, the first thing given is a description of the test aircraft. In the video, the test aircraft is the Aermacchi MB-326 Impala jet trainer.

Figure 6.1 Impala Jet Trainer

This aircraft is used by a number of air forces throughout the world and is used as a flight test trainer at the National Test Pilot School. The aircraft has the following features:

1. Weight: 7887 pounds (includes pilots and fuel)
2. Wing Area: 205.33 square feet
3. Maximum Lift Coefficient: $C_{L\,max} = 1.51$
4. Static maximum thrust at Mojave Airport: 2200 pounds
5. Drag coefficient: $C_D = 0.06$

NOTE:

The air density (ρ) at Mojave Airport is usually about 0.0022 slugs per cubic foot.

This information is necessary to calculate the takeoff performance.

4.0 Determining the Takeoff Speed

The ***minimum lift*** required for flight, that is to just become airborne, ***occurs at a speed where the lift and weight just become equal.***

NOTE:

At speeds above that where the lift and weight just become equal, the aircraft will be able to climb or accelerate. At speeds below this there will not be enough lift generated to become airborne.

To calculate the takeoff speed for the Impala, begin with the lift equation.

$$w = L = \tfrac{1}{2}pV^2SC_{L_{\max}} \qquad (6.1)$$

Taking the numbers from the aircraft description above and rearranging Equation 6.1, the speed to just become airborne, that is the takeoff speed, is found by the following:

$$V = \sqrt{\frac{2w}{pSC_L}}$$

$$V = \sqrt{\frac{2(7887lbs)}{\left(0.002\frac{slugs}{ft_3}\right)(205.33ft^2)(1.51)}}$$

V = 152.1 ft/sec (or 90 knots)

To propel the aircraft forward to achieve this speed, the aircraft must have enough thrust to overcome the drag.

5.0 Determining the Drag

Session 4 demonstrated that drag varies with speed. Recall the drag equation is written as:

$$D = \tfrac{1}{2} pV^2 SC_D \qquad (6.2)$$

However, as the aircraft accelerates for takeoff, the speed is constantly changing so what speed is entered into equation 6.2? Through experience, engineers have learned that if 70% of the takeoff speed is used to calculate the drag during takeoff, the results are very close to the actual drag. The takeoff speed for the Impala has just been calculated at 152.1 feet per second. Then to calculate the drag, a value of 106.5 feet per second (70% of 152.1) is used in equation 6.2. Using the values for density, drag coefficient and wing area, the drag which must be overcome during the takeoff is:

$$D = \tfrac{1}{2} pV^2 SC_D$$

$$D = \tfrac{1}{2}\left(0.0022 \tfrac{slugs}{ft^3}\right)(106.5 \tfrac{ft}{\sec})^2 (0.06)$$

$$D = 153.7 \text{ lbs}$$

This drag must be subtracted from the total thrust, since these forces act in opposite directions. Additionally, since the thrust and drag are not equal, the unbalanced force (thrust) will cause the aircraft to accelerate. To determine the rate of acceleration, Newton's second law is used.

6.0 Determining the Acceleration

To calculate the acceleration rate from the $F = ma$ equation, the forces must be inserted.

$$F = T - D = ma$$

This equation can be rearranged to solve for the acceleration:

$$a = \frac{T-D}{m} \qquad (6.3)$$

Recall from session 2 the mass is the aircraft's weight divided by the acceleration of gravity:

$$m = w/g$$

Then inserting this into equation 6.3 gives:

$$a = \frac{g(T-D)}{w} \qquad (6.4)$$

Using the appropriate values from the test aircraft description:

$$a = \frac{\left(32.2 \frac{ft}{\sec^2}\right)(2200 lbs - 153.7 lbs)}{(7887 lbs)}$$

$$a = 8.35 \text{ ft/sec}^2$$

This acceleration rate can be used as the slope of a straight line to construct a graphic plot of velocity (in feet/second) versus time (in seconds). Assuming this acceleration rate is constant, at the end of one second, the velocity is 8.35 ft/sec; at the end of 2 seconds, the velocity is 2 times 8.35, or 16.7 ft/sec. This method can be continued for as long as desired, but since the velocity at takeoff has already been calculated from equation 6.1 and found to be 152.1 ft/sec, then if the speed is divided by the acceleration rate, the time required to reach that speed can be determined.

$$time = \frac{\text{takeoff speed}}{\text{acceleration rate}}$$

$$time = \frac{152.1 \frac{ft}{\sec}}{8.35 \frac{ft}{\sec^2}}$$

$$time = 18.21 \text{ sec}$$

To construct the graph, perform the following:

- at time zero the speed is zero, then place a dot at the origin
- place another dot at the point where the speed is 152.1 ft/sec and the time is 18.21 sec
- connect the two dots

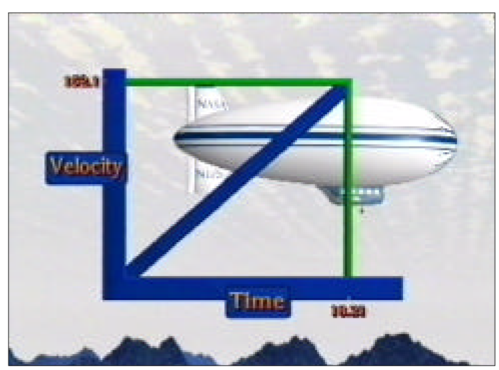


Figure 6.2 Plot of Velocity vs. Time

The predicted takeoff distance is simply the area under the line which was just drawn.

7.0 Determining the Takeoff Distance

Since a, the acceleration, is assumed to be constant, the resulting slope is a straight line. It can be seen in Figure 6.1 that by locating the point on the line associated with the takeoff speed then dropping straight down to the times axis, the area under this curve is equal to the area of a right triangle. The equation for the area of a right triangle is:

$$area = \tfrac{1}{2}(base \times height) = takeoff\ distance$$

The base of the acceleration plot is the "time" axis and the height is the "velocity axis." By substituting into the equation the appropriate values from the curve, the area is found to be:

$$area = \tfrac{1}{2}[(18.21\,\text{sec})(152.1\tfrac{ft}{\text{sec}})]$$

$$area = takeoff\ distance = 1384.9 \text{ feet}$$

Then the calculated takeoff distance is approximately 1385 feet. The actual takeoff distance exceeded the predicted distance by a considerable amount.

Figure 6.3 Takeoff Distance Determination from Chase Aircraft

After the flight, the test team realized they had forgotten to account for the rolling friction of the aircraft.

8.0 Rolling Friction

NOTE:

Further explanation of rolling friction can be found in Session 4's Operational Supplement.

It takes much less force to push a hockey puck across the ice than it does to push that same puck across asphalt. This is because there is less friction to resist the movement of the puck when it is pushed across the ice. Air hockey games blow air up through holes in the table surface so the puck then rides on a cushion of air. This eliminates almost all the friction allowing the puck to move with very little force applied. Each type of surface has certain friction factor called the "coefficient of friction" and given the Greek symbol "μ". This coefficient has been determined experimentally for each type of surface and the values placed in a table. In this application, the coefficient for rubber tires on a concrete runway is approximately 0.05.

The force required to overcome friction and move an object depends on the object's weight and the surface on which it rests as shown in the following equation:

$$Friction = \mu w \qquad (6.5)$$

Then for the Impala on a concrete runway, the friction force which must be overcome before movement can begin is:

$$Friction = (0.05)(7887 \text{ lbs})$$

$$Friction = 394.4 \text{ lbs}$$

Since the rolling friction resists movement, it actually acts in the same direction as the drag and must therefore be subtracted from the thrust. Including the friction into equation 6.4, the new acceleration can be calculated:

$$a = \frac{g(T - D - Friction)}{w} \qquad (6.6)$$

$$a = \frac{\left(32.2\tfrac{ft}{\text{sec}^2}\right)(2200lbs - 153.7lbs - 394.4lbs)}{7887lbs}$$

$$a = 6.74 \text{ ft/sec}^2$$

This acceleration rate is significantly slower than the 8.35 ft/sec^2 rate determined earlier. Using this new acceleration rate as the slope, a new curve can be generated in the same manner as the previous acceleration curve. The takeoff speed will remain the same since the speed depends on the aircraft weight, which hasn't changed. The final point then corresponds to a time of 22.6 sec.

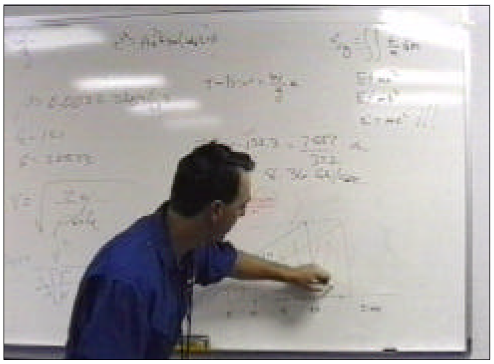


Figure 6.4 Plot of Speed vs. Velocity

NOTE:

The time required to accelerate to the takeoff speed can also be found using the following relation:

$$time = \frac{\text{velocity}}{\text{acceleration rate}}$$

$$time = \frac{(152.1\,\frac{ft}{\text{sec}})}{\left(6.74\,\frac{ft}{\text{sec}^2}\right)}$$

$$time = 22.6 \text{ sec}$$

To calculate the revised takeoff distance using the same relationship as before (recall the area for a right triangle), the distance should be:

$$area = \tfrac{1}{2}(base \,\%\, height) = takeoff\ distance$$

$$area = \tfrac{1}{2}[(22.6\,\text{sec})(152.1\,\tfrac{ft}{\text{sec}})]$$

$$takeoff\ distance = 1718.7 \text{ feet}$$

A subsequent takeoff test revealed the takeoff distance to be approximately 1750 feet so the theory appears correct.

Figure 6.5 Takeoff distance determination from chase aircraft

9.0 Summary

Takeoff performance is mainly concerned with the distance required to accelerate the aircraft to a speed where the lift just begins to exceed the weight. The weight, drag and thrust of the aircraft are used in the $F = ma$ equation to determine the acceleration rate. Neglecting the rolling friction yields an acceleration rate which is too high, since the friction acts as a drag force. Assuming the calculated acceleration rate is constant, it is used as the slope of a line on a graph of speed versus time. The lift equation determines the takeoff speed and the time required to accelerate to that speed is found by intersecting the acceleration line at that speed and dropping down to the "time" axis. The area of the triangle formed by this procedure is equal to the takeoff distance.

Warning:

In all of the above calculations, there has been more thrust available than required. In other words, there is excess thrust available. This excess thrust is used to accelerate the aircraft above takeoff speed and is also used to allow the aircraft to climb.

Once the aircraft finally becomes airborne, it begins to climb to altitude. The forces involved in climbs and descents are the subject of the next session.

Session 6
Takeoff Performance

10.0 Measures of Performance

1 Define "takeoff speed."

ANSWER

The speed where the lift generated just equals the weight of the aircraft.

2 Why do engineers use 70% of the predicted takeoff speed to determine the aerodynamic drag during the takeoff?

ANSWER

The drag is a function of speed, and the speed is constantly changing during the takeoff roll. Experience has shown that a speed of 70% of the takeoff speed provides a good estimation of the average drag encountered during the takeoff.

3 Why is the acceleration rate the slope of a line on the graph of velocity versus time?

ANSWER

By definition, acceleration is the rate of change of velocity for a given time period. In this application the acceleration is assumed to be constant, therefore the slope is a straight line.

4 Why does rolling friction increase the takeoff distance?

ANSWER

Friction is an additional term which acts in the same direction as the drag. Since this is opposite the thrust, the amount of friction must be subtracted from the thrust. The resulting lower thrust decreases the acceleration rate, increasing the takeoff distance.

5 If an aircraft only has enough thrust to accelerate it to takeoff speed, what is the consequence?

ANSWER

There is no excess thrust, the thrust exactly equals the drag and friction, therefore the aircraft cannot accelerate or climb.

11.0 Example

***Problem*:**

Determine the takeoff distance for an aircraft with the following characteristics:

Weight (w): 18,500 lbs
Thrust (T): 6000 lbs
C_{Lmax}: 1.15
C_D: 0.05
Wing area (S): 342 ft^2
Coefficient of friction (μ): 0.06
Air density (ρ): 0.0023 slugs/ft^3

Solution:

Step 1: Determine the takeoff speed.

$$w = L = \tfrac{1}{2} pV^2 SC_L \qquad (6.1)$$

$$V = \sqrt{\frac{2w}{pSC_L}}$$

$$V = \sqrt{\frac{2(18,500 lbs)}{(0.0023\frac{slugs}{ft^3})(342 ft^2)(1.15)}}$$

$$V = 202.2 \text{ ft/sec (or 119.7 knots)}$$

Step 2: Determine the drag by using the drag equation and 70% of the takeoff speed calculated in step 1.

$$V = 0.7\ (202.2 \text{ ft/sec})$$

$$V = 141.5 \text{ ft/sec}$$

$$D = \tfrac{1}{2} pV^2 SC_D \qquad (6.2)$$

$$D = \tfrac{1}{2}\left(0.0023\frac{slugs}{ft^3}\right)\left(141.5\frac{ft}{\sec}\right)^2 (342 ft^2)(0.05)$$

$$D = 393.7 \text{ lbs}$$

Step 3: Determine the friction force based on the aircraft weight and a coefficient of friction of 0.06.

$$F_{friction} = (\mu)\ w$$

$$F_{friction} = (0.06)\ (18,500 \text{ lbs})$$

$$F_{friction} = 1100 \text{ lbs}$$

Step 4: Determine the acceleration by subtracting the forces calculated in steps 2 and 3 from the total thrust.

$$F = ma$$

$$F = T - D - F_{friction} = \frac{w}{g} a$$

$$a = \frac{g}{w}F = (T - D - F_{friction})$$

$$a = \frac{\left(32.2\frac{ft}{\sec^2}\right)}{(18,500lbs)}(6000lbs - 393.7lbs - 1100lbs)$$

$$a = \text{ 7.84 ft/sec}^2$$

Step 5: Determine the time required to accelerate to the takeoff speed.

$$time = \frac{\text{takeoff speed}}{\text{acceleration rate}}$$

$$time = \frac{202.2\frac{ft}{\sec}}{7.84\frac{ft}{\sec^2}}$$

$$time = 25.79 \text{ sec}$$

Step 6: Determine the takeoff distance assuming the acceleration calculated in step 5 is a constant.

NOTE:

This assumption results in a straight line slope which permits the use of the right triangle formula to calculate the area under the curve.

$$area = \tfrac{1}{2}(base \,\%\, height)$$

$$takeoff\ distance = \tfrac{1}{2}(takeoff\ speed \,\%\, time\ to\ reach\ takeoff\ speed)$$

$$area = \tfrac{1}{2}\{(25.79\,\sec) \,\%\, (202.2\tfrac{ft}{\sec})\}$$

$$area = takeoff\ distance = 2607.5\ ft$$

Problem 1: An aircraft with the following characteristics is operated from a runway 3000 feet long. Can the aircraft takeoff ?

Weight (w): 1800 lbs
Thrust (T): 850 lbs
$C_{L\,max}$: 1.21
C_D: 0.05
Wing area (S): 48 ft^2
Coefficient of friction (μ): 0.03
Air density (ρ): 0.0021 slugs/ft^3

ANSWER

Step 1: Determine the takeoff speed. The lift equation is used since this is the speed where the lift is just equal to the weight.

$$w = L = \tfrac{1}{2}pV^2SC_L \qquad (6.1)$$

$$V = \sqrt{\frac{2w}{pSC_L}}$$

$$V = \sqrt{\frac{2(1800lbs)}{\left(0.0021\frac{slugs}{ft^3}\right)(48ft^2)(1.21)}}$$

$$V = 171.8 \text{ ft/sec (or 101.7 kts)}$$

Step 2: Determine the drag by using the drag equation and 70% of the takeoff speed calculated in step 1.

$$V = .7\ (171.8 \text{ ft/sec})$$

$$V = 120.3 \text{ ft/sec}$$

$$D = \tfrac{1}{2}pV^2SC_D \qquad (6.2)$$

$$D = \tfrac{1}{2}\left(0.0021\frac{slugs}{ft^3}\right)(120.3\tfrac{ft}{\sec})^2(48ft^2)(0.05)$$

$$D = 36.4 \text{ lbs}$$

Step 3: Determine the friction force based on the aircraft weight and a coefficient of friction of 0.03.

$$F_{friction} = (\mu)\ w$$

$$F_{friction} = (0.03)\ (1800 \text{ lbs})$$

$$F_{friction} = 54 \text{ lbs}$$

Step 4: Determine the acceleration by subtracting the forces calculated in steps 2 and 3 from the total thrust.

$$F = ma$$

$$F = T - D - F_{friction} = \frac{w}{g}a$$

$$a = \frac{g}{w}(T - D - F_{friction})$$

$$a = \frac{32.2\frac{ft}{\sec^2}}{1800lbs}(850lbs - 36.4lbs - 54lbs)$$

$$a = 13.59 \text{ ft/sec}^2$$

Step 5: Determine the time required to accelerate to the takeoff speed.

$$time = \frac{\text{takeoff speed}}{\text{acceleration rate}}$$

$$time = \frac{171.8\frac{ft}{\sec}}{13.59\frac{ft}{\sec^2}}$$

$$time = 12.64 \text{ sec}$$

Step 6: Determine the takeoff distance assuming the acceleration calculated in step 5 is a constant.

NOTE:

This assumption results in a straight line slope which permits the use of the right triangle formula to calculate the area under the curve.

$$area = \tfrac{1}{2}(base \times height)$$

$$area = \tfrac{1}{2}\{(12.64\text{ sec}) \times (171.8\tfrac{ft}{\sec})\}$$

$$area = takeoff\ distance = 1085.9 \text{ feet}$$

YES; there is enough runway to takeoff.

Problem 2: How much thrust is required to takeoff on a 3500 foot runway if an aircraft has the following characteristics:

- Weight (w): 3500 lbs
- Thrust (T): **?** lbs
- C_L: 1.21
- C_D: 0.05
- Wing area (S): 90 ft^2
- Coefficient of friction (μ): 0.03
- Air density (ρ): 0.0021 slugs/ft^3

ANSWER

Step 1: Determine the takeoff speed.

$$w = L = \tfrac{1}{2}\rho V^2 S C_L \qquad (6.1)$$

$$V = \sqrt{\frac{2w}{\rho S C_L}}$$

$$V = 175 \text{ ft/sec (or 103.6 knots)}$$

Step 2: Since the takeoff distance must be at least 3500 feet, the acceleration can be determined from the following relationship:

$$area = \tfrac{1}{2}(base \times height)$$

$$area = takeoff\ distance$$

$$takeoff\ distance = \tfrac{1}{2}(takeoff\ speed \times time\ to\ reach\ takeoff\ speed)$$

$$3500 \text{ feet} = 1/2\ (175 \text{ ft/sec})(time)$$

$$time = 40 \text{ sec}$$

Since the time required to accelerate to takeoff speed is defined as:

$$time = \frac{\text{takeoff speed}}{\text{acceleration rate}}$$

Then the acceleration rate is found by:

$$acceleration\ rate = \frac{takeoff\ speed}{time}$$

$$a = \frac{175\frac{ft}{\sec}}{40\sec}$$

$$a = 4.38 \text{ ft/sec}^2$$

Step 3. Calculate the friction force:

$$F_{friction} = (\mu)w$$

$$F_{friction} = (0.03)\ (3500 \text{ lbs})$$

$$F_{friction} = 105 \text{ lbs}$$

Step 4: Determine the drag by using the drag equation and 70% of the takeoff speed calculated in step 1.

$$V = 0.7\ (175 \text{ ft/sec})$$

$$V = 120.3 \text{ ft/sec}$$

$$D = \tfrac{1}{2}\rho V^2 S C_D \qquad (6.2)$$

$$D = \tfrac{1}{2}\left(0.0021\tfrac{slugs}{ft^3}\right)(120.3\tfrac{ft}{\sec})^2(90ft^2)(0.05)$$

$$D = 68.4 \text{ lbs}$$

Step 5: Determine the thrust required by rearranging the $F = ma$ equation.

$$F = ma$$

$$F = T - D - F_{friction} = \frac{w}{g}a$$

$$T = \frac{w}{g}(a) + D + F_{friction}$$

$$T = \frac{3500lbs}{32.2\frac{ft}{\sec^2}}\left(4.38\tfrac{ft}{\sec^2}\right) + 68.4lbs + 105lbs$$

$T = 649.6$ lbs

NOTE:

This thrust will EXACTLY equal all of the opposing forces. Therefore, the aircraft will never accelerate beyond takeoff speed. In order for the aircraft to climb or accelerate, MORE thrust is required.

Session 7
Climb and Descent Performance

1.0 Definitions

Rate-of-climb - The straight-up vertical velocity, measured in feet per second. The abbreviation for rate-of-climb is *RC*.

Climb angle - The number of degrees between the horizon and the flightpath of the aircraft. The abbreviation for climb angle is the Greek letter gamma, γ.

2.0 Introduction

The climb performance of an aircraft is an important safety of flight consideration as it determines the capability of the aircraft to clear an obstacle after takeoff, enroute terrain avoidance, and go-around capability from an aborted landing. Due to the safety of flight considerations, the Federal Aviation Administration (F.A.A.) has minimum angle of climb criteria for those flight modes close to the ground such as takeoff and landing and also for the engine out (emergency) phase of flight for multi-engine aircraft.

Figure 7.1 Takeoff

3.0 Theory

From basic trigonometry,

$$\sin \gamma = RC/V$$

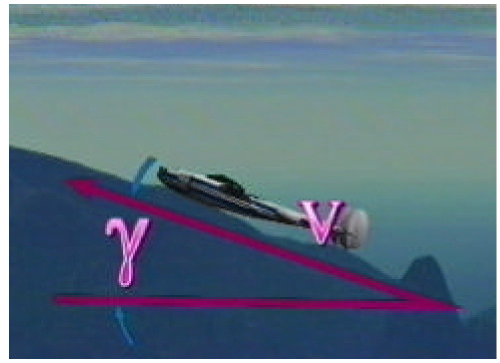


Figure 7.2 Climb Angle

Video Example: aircraft is flying at 200 knots [or 338 ft/sec]. If the climb <u>angle</u> is 10 degrees, then $RC = V \sin g = 338[ft/sec] \sin 10° = 58.7[ft/sec]$

3.1 Simplifying Assumptions

The first simplifying assumption is used only at the basic level to illustrate the principle factors in climb performance: **The aircraft's angle of attack is small.**

Figure 7.3 Angle of Attack is Small

NOTE:

A non-trivial angle of attack complicates the equations a bit, but is used for all accurate analysis: Summing up the forces <u>along the direction of the flightpath</u>, the tilt of the thrust line must be included:

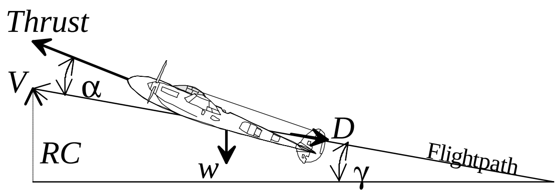


Figure 7.4 $\Sigma F = T \cos \alpha - D - w \sin \gamma$

Along the flightpath:

$$\Sigma F = T \cos \alpha - D - w \sin \gamma$$

Figure 7.5 Mass is Constant

The next simpifying assumption: **The aircraft's mass is constant** is quite reasonable for all propeller aircraft and most jets. This assumption simplifies the equation to:

$$T - D - w \sin \gamma = m\, \Delta V/\Delta t$$

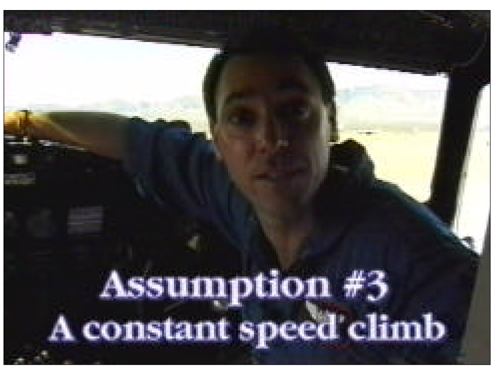


Figure 7.6 Airspeed is Constant

The equations used if the "constant mass" assumption is not valid, can be found in the "energy method" section of this guide. This section also shows how to look at a plane's ability to climb and accelerate at the same time. To avoid complication, the video made a third simplifying assumption: **The aircraft is climbing at a constant speed** so that $\Delta V/\Delta t = 0$.

With no mass or velocity change, the sum of the forces is zero:

$$T - D - w \sin \gamma = 0 \quad \text{or} \quad [T - D]/w = \sin \gamma.$$

The numerator $[T - D]$ is called the excess thrust because it is the extra thrust available after the aircraft's drag is overcome.

The video stated that there are several ways to measure the drag; it repeated the method in Session 4 with the glider and calculated its drag at a particular airspeed. This gliding test works great on sailplanes, but not on big airplanes because it isn't safe to shut down all the engines. In this case, flight testers use knowledge of engine thrust which is opposite to the drag force. The thrust prediction provided by the manufacturer is usually very complicated, but a simplified version was used in the video.

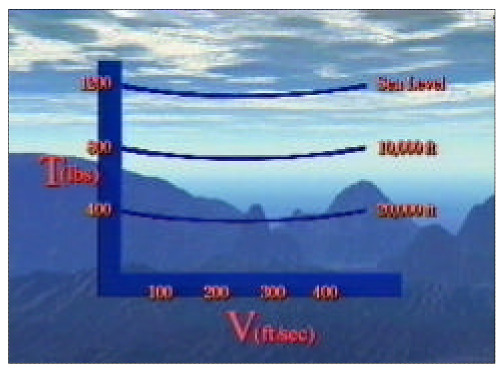


Figure 7.7 Thrust Prediction

At any one altitude, the thrust changes just a little as the airspeed increases. For any given airspeed, the thrust gets smaller as altitude increases. For the example in the video, the flight condition was 300 ft/sec and 5,000 feet altitude. The thrust was 1000 lbs., the drag was 400 lbs. and the aircraft weight was 4200lbs. Putting all of this together gave:

$$[1000 \text{ lbs.} - 400 \text{ lbs.}]/4200 \text{ lbs.} = \sin \gamma$$

$$\text{or} \quad 0.1428 = \sin \gamma$$

$$\text{solving for } \gamma \text{ gave} \quad \gamma = \sin^{-1} 0.1428 = 8.2^{\circ}$$

The above relation for sin γ can be inserted into the rate-of-climb equation ($RC = V \sin \gamma$) to give

Climb Rate Equation:

$$RC = V\frac{[T-D]}{w}$$

so, $RC = 300$ *ft/sec* (0.1428) = 42.8 *ft/sec.*

3.2 Climb Rate vs Velocity

Drag increases with the square of velocity. Compare the drag to the engine thrust available at sea level. The vertical distance between the two curves is the excess thrust, $F - D$. As the airspeed increases, the excess thrust gets smaller and smaller. At very low speeds there is a lot of excess thrust, but the velocity is small, so the climb rate is moderate. At medium speeds, there is not quite as much excess thrust, but multiplying it by the higher speed gives a good climb rate. Finally, at high speed, the excess thrust is very small. Even though the speed is high, the product of the two yields a poor climb rate. This should sound reasonable since most of the available thrust is needed just to overcome the drag, leaving little excess thrust for climbing.

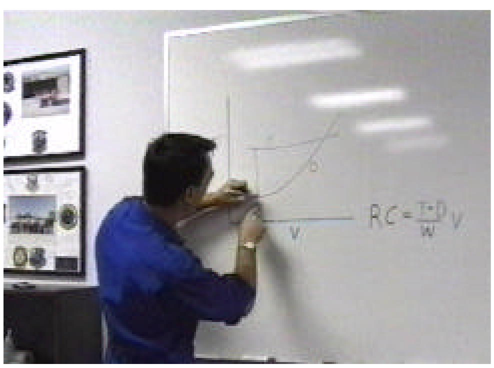


Figure 7.8 Execss Thrust

To determine the altitude effect on climb performance, first go back to the engine chart. Since the air is less dense at high altitude, the maximum thrust of the engine will also be less. At 23,000 feet where the density is half of that at sea level, the thrust will also be about half of the sea-level value. Of course, the profile drag will also be about half of the sea-level value. Since both the thrust and drag are reduced by 50%, then the excess thrust reduction will be the same. Finally, the climb rate and angle will be about half of the sea level climb capability.

4.0 Power Method

Climb performance is directly related to the excess **power** available. This is the difference between the power required for level flight and the power available from the propulsion system at a particular airspeed and density altitude. The video showed that climb performance is a function of excess thrust available, which is also true. The connection between the two is quite simple: thrust times velocity equals power ($P = TV$).

The video showed that climb rate is $RC = V\frac{[T-D]}{w}$ where $\frac{[T-D]}{w}$ is the specific excess thrust. Climb rate is velocity *times* specific excess thrust or simply specific excess power. In a similar fashion, since the sine of the climb angle is the specific excess thrust, then it is also the specific excess power *divided* by the speed.

$$\sin c = \frac{F-D}{w} = \frac{(F-D)V}{w}\frac{1}{V}$$

Figure 7.7 illustrates this for both a jet and a propeller aircraft. The excess *power* can be used to either climb or accelerate the aircraft; therefore, knowledge of the excess power available at each altitude and airspeed will define the aircraft climb performance, level acceleration performance, or any combination of the two. Conversely, measurement of the climb and/or acceleration performance of an aircraft will define the specific excess power.

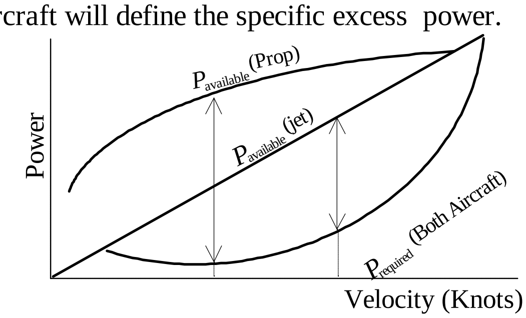


Figure 7.9 Maximum Rate of Climb, Prop and Jet

5.0 Energy Method for Climb Performance

If an energy approach is used where the total energy of an aircraft is expressed as the sum of the potential and kinetic energy, basic physics states that a change in energy requires that work be done (Figure 7.8).

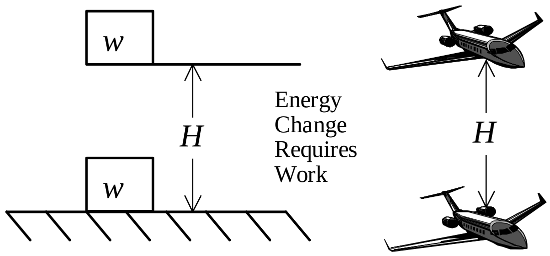


Figure 7.10 Power Available - Physics

The rate of change of energy requires the application of power which is the work done per time interval. Since the total energy of the aircraft is changed by the excess power available then:

Excess Power Available

$$= \frac{d}{dt}\text{ [Potential Energy + Kinetic Energy]}$$

$$= \frac{d}{dt}\left[wH + \frac{w}{2g}V^2\right]$$

$$= w\frac{dH}{dt} + H\frac{dw}{dt} + V\frac{w}{g}\frac{dV}{dt} + \frac{V^2}{2g}\frac{dw}{dt}$$

This complete equation is needed for rockets and aircraft with extreme fuel flow rates such as the F-22 in full afterburner. For most general aviation commercial transport aircraft however, the rate of change of weight $\frac{ds}{dt}$ is very small and can be neglected with the result that:

$$\text{Excess Power} = X_s P = \left[w\frac{dH}{dt} + \frac{wV}{g}\frac{dV}{dt}\right]$$

where $\frac{dH}{dt}$ is the time rate of change of altitude, $\frac{dV}{dt}$ is the time rate of change of true velocity in ft/sec and V is the velocity in ft/sec The unit of power is ft-lb per second. Since an aircraft has a fixed amount of excess power at any given flight condition, this equation can be used to show the plane's ability to climb at constant velocity, accelerate at constant altitude, or some combination of both climb and acceleration.

To **measure** the excess power available at any altitude, it is necessary to measure the rate of climb, $\frac{dH}{dt}$ and the flight path acceleration, $\frac{dV}{dt}$. The common technique is to keep one of the variables constant and measure the rate of change of the other. The excess power can thus be measured by the rate of climb (sawtooth climb) test or by the level acceleration. The term "sawtooth" climb is used to describe a series of climbs where the pilot climbs through an altitude band at some constant airspeed (so that $\frac{dV}{dt} = 0$), then descends, then repeats the climb at another constant airspeed and so forth.

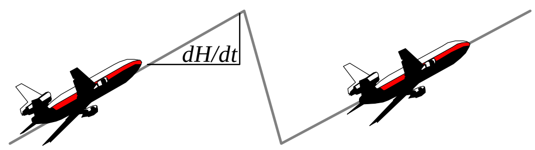


Figure 7.11 Sawtooth Climb Technique

During each climb, the pilot records the airspeed and weight and times the ascent with a stopwatch to get the climb rate. The results of a series of sawtooth climb tests can be plotted as shown in Figure 7.11.

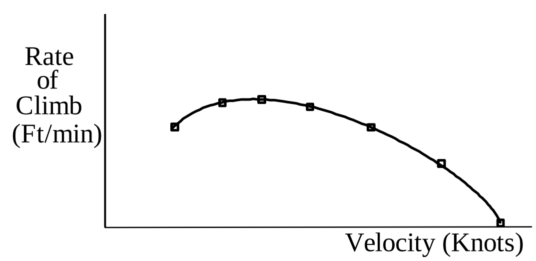


Figure 7.12 Plot of Sawtooth Climb Data

6.0 Data Analysis

When the rate of climb data is taken at different altitudes, corrected it can be presented as seen in Figure 7.11. The top of each curve gives the maximum rate of climb at particular altitudes and the speed that must be held to obtain that maximum rate of climb. The tangents from the origin give the velocities for the maximum angle of climb. The speeds for maximum angle of climb and maximum rate of climb are defined as V_x and V_y respectively. A typical plot of the variation of V_x and V_y with altitude is given in Figure 7.12 where it can be seen that at the absolute ceiling of the aircraft $V_x = V_y$.

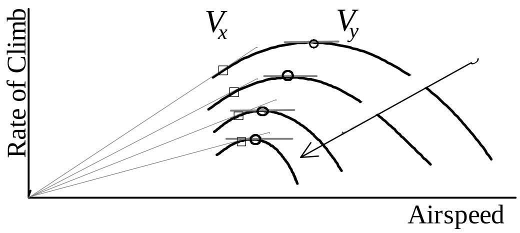


Figure 7.13 Climb Data as a Function of Altitude

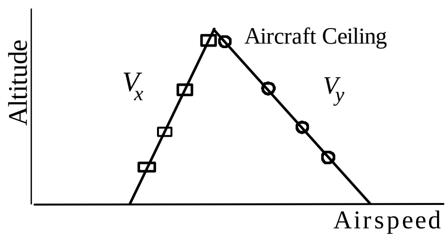


Figure 7.14 Variation of V_x and V_y with Altitude

7.0 Descents

This very same rate and angle of climb equations also work for an airplane that is descending at a constant airspeed. All the pilot has to do is decrease the thrust until it is less than the drag. This means that the excess thrust is a *negative* value. Substituting a negative value into the climb rate equation means the aircraft is descending. If the excess thrust is a *large* negative value, then the airplane will descend faster. This concept was shown with the glider in Session 4 (although the intent of that video segment was to illustrate the change in drag). The brakes added more drag thereby making a more negative excess thrust.

8.0 Summary

Assuming a small angle of attack,

$$RC = V \sin \gamma$$

Then, starting from Newton's second law and assuming a constant mass and velocity, simple calculations give the equation for predicting climb angle. Note that the climb angle is directly related to the *specific excess thrust*.

$$[T - D]/w = \sin \gamma$$

And finally, combining these two gave the rate of climb equation. Note that the climb rate is directly related to the *specific excess power*.

$$RC = V[T - D]/w$$

Examples 5.1 and 5.2 in the textbook give further illustrations of these lessons.

9.0 Measures of Performance

1 What happens to the climb rate and climb angle of an aircraft if the weight increases?

ANSWER:

Since $RC = \frac{V(T-D)}{w}$ then a larger weight **reduces** climb rate. Similar result of climb angle.

2 Why does the climb rate decrease at high altitudes?

ANSWER:

Because less thrust is available at low air density.

3 What climb measurement is directly related to specific excess thrust?

ANSWER:

Specific Excess Thrust = $\frac{T-D}{w} = \sin c$ it's related to climb **angle**.

4 For the simplified math presented in the video, what were the assumptions?

ANSWER:

Negligible angle of attack, weight change and velocity change

Session 8
Cruise Performance

1.0 Definitions

Endurance - A measure of how long an aircraft is able to remain airborne on a given amount of fuel.

Fuel flow - The number of gallons (or pounds) of fuel used per hour of flight time.

Maximum endurance airspeed - The airspeed (for a given weight and altitude), where the fuel flow is the minimum. The low fuel flow permits the aircraft to remain aloft longest.

Range - A measure of how far an aircraft can go with a given amount of fuel.

Maximum range airspeed - The airspeed that results in the best ratio of fuel flow to airspeed. This airspeed results in the maximum distance for a given amount of fuel.

2.0 Introduction

Many performance parameters tested on a car or an airplane are ancillary to the overall purpose of the vehicle. Evaluating the horsepower available, the takeoff distance, or the acceleration rate are all secondary factors for the real purpose of a motorized vehicle; the primary being how efficiently does it get from point ***A*** to point ***B***. That efficiency is usually measured in miles per gallon because it directly relates to miles per dollar. If the fuel mileage of the vehicle is low, it costs more dollars per mile to operate. So even though an aircraft may be able to achieve Mach 2, it cannot remain at Mach 2 for very long because it uses a lot of fuel when flying at high speeds.

The cruise performance of an aircraft is measured in two specific areas; 1) how **long** can the aircraft remain airborne on a specific amount of fuel (commonly referred to as its *endurance*) and 2) how **far** can the plane travel on a given amount of fuel (referred to as the aircraft's *range*). This session investigates the factors influencing an aircrafts cruise performance and describes how to determine the best cruise speed for both endurance and range.

NOTE:

See section 7.4 of the accompanying text for additional information on determining cruise performance of an aircraft.

3.0 What is Range Performance?

Range performance can be presented in the form of a ratio between distance travelled and fuel used. A ratio is found by dividing one term by another. Ratios can also be graphically represented by plotting the numerator on the ordinate and the denominator on the abscissa. From this graph, the slope of the line drawn is the ratio. In other words, the ratio gives the rate of change of one parameter with reference to another. For example, to find a car's fuel mileage experimentally, you might proceed in the following manner:

a) fill the car with fuel
b) drive for 100 miles
c) refill the car with fuel
d) divide the number of miles travelled (100) by the amount of fuel you just put in (possibly 4 gallons)
e) the result would be 25 miles per gallon

To look at this problem graphically, plot miles on the vertical axis and gallons on the horizontal axis. Then place a mark at the point which corresponds to 100 miles and 4 gallons. Drawing a line from the origin to this point graphically shows the ratio between miles driven and fuel used, as shown in Figure 8.1.

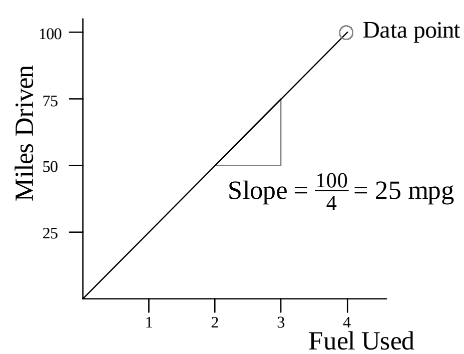


Figure 8.1 Ratio of Miles Driven vs. Fuel Used

The slope of the line is the "miles-per-gallon." The usefulness of a graph like this can be seen when considering a trip of less than 100 miles. To determine the amount of gas need for a 58 mile trip, enter the graph on the vertical axis at "58," go across until it meets the slope line, then drop down to the horizontal axis. The value will be 2.32 gallons (Figure 8.2).

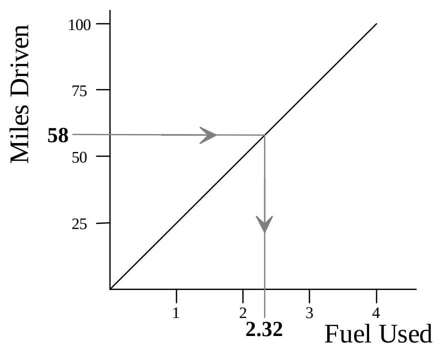


Figure 8.2 Determining Fuel for Trip

However, the above test was performed at only one speed. To find the *best* speed, a similar test would have to be accomplished for several speeds resulting in a series of graphs. Determine the slope of the graph for each speed tested as in Figure 8.1.

To determine the best speed to travel for the maximum gas mileage, make another graph. For each of the speeds tested plot the slope versus the speed from which it came. This procedure yields a curve similar to that shown in Figure 8.3.

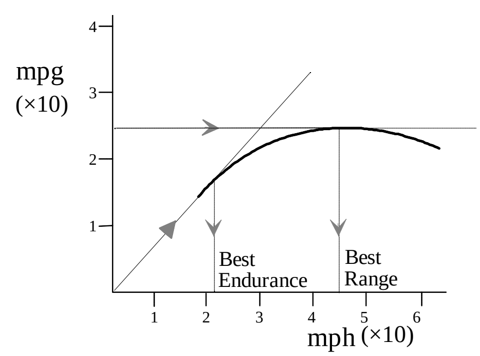


Figure 8.3 Miles per Gallon
vs. Miles per Hour

Two valuable pieces of information are available from this curve: a tanget from the origin to the curve (that is to a point where the line just touches the curve) shows the speed and fuel mileage which will result in the car's best *endurance*. In other words, for a given amount of fuel, traveling at this speed will result in the *longest time* between fuel stops.

The point at top of the curve is the speed and fuel mileage for the car's *best range*. This allows the *farthest* distance between fuel stops.

You've just completed a test to find the cruise performance of your car and presented the data in a manner which is useful to the owner.

Caution:

Each of the tests must be conducted on the same section of road. Conducting one test on a flat road and another on a hill will obviously interfere with the results. Additionally, if the tests are conducted on the same day, atmospheric effects (wind and density changes) and road surface conditions (wet, icy, etc.) can be minimized.

Although the terminology is a little different, the cruise performance of an aircraft is determined in much the same manner.

4.0 Determining the Maximum Endurance Airspeed

When an aircraft is in level, unaccelerated flight, it is usually thought of as being in a cruise condition. Since the plane is not accelerating in any given direction, Newton says the forces acting on the airplane are balanced. Recall from earlier sessions this means the lift equals the weight and the thrust equals the drag. Then the force equations which describe cruise flight are written as:

$$L = w = {}^{1}\!/_{2}\, \rho\, V^2\, SC_L \qquad (8.1)$$

and

$$D = T = {}^{1}\!/_{2}\, \rho\, V^2\, SC_D$$

The way to determine how efficient the aircraft is in the cruise configuration is look at the amount of drag at some weight. This is logical because more thrust required means more fuel burned, which in turn costs more money. To accomplish this, a ratio of lift to drag (or weight to thrust) is created. Equation 8.1 can be used to show the similarity between the lift-to-drag ratio and the C_L to C_D ratio:

$$\frac{L}{D} = \frac{\frac{1}{2}\rho V^2 S C_L}{\frac{1}{2}\rho V^2 S C_D} = \frac{C_L}{C_D}$$

The values are put into the coefficient form because it is a more general reflection of the relationship between lift and drag. Rearranging Equations 8.1 to solve for the coefficients of lift and drag yields the following relationships:

$$C_L = \frac{2w}{\rho V^2 S} \qquad (8.2)$$

and

$$C_D = \frac{2T}{\rho V^2 S}$$

By putting these measurements into Equation 8.2 the lift and drag coefficients can be determined. Plotting the values of C_L and C_D for a plane results in a curve which takes on the shape of a parabola, as shown below. From this drag curve we can obtain the same information for endurance that we did for the car.

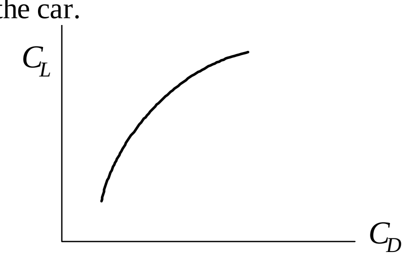


Figure 8.4 Drag Curve

Drawing a line from the origin to the tangent, the point of intersection occurs where the ratio of C_L to C_D is the maximum. This is illustrated below:

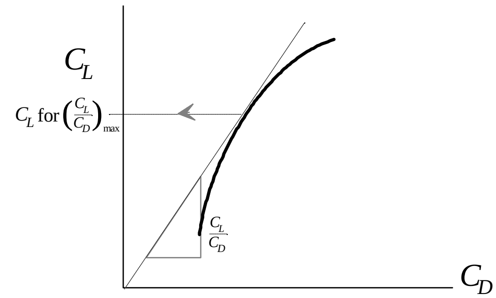


Figure 8.5 Determining Tangent

The slope of the tangent line is the maximum C_L to C_D ratio. By drawing a line from the tangent point to the C_L axis, the *optimum* lift coefficient is determined. Inserting this value of C_L into Equation 8.2, the optimum velocity is found. This speed yields the maximum C_L to C_D ratio.

$$V = \sqrt{\frac{2w}{\rho S C_L}} \qquad (8.3)$$

This velocity is the "maximum endurance airspeed" and gives the pilot the greatest amount of time airborne for a given amount of fuel. This is the speed the pilot would fly if stuck in a holding pattern.

NOTE:

After considering the problem, it should seem logical that the best endurance occurs at the plane's best lift-to-drag ratio (same as C_L/C_D ratio):

1. The best endurance occurs when the fuel flow is as low as possible.
2. Since fuel flow is directly related to thrust, the best endurance should come at the condition for minimum thrust.
3. Since thrust equals drag in cruising flight, the thrust (and fuel flow) will be lowest when the drag is lowest.
4. For any given weight, the lowest drag occurs when the lift-to-drag ratio is highest.
5. Since $L/D = C_L/C_D$, then the highest C_L/C_D ratio (tangent point) yields the best endurance.

Consider the following example:

***Example 1*:**

During a test flight, the following data is collected:

Velocity (ft/sec)	**Weight** (lbs)	**Thrust** (lbs)
591	10,000	1,350
513	9,850	925
440	9,700	600
366	9,550	400
293	9,400	250
257	9,250	215
220	9,100	200
205	8,950	205
190	8,800	220
184	8,650	240

If the aircraft has a wing area of 205.33 square feet and is flying at an altitude where the density is 0.002 slugs per cubic foot, what is the maximum endurance lift coefficient and airspeed for a 9000 pound aircraft?

***Answer*:**

***Step 1*:** Compute the lift coefficient (C_L) and drag coefficient (C_D) for each point in the table above using Equation 8.2.

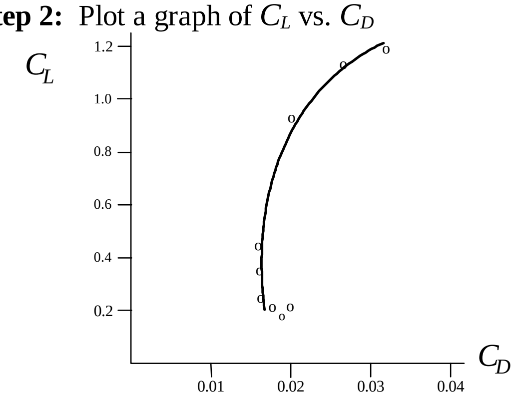
$$C_L = \frac{2w}{\rho V^2 S} \qquad (8.2)$$

and

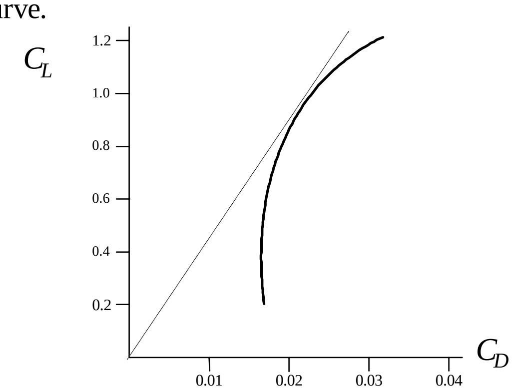
$$C_D = \frac{2T}{\rho V^2 S}$$

Velocity (ft/sec)	**Weight** (lbs)	**Thrust** (lbs)	C_L	C_D
591	10,000	1,350	0.14	0.019
513	9,850	925	0.18	0.017
440	9,700	600	0.24	0.015
366	9,550	400	0.35	0.015
293	9,400	250	0.53	0.014
257	9,250	215	0.68	0.016
220	9,100	200	0.92	0.02
205	8,950	205	1.04	0.024
190	8,800	220	1.19	0.03
184	8,650	240	1.24	0.035

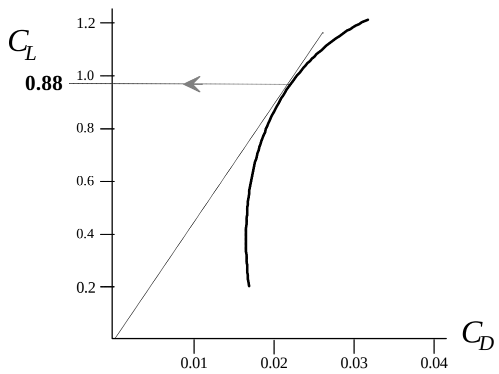
Step 2: Plot a graph of C_L vs. C_D



Step 3: Draw a tangent line from the origin to the curve.



Step 4: From the tangent point, determine the optimum C_L by drawing a line to the vertical axis.



Step 5: For this value of C_L, use Equation 8.3 to determine the velocity for an aircraft which weighs 9000 pounds.

$$V = \sqrt{\frac{2w}{\rho S C_L}} \qquad (8.3)$$

$$V = \sqrt{\frac{2(9000\ lbs)}{(0.002\ slugs/ft^3)(205.33\ ft^3)(0.88)}}$$

$V =$ 223 ft/sec (152 miles/hr)

However, this speed only applies to the weight entered into Equation 8.3. As fuel is burned, the weight will decrease, therefore the lift required also goes down. The result is the speed associated with the optimum C_L is lower. Therefore, to achieve the maximum endurance, the aircraft should fly at a slower speed.

While traveling large distances, instead of maximizing the *time* spent airborne, the mileage is the main area of interest. To optimize the mileage, an aircraft will fly at the optimum airspeed for range.

5.0 Determining the Maximum Range Airspeed

When an aircraft flies at its maximum range airspeed, it travels the maximum distance for a given amount of fuel. In other words, it yields the best fuel mileage and is therefore most cost efficient. Through experience, engineers have determined that the C_L for maximum range airspeed is approximately equal to 70% of the C_L for maximum endurance. By inserting this value into the lift equation the maximum range airspeed is calculated.

NOTE:

The details of this approximation are outside the scope of this course. There may be some question as to why the point of best C_L/C_D is not also the best range condition. The qualitative explanation is: The condition of best C_L/C_D is the absolute lowest fuel flow possible. Flying at this condition turns out to be a fairly slow speed.

If the pilot adds more thrust, then both the fuel flow and speed increase. The key is that the fuel flow increases only a little, but the speed increases a lot. This means an increase in mileage. If, however, the pilot adds **too** much more thrust, then just the opposite happens and the milage goes down. Experience and analysis shows that the proper amount of extra thrust occurs when the C_L is only 70% of the C_L for best endurance

The following example will highlight the relationship between maximum endurance and maximum range airspeeds.

***Example 2*:** Using the same aircraft and test data from Example 1, what is the maximum range airspeed for the 9000 pound aircraft?

Answer:

Step 1: Since the C_L for maximum endurance has already been determined then simply multiply this number by 0.70.

(0.88) (0.70) = 0.62

Step 2: Place this value of C_L into Equation 8.3.

$$V = \sqrt{\frac{2w}{\rho S C_L}} \qquad (8.3)$$

$$V = \sqrt{\frac{2(9000\ lbs)}{(0.002\ slugs/ft^3)(205.33\ ft^3)(0.62)}}$$

$V =$ 265 ft/sec (181 miles/hr)

Notice how this speed is higher than that for maximum endurance. Because the lift coefficient is smaller, the speed must be higher.

Here again, as fuel is used and the weight decreases, so airspeed for maximum range also decreases. Then whether the flight is made at maximum endurance airspeed or maximum range airspeed, as fuel is burned off, the plane's speed should decrease.

The whole reason we fly in aircraft from point **A** to point **B** is to be there quicker. For this reason, its not really desirable to fly *slower* as the plane lightens. In order to keep the speed high, a reexamination of the lift equation is in order.

6.0 How to Keep the Cruise Speed High

As the weight (and subsequently the lift required) decreases and the velocity is kept the same, what other items can be changed to maintain the optimum C_L? To answer this, look again at equation 8.1.

$$L = W = {}^1/_2\,\rho\, V^2\, SC_L \qquad (8.1)$$

Obviously, it's difficult to change the wing area, S, and the goal is to still fly at the optimum C_L. Therefore, to keep the velocity the same, the only factor left to decrease is the air density. The pilot can decrease the density quite easily by flying at a higher altitude. To illustrate this, look at the following example.

***Example 3*:**

Again using the aircraft in Example 1, assuming the aircraft maintains the optimum C_L and airspeed for maximum range, what should the flying altitude be if 2000 pounds of fuel is burned?

Answer:

Step 1: Rearrange Equation 8.1 to solve for the density.

$$L = w = {}^1/_2\,\rho\, V^2\, SC_L \qquad (8.1)$$

$$\rho = \frac{2w}{V^2 SC_L}$$

Step 2: Insert the appropriate values from Example 2.

$$\rho = \frac{2(7000\ lbs)}{(265\ \frac{ft}{\sec})^2(205.33 ft^2)(0.62)}$$

$$\rho = 0.0016\ slugs/ft^3$$

Step 3: The engineers would then use the "Standard Atmosphere Chart" to determine what altitude corresponds to this density. Recall the original density was 0.002 slugs/ft^3. From the table this corresponds to an altitude of 5,000 feet. Similarly, a density of 0.0016 slugs/ft^3, corresponds to an altitude of 12,000 feet. As a result, the pilot would climb to 12,000 feet in order to keep the optimum C_L and airspeed for the maximum range.

This is the procedure aircraft use when flying long distances. The pilot will cruise at a specific altitude until he burns a certain amount of fuel. Then he will climb to another altitude until more fuel is burned, then repeat the process until the point is reached where the descent for landing should begin. This procedure is called a "step climb" profile and works especially well for jet aircraft which burn large amounts of fuel.

The descriptions in Sections 3, 4, and 5 describe how engineers predict the cruise performance of an aircraft. Flight testers then verify these predictions using a slightly different technique.

7.0 Flight Testing Cruise Performance

The test instrumentation used on a test flight greatly assists in verifying the engineering predictions. A test aircraft will be outfitted with a fuel flow meter which measures the amount of fuel the engine (or engines) use per hour.

Figure 8.6 Fuel Flow Meters on Test Aircraft

At each altitude, the aircraft is flown at various airspeeds and the fuel flow at each of those speeds is recorded. This data is then plotted to create a graph similar to the one shown below.

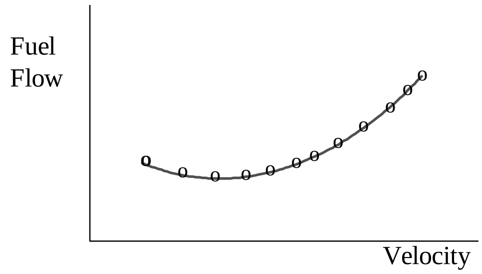


Figure 8.7 Plot of Flight Test Data

This single curve will verify the predicted speeds for both maximum endurance and maximum range. Draw a tangent from the origin tangent to the curve. At the tangent point, a line is draw straight down to the "airspeed" axis and another line is drawn over to the "fuel flow" axis. The axis values are the maximum range airspeed and the fuel flow associated with that airspeed. Figure 8.8 illustrates this procedure:

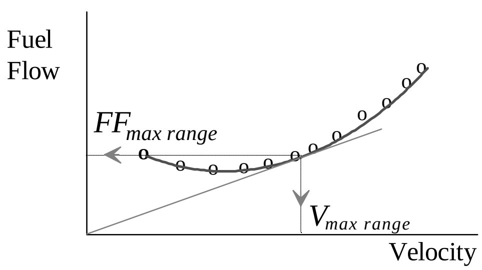


Figure 8.8 Determining Maximum Range Airspeed and Fuel Flow

The bottom of the curve shows the maximum endurance airspeed and fuel flow, as shown in Figure 8.9.

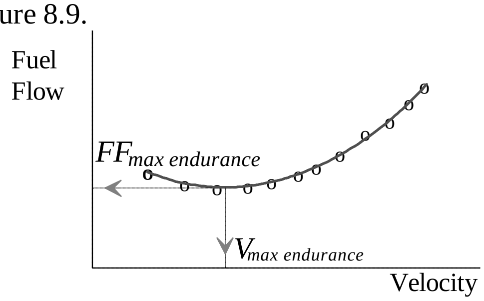


Figure 8.9 Determining Maximum Endurance Airspeed and Fuel Flow

Each altitude will have a curve constructed so the pilot can determine how much fuel he will need and what airspeed he should fly at that altitude.

8.0 Summary

Predicting cruise performance is really straight forward once you realize that the critical fuel flow value is related directly to thrust and drag. Since the forces are balanced in cruise flight, thrust equals drag. The lowest fuel flow occurs at the speed for lowest drag. The best range occurs at the lowest ratio of drag to velocity (Figure 8.8). During the course of a flight test, by measuring the velocity and fuel flow, a graph can be quickly generated to verify the predicted results. Once it has been determined how to efficiently cruise to your destination, the next step is to land. Determining landing performance is the topic of the next session.

9.0 Measures of Performance

1 What is the definition of endurance?

ANSWER

Endurance is how *long* an aircraft can remain airborne on a given amount of fuel.

2 What is the definition of range?

ANSWER

Range is how *far* a vehicle can go on a given amount of fuel.

3 As fuel is burned and the aircraft's weight decreases, what is the best course of action a pilot can take?

ANSWER

As the weight decreases, so does the lift required. As the lift required decreases, equation 8.1 shows that either the speed, lift coefficient, or the density can be decreased. To keep the lift coefficient at the optimum and the speed high, the density should be decreased by climbing to a higher altitude.

10.0 Problems

1. A jet airplane yields the following flight test data. The aircraft weighs 3600 pounds, has a wing area of 125 square feet, and is flown at an altitude where the density is 0.0019 slugs/ft^3. What are the predicted maximum range and endurance airspeeds? Neglect any changes in weight.

Answer:
Step 1: Determine the lift and drag coefficients for each velocity using Equation 8.2.

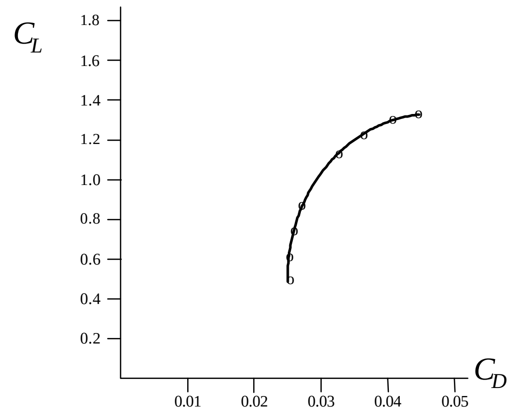
$$C_L = \frac{2w}{\rho V^2 S} \qquad (8.2)$$

and

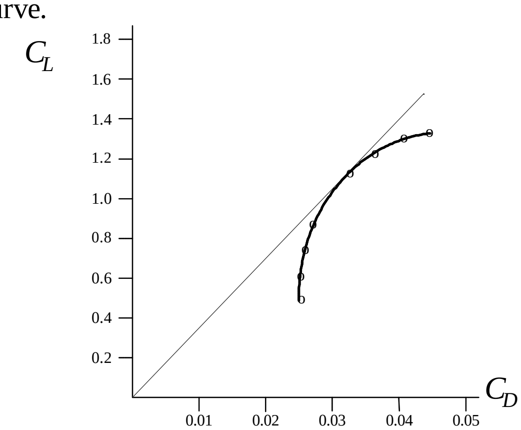
$$C_D = \frac{2T}{\rho V^2 S}$$

Velocity (ft/sec)	**Thrust** (lbs)	C_L	C_D
132	115	1.74	0.05
147	110	1.4	0.04
161	90	1.17	0.029
176	120	0.98	0.032
190	140	0.84	0.033
205	180	0.72	0.036
220	220	0.63	0.038

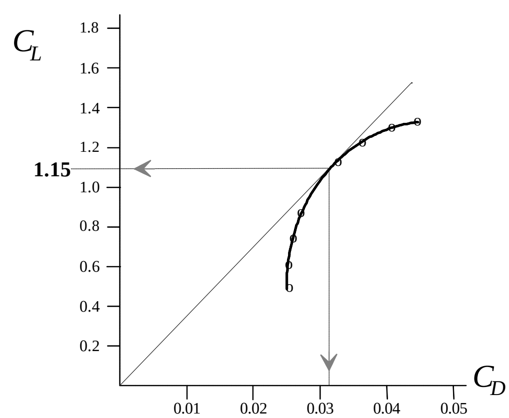
Step 2: Make a graph of C_L vs. C_D



Step 3: Draw a line from the origin tangent to the curve.



Step 4: Draw a line from the tangent point to the vertical axis and read the C_L for maximum endurance.



Step 5: Put the C_L found in step 4 into Equation 8.3 and solve for the best endurance airspeed.

$$V = \sqrt{\frac{2w}{\rho S C_L}} \qquad (8.3)$$

$$V = \sqrt{\frac{2(3600\ lbs)}{\left(0.0019\frac{slugs}{ft^3}\right)(125\ ft^2)(1.15)}}$$

$$\boxed{V = \ 162 \text{ ft/sec (111 miles/hr)}}$$

Answer:

Step 1: Since the C_L for maximum endurance has already been determined then simply multiply this number by 0.70.

$$(1.15)\ (0.70) = 0.85$$

Step 2: Place this value of C_L into Equation 8.3.

$$V = \sqrt{\frac{2w}{\rho S C_L}} \qquad (8.3)$$

$$V = \sqrt{\frac{2(3600\ lbs)}{\left(0.0019\frac{slugs}{ft^3}\right)(125\ ft^2)(0.85)}}$$

$$\boxed{V = \ 194 \text{ ft/sec (132 miles/hr)}}$$

Session 9
Landing

1.0 Definitions

Descent: moving from the cruising altitude to just above the runway.

Approach phase: The point where the pilot guides the airplane around to join the airport traffic pattern and lowers the landing gear and the flaps.

Traffic pattern: an invisible path in the sky around runways that pilots use to smooth traffic flow

Flare: the process of increasing the angle of attack of the wing and its lift. The purpose of the flare is to arrest the sink rate of the airplane just above the runway.

Rollout phase: starts when the plane touches down and ends when it stops rolling.

2.0 Introduction

The landing process has three distinct steps or phases: the approach, the flare to touchdown, and the ground roll after touchdown. Most of the math in this session is simply a variation of what was already described in the sessions on lift, drag, takeoff, and descents.

3.0 Theory

3.1 Descents

The technical look at descents is exactly the same as that for climbs except that the excess thrust is a negative value and therefore gives a negative climb rate. Although descents are not part of the landing session video, it is useful to understand the basic methods used to get down to the airport vicinity.

Consider yourself flying at cruising altitude until you see the runway below. If you simply push the plane into a dive straight for the runway and watch what happens, you'll see the airspeed increase very rapidly. That is because from the moment you push the nose of the airplane downhill, you get the extra thrust due to the weight of the aircraft. This was discussed in more detail in the previous sessions on drag and climb performance. The thrust force <u>was</u> set to exactly cancel the drag force during the level cruise, but that balance is upset in the dive.

You may have guessed that a steeper dive generates a greater thrust-due-to-gravity, and therefore gives a faster acceleration and a higher diving <u>speed</u>. This kind of high speed descent may create a problem. By doing this, you might either overspeed the plane by diving too steeply, or, more likely, end up right near the runway with too much airspeed.

You can have a real problem if you try to land with too much speed on the plane. One problem is that a fast vehicle of any kind is more difficult to control than a slow one. Another problem is that if you land fast, then you'll need more runway to stop.

If you fly near the airport and just push over, you'll end up too fast. Instead, pull the throttle back to idle when pushing over. By reducing the engine's thrust force, you can cancel out the extra thrust force from gravity. Reducing the thrust reduces the tendency to speed up in a descent. You still could fly too fast by nosing over *too* much, but it *is* easier to keep things under control this way.

There is another way of descending that you have already experienced in an airliner. Airline pilots fly along at cruise altitude until they're about 100 miles away from the destination airport. At that point you may hear the plane's engines reduce power slightly.

At the same time the captain will nose over so the speed doesn't change at all. Of course the plane will start descending because it’s now pointed downward. This kind of descent begins long before you see the airport and can take 20 or 30 minutes.

To summarize descents, there are several ways of getting down: You can drop down steeply with idle power and high speed, or descend gradually with partial power and moderate speed, or descend by nosing over to a high speed with full power. Whichever way you get down, the descent phase is complete when you're close enough to the airport to prepare to land.

3.2 Approach

Session 9
Landing

The next step, called the approach phase, is the point where the pilot guides the airplane around to join the airport traffic pattern. Sometimes pilots fly "straight-in" approaches rather than fly in a pattern. This is the usual airline approach. To be sure of being over the runway with just the right combination of speed, altitude and sink rate, the pilot must be considerably more precise when flying the approach as compared to descending. While in the approach phase, the pilot also has to lower the landing gear and the flaps.

Figure 9.1 The Approach

All of the detailed procedures the pilot must follow in the approach phase are designed to do one thing: get the plane into position for the landing flare. To accomplish a safe flare, the plane must be within a range of values for speed, sink rate, and height above the runway. This "window" of numbers must be consistently attainable. To help the pilots be consistent, the approach phase is broken into several steps such as first getting to a specified speed, then lowering the gear, then lowering partial flaps, then slowing to another speed and so forth.

Part of the video discusses the invention and application of flaps. Review the lift discussion in Session 3 where the lift is affected by the wing's velocity, angle of attack and curvature. To get lift at the normal flying speed, the wing has a little bit of curvature and the pilot flies with a little angle of attack. To land, the pilot would want to slow down. To fly slower and still create the same lift, the pilot has to increase the angle of attack. This simple procedure works for typical, light aircraft because they have a lot of wing surface that allows them to fly very slow.

The stall speeds are faster for heavier aircraft than for light ones. The reason for this is the ratio of the weight (*w*) compared to the wing area (*S*). This ratio (*w*/*S*) is called the "wing loading." A sheet of paper (with a lot of area compared to its weight) will easily be lifted by a gentle wind, but a (bound) pad of paper will not because it weighs 50 or 100 times as much. This is the principle of wing loading and is applied to minimum flying speed for heavy and light aircraft. A typical general aviation aircraft (such as the Cessna 172) may have a wing loading of only 11 lbs/ft^2 and a typical airliner may be more like 120 lbs/ft^2. This yields a considerable difference in minimum flying speed.

To further illustrate this idea, consider a simple wing that, due to its cross-sectional shape and maximum angle of attack, has a maximum lift coefficient, C_{Lmax}, of 1.6. We can use the definition of C_L to calculate the minimum flying speed for various wing loading ratios:

Since *by definition* $C_L = 2w/\rho V^2 S$, then

$$V_{min} = [2/\rho C_{Lmax}]^{1/2} \times [w/S]^{1/2}.$$

Picking the standard sea-level value of .002377 for density (ρ), we can calculate the minimum speed for the Cessna as

$$V_{min} = [2/(.002377 \text{ x } 1.6)]^{1/2} \times [11]^{1/2} = 76 \ ft/sec$$

Using the same maximum C_{Lmax} and density, the effect of the higher wing loading is a stall speed of:

$$V_{min} = [2/(.002377 \text{ x } 1.6)]^{1/2} \times [120]^{1/2} = 251 ft/sec$$

This considerably higher stall speed leads to higher landing speeds and to two problems; more difficult handling as the pilot tries to precisely guide the aircraft at high speeds, and greater runway requirements for the ground roll.

To get slower stall speeds, the first idea may be to decrease the wing loading by putting on a much larger wing. A modern transport would look unusual if the wing was four or five times its current size. More importantly, it would have huge amounts of drag and would therefore fly very slowly. Designers had to go back to Newton's laws. They knew that they could use more curvature on the wing to create more lift at lower speeds, but then, they would have too much drag at high speeds. This drag discussion was covered in Session 4.

Of course, the answer was the development of flaps that could be used to change the camber (curvature) of the wing only when it was desired. Flaps don't weigh much and are very useful for increasing the value for C_{Lmax} anywhere from 20% to 60% (Figure 9.2). The advantage in decreased stall speed can be calculated using the previous equation.

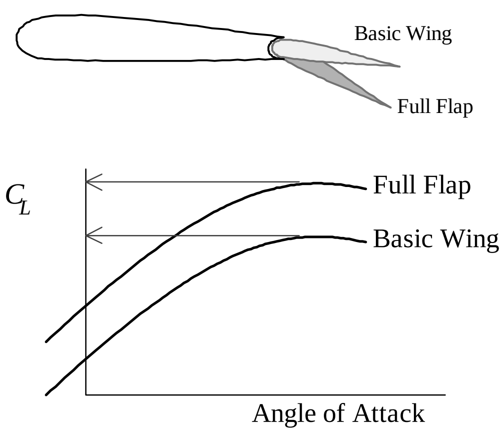


Figure 9.2

To ensure a smooth flight, most flight manuals call for a series of steps where the pilot incrementally extends the flaps, changes speed, and steers the plane around until it's lined up with the runway about 50 ft. above the ground and ready for the next phase, the flare.

3.3 Flare

The landing flare is the simplest to talk about, but the most difficult to do, and takes a lot of practice to be good at. The flare procedure goes something like this: the plane is approaching the runway at 80 mph on a 3 degree downhill slope (also known as the glideslope). Once in this position the pilot begins the flare about 50 feet above the runway by pulling on the wheel to smoothly increase the angle of attack and the lift of the airplane. This extra lift stops the plane's descent. This increase in angle of attack (and lift) from the approach to the flare is illustrated below.

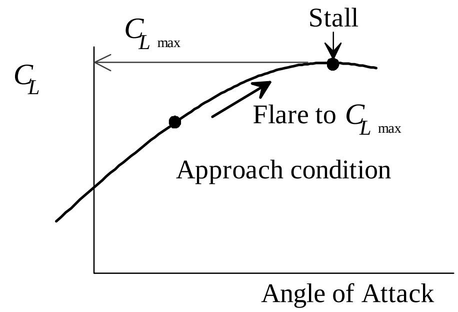


Figure 9.3

While doing this, gradually decrease the thrust to idle. With no thrust, the plane can't sustain flight a foot above the runway because the drag force acting on the plane's mass wants to decelerate it. Typically what a pilot will do is let the plane decelerate all the way to stall speed and gradually sink the last foot. The pilot’s timing is crucial. The pilot has to judge just when and how much to pull the wheel and throttle. Depending on its size, speed, and handling characteristics, each plane has its own method.

Figure 9.4 The Flare

If the pilot pulls too aggressively, then the plane might "balloon up" back into the air and might even come crashing back to the ground if he doesn't react quickly. To recover from this situation, the pilot would have to add power to keep the plane from slowing and/or sinking too quickly. If the pilot doesn't pull enough during the flare, then he won't stop the sink rate, and the plane might hit the ground with the nose gear first or too hard.

Figure 9.5 The Balloon

Another complicating factor is turbulence. A gust of wind can upset any part of the approach or flare phase. It's just like riding a bike or driving a car with precision - the faster you're going, the harder you have to work at it (see Suggested Activities for a demonstration of this).

One of the characteristics of a flare is that you pass by a lot of runway before touching down. If you point at the end of the runway during the approach but level off just above the runway, then you'll be flying along it, passing it by (Figure 9.6(a)).

This is acceptable for most kinds of flying because the runways are long. But suppose you don't have a long runway? Suppose you want to land on an aircraft carrier? There's no room to flare. The pilot flies the airplane straight onto the ship with a pretty high sink rate, (Figure 9.6(a)). Eliminating the flare gives the pilot pinpoint landing capability, but every landing is a hard "controlled crash." All carrier capable aircraft are built with a super strong structure and landing gear so they can slam onto the deck without being destroyed. Of course, the aircraft do have limits on how much sink rate they can handle.

3.4 Rollout

The last part of the landing, the ground roll, is least susceptible to pilot technique and so it is easiest to determine using Newton's Laws. The forces are similar to those for the session on takeoff performance.

Once the wheels touch, the wing doesn't have to support the aircraft's weight any more, so the pilot can feel free to decrease the angle of attack and speed as quickly as he wants to. The next task is decelerate the airplane to a full stop. To get an idea of the ability to slow down, go to Newton's second law, $F = ma$. Since we're looking for a deceleration, a should be negative. This means that to get the most possible deceleration, we would like the largest forces possible in the negative (or drag) direction and the smallest mass possible. Since we can't change the mass of the plane on most flights, we need to concentrate on the decelerating drag forces. To create drag forces, we have the brakes, the air, and the engine.

Brakes generate a drag force by converting the momentum of the plane into heat. Calculating the drag force they generate is simple: The braking drag is the braking coefficient μ times the weight on the wheels. μ decreases if you're braking on snow or ice (μ= .25), but is more or less a constant number for normal tires on normal, dry runways (μ=.75). In the takeoff session, μ was used to illustrate brakeless rolling friction and is typically about .05.

A class experiment to illustrate the concept of friction coefficient was discussed in the teacher's guide in Session 4 (Drag). Technically, the experiment discussed was for sliding friction - which is the case for a plane that is skidding, not rolling with braking force, as is normal.

If the wing isn't lifting at all, then the entire weight of the plane is supported by the wheels. A

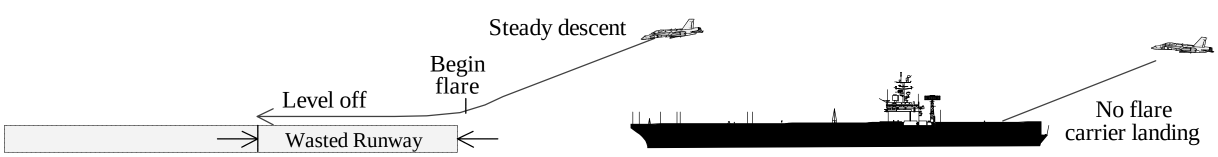


Figure 9.6

heavy weight on the wheels gives a lot of braking force to the plane. This wheel weight is often called the "reaction" force (R) in reference to Newton's third law. Some aircraft wings are still lifting after the plane touches down. This means that the brakes are not getting the full weight and are therefore not as effective since $F_{brakes} = \mu R$. As the plane slows further after touching down, the wing lift decays and the reaction force increases, thereby increasing the braking force.

Most of the time we're trying to minimize air drag force as much as possible, but to improve deceleration, we **like** drag. So, instead of keeping the airplane aerodynamically clean, the big heavy planes have spoilers (or air brakes) that pop out at touchdown. This desire for drag also encourages the use of full flaps for landing: planes can descend slower and steeper without speeding up if they have extended flaps. This extra drag is why planes <u>don't takeoff</u> with full flaps, although some takeoff with partial flaps. Some military planes even use parachutes called "drag chutes" to help slow them down. This is done only on military planes because it is expensive to have extra ground crew to pick up, pack and reload the chutes.

Another drag force can be created by the engine. Some propeller aircraft can be put in a "reverse" mode which changes the blade angle so it accelerates the air forward and slows the plane. These are a little complicated and more expensive to build, so not all planes have them. In a jet engine, they're called thrust reversers. Basically, a reverser "bucket" forces the exhaust towards the front in the direction opposite of the plane's motion (Figure 9.7). Newton's law about "equal and opposite reaction" shows that if the bucket forces the air to the front, then the air forces the bucket -and the rest of the plane- to the rear, the drag direction. Again, thhis can be seen on airliners and other transport aircraft because those types of planes need the most help to decelerate.

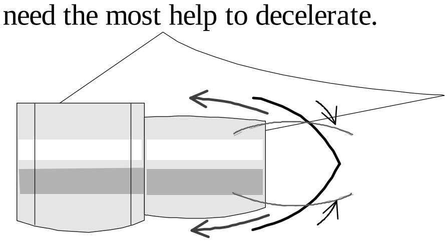


Figure 9.7 Bucket Swings

NOTE:

Session 4 of the text provides expanded information on kinematics.

Use Newton's second law to help see why it is important for large aircraft to use everything available to slow down the plane. Keep in mind that the biggest problem is the length of the runway. If its too short, then the plane can't fly in. To relate Newton's second law to required runway distance, review basic kinematics in class.

If desired, the kinematic relationship for any object can be developed as follows: From an initial speed (V_o) **assume** a constant deceleration (a) all the way to a stop. This can be illustrated graphically as follows:

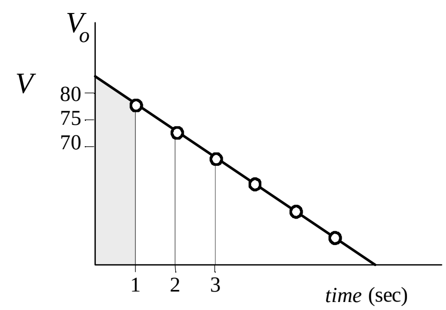


Figure 9.8 Kinematic Relationship

For the first second of travel, the average speed is 80 ft/sec which gives a distance of {80ft/sec x 1 sec} = 80 ft. For the next second, the average speed is 75 ft/sec which gives a distance of {75ft/sec x 1 sec} = 75 feet traveled and a cumulated distance of 80 + 75= 155 ft. This process can be continued step-by-step to get the total distance traveled. Note that for each time slice, the distance is the area under the curve.

A simple method is to recognize that the total <u>area</u> under the curve is the total distance traveled during the deceleration. Since the area of a triangle is $^1/_2$ *base x length*, then the distance $S = {}^1/_2 V_o$ *x time to stop*. Since we know $\Delta V/\Delta t = a$, then the time to stop is $t = V_o /a$. Combining gives the kinematic equation for distance traveled:

$$S = V_o^2/2a.$$

To apply this calculation to Newton's second law, determine the acceleration which gives $a = F/m$. Putting it all together gives a neat little equation for estimating ground roll distance:

$$S_{ground\ roll} = mV_{TD}^2/2\ F_{\mathrm{Drag}}$$

This says that the landing distance increases with the mass and the square of touchdown velocity (V_{TD}). The distance decreases as the drag forces go up. It's important to realize that this equation is valid only if the decelerating forces are constant. In reality, all of the forces in the drag direction change a little, so the equation is not exact.

The old biplanes were so light and landed at such low speeds that most of them didn't even need brakes - especially since they landed in grass fields that created lots of drag on the wheels. Approximate values to show a calculation of this for a Fokker Triplane are V_{TD} = 60 ft/sec, w = 1200 lbs, average drag from rolling wheels over grass = 90 lbs, average aerodynamic drag = 80 lbs.

$$S = [1200/32.2]\{60^2\}/2[90+80] = 395 \text{ ft}$$

A big transport on the other hand, has a lot of mass and a high landing speed like 150 mph. The "velocity squared" effect shows that big planes would have huge landing distances unless they created a lot of drag. That's why we put big brakes, big spoilers, and thrust reversers on them. The landing distance equation is one of the primary reasons that we're trying to land as slowly as possible--- to shorten the required runway distance.

In reality, each of these drag forces changes a little during the ground roll. You can feel this when you get jerked around in your seat after touchdown. That jerking around is you experiencing Newton's first law: bodies in motion tend to remain in motion unless disturbed by an outside force. You are the body in motion. The seatbelt -which is attached to the rest of the plane- exerts an outside force on you that slows you down along with the plane.

4.0 Summary

To land an airplane you need to descend *to* the airport, reconfigure the airplane for landing then approach the end of the runway with the proper sink rate, flare the plane *just* over the runway to stop the descent then allow it to land in the last foot, and finally, decelerate the plane on rollout. Each step can be explained with basic physics.

It's the test pilot's job to figure out the best procedures for descending, that means measuring the dive angle when thrust is reduced by 10 or 20%. During the approach it means figuring out the safest speed to fly each step of the way when the gear goes down and when the flaps go down. During the flare it means figuring out just when to throttle the engines and start the flare. Finally, a test pilot has to perform a series of ground rolls to see how much runway the plane <u>really</u> needs, not what is predicted from approximations.

5.0 Measures of Performance

1 What are the three phases of the landing process?

ANSWER

The approach, the flare to touchdown, and the ground roll after touchdown.

2 During the flare, why does the aircraft descend?

ANSWER

With the bulk of the thrust removed, the drag is greater than the thrust causing the aircraft to decelerate. As the aircraft decelerates, lift decreases causing the aircraft to descend to the runway.

3 What are three ways to create drag forces to decelerate?

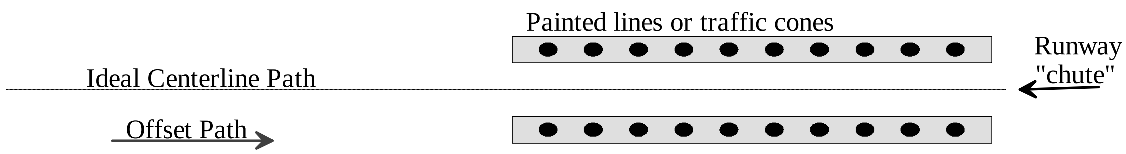
ANSWER

1. Brakes, to generate rolling drag.
2. Airbrakes to increase aerodynamic drag.
3. Engine by "reversing" the thrust; that is directing the thrust forward.

6.0 Suggested Activities

The difficulty of landing at high speeds can be demonstrated by having students ride bikes into a runway-like "chute" at different speeds. Like landing on a calm day, this is not much of a challenge if the rider is lined up with the chute long before he or she gets there, even at high speed. The task is made challenging (**especially** at high speed) if the rider follows a path that is laterally offset from the ideal path and is allowed to maneuver into position only immediately before entering the zone. This is like having a plane get bumped off-track by a wind gust.

To simulate a pilot flying in the weather and "breaking out of the clouds" just before touching down, the students can ride approximately towards the chute with eyes closed until someone shouts "BREAKOUT!" just prior to entering the chute. This simple exercise will illustrate the benefits of slow approach speeds in poor weather.



Session 10
Summary and Review

1.0 Introduction

The previous sessions showed that Newton's Laws of Motion are used during aircraft flight testing. During this final session, the same techniques used to evaluate a full size aircraft are used to predict the performance of a radio controlled (R/C) model aircraft. The scope of testing is limited because there is no pilot on board and instrumentation, such as airspeed and fuel flow, is not available. As a result, the focus of this session is on weight and balance, thrust determination, and takeoff performance. The procedures described can be accomplished by any student having access to a R/C model.

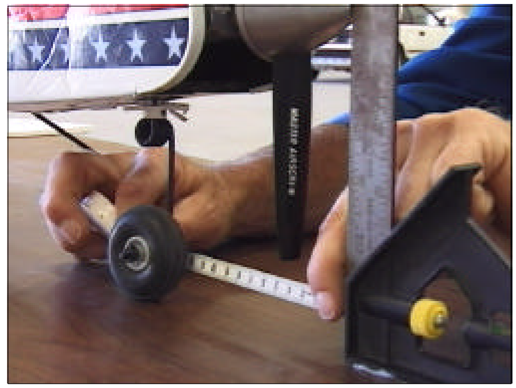
2.0 Weight and Balance of the Model

To determine the aircraft's weight and location of the center of gravity, use the same procedures described in Session 2. Begin by establishing a Reference Datum Line (RDL) at the forward end of the propeller hub. This can be done by placing a carpenter's square at the end or by placing the model flush against a wall. From the RDL, measure the horizontal distance to the point where the nose wheel (or tail wheel, depending on the type of model) touches the ground. This is the arm length for the nose gear. Accomplish the same procedure for the main landing gear.

NOTE:
Since the assumption is that the aircraft is symmetric, only one main gear need be measured.

The following lengths were found using the model shown in the video:

Nose Landing Gear Arm = 5.75 inches
Main Landing Gear Arm = 14.25 inches



Measuring Landing Gear Arms

The aircraft is then weighed. Recall from Session 2 that a scale is placed under each landing gear, the weights are recorded and then added together to obtain the total aircraft weight.

Caution:

It is important that the aircraft be level to achieve the proper weight distribution on each landing gear.

Weighing the Aircraft

Now, to determine the *cg* location, the weight recorded at each landing gear is multiplied by the arm length from the RDL for that gear. For the model under evaluation, this yields the following:

Item (Gear)	Arm	Weight	Moment
Nose	5.75 in	0.88 lbs	5.06 in-lbs
Left Main	14.25 in	1.75 lbs	24.93 in-lbs
Right Main	14.25 in	1.56 lbs	22.23 in-lbs
Total		4.19 lbs	52.22 in-lbs

Then, divide the total moment by the total weight to determine the location of the *cg*. Here, the *cg* is located 12.46 inches from the RDL, which locates it on the wing within the range of *cg*'s specified by the model maker.

3.0 Determining Thrust of the Model

The next step in the flight test sequence is to predict the thrust available from the engine. Session 5 gave the relation:

$$T = \left[\rho\left(\frac{\pi d^2}{4}\right)\right]2\left[2Vk(RPM) + k(RPM)^2\right]$$

The model maker has provided the following information for use in this equation:

Propeller diameter (d): 9 inches (0.75 ft)
Propeller efficiency (k): 0.00066
Max engine RPM: 1250 RPM

NOTE:
For most models, propeller constants range between 0.00044 and 0.00070. Tests to determine actual k values are very involved. Therefore, should you decide to conduct a test similar to the one shown in the video, an average propeller constant of 0.00057 can be used.

The air density at the test site is 0.002 slugs/ft^3. By examining the equation, we see a velocity term in the second set of brackets. However, for a static thrust check, the velocity is zero. Therefore, substituting into the thrust relationship gives:

$$T = \left[0.002\frac{slugs}{ft^3}\left(\frac{\pi 0.75ft}{4}\right)^2\right]2\left[0 + 0.00066(1250)^2\right]$$

$$T = 1.82 \text{ lbs}$$

To verify this value, a spring scale is attached to the model. With the engine operating at maximum RPM, the scale reading is 1.5 lbs. To predict the takeoff performance, the 1.5 pounds of static thrust determined experimentally should be used in the calculations.

Measuring Engine Thrust

NOTE:
This is 21% less power than predicted. This highlights an important aspect of testing. The numbers for RPM and propeller efficiency provided by the manufacturer are for a brand new engine under carefully controlled test conditions. The engine on the model is a number of years old and the propeller has a considerable number of "nicks" on the blades. Each of these factors detracts from the amount of thrust the engine can produce. This is why we test the thrust using a scale.

4.0 Determining Takeoff Speed

Recall from Session 6 that the lowest speed at which the lift just equals the aircraft's weight is the takeoff speed. This speed is determined by the relationship:

$$w = L = \tfrac{1}{2}\rho V^2 S C_{L_{\max}}$$

Rearranging this equation to solve for takeoff speed gives:

$$V = \left(\frac{2w}{\rho S C_L}\right)^{\frac{1}{2}}$$

In order to determine the wing area, the video depicted measuring;
- the chord length, c, (distance from leading edge to the trailing edge of the wing)
- the wing span, b, (distance from one wingtip to the other)

To determine the area of the rectangular wing, simply multiply the chord length times the span length, or

$$S = c \times b$$

For the model being tested, the chord length is 8.5 inches (0.7083 ft) and the wing span is 52 inches (4.33 ft). Multiply these values and we find the wing area is 3.07 square feet.

Determining Wing Area

Caution:

It's important to convert all units into feet and pounds prior to performing the calculations for takeoff speed, lift, drag, and thrust.

The next item needed for the takeoff speed calculation is the air density. For the temperature and air pressure measured on the day of the test, the density, ρ, was found to be 0.002 slugs/ft^3. This may vary for your test. All we need now is the overall lift coefficient.

The lift coefficient is usually found by wind tunnel analysis. In this case, the model maker didn't provide this data. A conservative number for an aircraft without flaps and a rectangular wing is C_L = 1.1. This is a reasonable assumption and can be used for most model applications.

Applying these numbers to the takeoff speed equation:

$$V = \left(\frac{2w}{\rho S C_L} \right)^{\frac{1}{2}}$$

$$V = \left(\frac{2(4.19 lbs)}{\left(0.002 \frac{slugs}{ft^3}\right)(3.07 ft^2)(1.1)} \right)^{\frac{1}{2}}$$

$$V = 35.2 \text{ ft/sec (24 MPH)}$$

This speed will be used in determining the drag on the aircraft during the takeoff roll.

5.0 Determining the Drag

Session 6 said that engineers have learned through experience, if seventy percent of the takeoff speed is used to calculate the drag during takeoff, the results are very close to the average drag. 70% of the speed just calculated is 0.70 times 35.2 ft/sec or 24.6 ft/sec.

Next the drag coefficient, C_D, should be determined. Again, this is usually found in a wind tunnel. However, for the type of model used in this test, a good estimated value of C_D is 0.06.

The drag equation is:

$$D = \tfrac{1}{2} \rho V^2 S C_D$$

Using the values for wing area, density, 70% of takeoff speed, and drag coefficient, the predicted average drag during the takeoff is:

$$D = \tfrac{1}{2}\left(0.002 \frac{slugs}{ft^3}\right)\left(24.6 \frac{ft}{\sec}\right)^2 (3.07 ft^2)(0.06)$$

$$D = 0.1115 \text{ lbs}$$

This value and the measured value for thrust are used to calculate the expected acceleration during takeoff.

6.0 Determining the Acceleration

Using Newton's $F = ma$ equation, we can define acceleration in the same manner as outlined in Session 6. This yields:

$$F = T - D = ma$$

when we rearrange the equation, we can solve for the acceleration:

$$a = \frac{T - D}{m}$$

$$a = \frac{g(T - D)}{w}$$

Inserting the appropriate values (remember $g = 32.2\frac{ft}{\sec^2}$) gives:

$$a = \left(32.2\frac{ft}{\sec^2}\right)\left(\frac{1.5lbs - 0.1115lbs}{4.19lbs}\right)$$

$$a = 10.67\frac{ft}{\sec^2}$$

To determine the time required to accelerate to takeoff speed, use the following relationship;

$$time = \frac{takeoff\ speed}{acceleration\ rate}$$

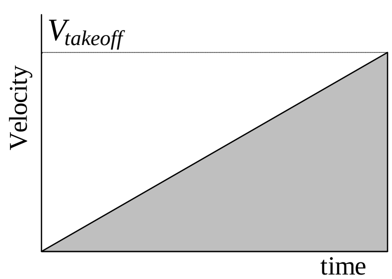
$$time = \frac{35.2\frac{ft}{\sec}}{10.67\frac{ft}{\sec^2}}$$

$$time = \ 3.3 \sec$$

In Session 6, it was stated if we assume the acceleration rate is constant, a plot of velocity versus time can be constructed. The acceleration rate is simply the slope of this curve. So if we use this relationship, the takeoff distance is determined.

7.0 Determining Takeoff Distance

Since the acceleration is assumed to be constant, the slope of the plot is a straight line. Using the right triangle equation, Session 6 showed that the area under the triangle is equal to the estimated takeoff distance.



$$area = \tfrac{1}{2}(base \times height)$$

$$area = \tfrac{1}{2}(takeoff\ speed \times time\ required\ to\ takeoff)$$
$$= takeoff\ distance$$

Substituting the appropriate values into this equation:

$$area = \tfrac{1}{2}(35.2\tfrac{ft}{\sec} \times 3.3 \sec)$$

$$area\ =\ takeoff\ distance\ =\ 58\ ft$$

During the first takeoff of the model, the takeoff distance was measured at 85 feet. To account for the increased takeoff roll, we must account for rolling friction.

Rolling Friction

From Session 6, the rolling friction is given as:

$$Friction = \mu w$$

where μ is the coefficient of friction for the surface the aircraft is rolling over. The surface of the "runway" used in the video is dirt with rocks and holes throughout. The handbook value of μ is 0.1, for surface conditions of the runway. To account for friction we use Newton's equation again:

$$F = (T - D - \ \mu w) = ma$$

and calculate the new acceleration rate:

$$a = \frac{g(T - D - \ \mu w)}{w}$$

$$a = \frac{32.2\frac{ft}{\sec^2}[1.5lbs - 0.115lbs - 0.1(4.19lbs)]}{4.19lbs}$$

$$a = 7.45\frac{ft}{\sec^2}$$

So, to estimate the new time required to accelerate to takeoff speed:

$$time = \frac{takeoff\ speed}{acceleration\ rate}$$

$$time = \frac{35.2\frac{ft}{\sec}}{7.45\frac{ft}{\sec^2}}$$

$$time = 4.72 \sec$$

Taking these factors into consideration, the new estimated takeoff distance should be:

$$area = \tfrac{1}{2}(takeoff\ speed \% time\ required\ to\ takeoff)$$
$$= takeoff\ distance$$

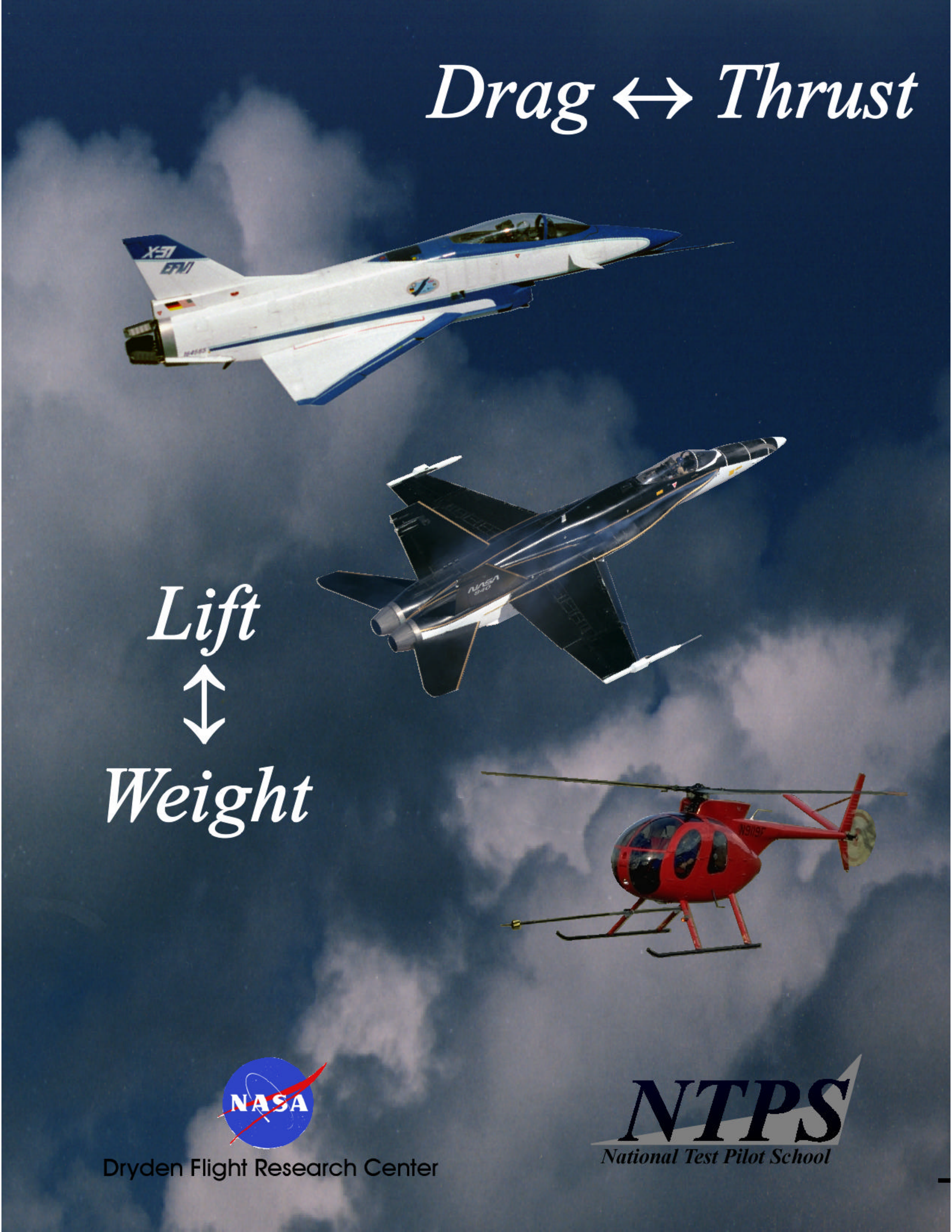
$$area = \tfrac{1}{2}(35.2\tfrac{ft}{\sec} \% 4.72 \sec)$$

$$takeoff\ distance = 83.1 \text{ ft}$$

This takeoff distance was within 2 feet of the actual distance required for the first takeoff. On a subsequent takeoff the distance required was 86 feet. This further substantiates our analysis.

8.0 Conclusion

The techniques used to flight test aircraft rely heavily upon Newton's Three Laws of Motion. Although some simplifying assumptions have been made to the aerodynamic relationships, the basic concepts remain valid regardless of the size of the aircraft. We demonstrated this by testing of a R/C Model. Further experiments are outlined in the section titled "Culminating Activities." We hope you find them interesting and challenging.


Drag ↔ Thrust
Lift
↕
Weight
X-31
EFM
NASA
Dryden Flight Research Center
NTPS
National Test Pilot School