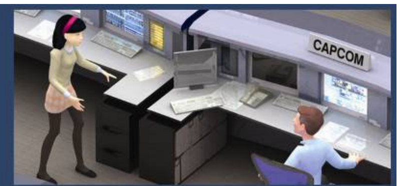
MATH AND SCIENCE @ WORK

AP* PHYSICS Educator Edition



SPACE SHUTTLE ROLL MANEUVER

Instructional Objectives

Students will analyze space shuttle schematics and data to:

- demonstrate graph and schematic interpretation skills;
- apply integration techniques to evaluate impulse and angular momentum; and
- evaluate the rotational kinematics, torque and energy associated with a roll maneuver.

Degree of Difficulty

This problem requires a strong integrated understanding of rotational dynamics, angular momentum and energy.

- For the average student in AP Physics C: Mechanics, this problem is at a moderate difficulty level.

Class Time Required:

This problem requires 60 minutes.

- Introduction: 10 minutes
- Student Work Time: 40 minutes
- Post Discussion: 10 minutes

Background

This problem is part of a series of problems that apply Math and Science @ Work in NASA's Mission Control Center.

Since its conception in 1981, NASA has used the space shuttle for human transport, the construction of the International Space Station (ISS), and to research the effects of space on the human body. One of the keys to the success of the Space Shuttle Program is the Space Shuttle Mission Control Center (MCC). The Space Shuttle MCC at NASA Johnson Space Center uses some of the most sophisticated technology and communication equipment in the world to monitor and control the space shuttle flights.

Within the Space Shuttle MCC, teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle. They work together as a

Grade Level
11-12

Key Topic
Rotational Motion and Dynamics

Degree of Difficulty
Physics C: Mechanics: Moderate

Teacher Prep Time
10 minutes

Class Time Required
60 minutes

Technology
Calculator

AP Course Topics
Newtonian Mechanics:
- Circular Motion and Rotation

NSES Science Standards
- Physical Science
- Science and Technology
- History and Nature of Science

powerful team, spending many hours performing critical simulations as they prepare to support preflight, ascent, flight, and reentry of the space shuttle and the crew. The flight controllers provide the knowledge and expertise needed to support normal operations and any unexpected events.

One of the flight controllers in the Space Shuttle MCC is the Guidance, Navigation, and Control (GNC) officer. To understand the roles of the GNC officer, one must first understand the basics of the GNC system. Guidance equipment (gyroscopes and accelerometers) and software first compute the location of the vehicle and the orientation required to satisfy mission requirements. Navigation software then tracks the vehicle's actual location and orientation, allowing the flight controllers to use hardware to transport the space shuttle to the required location and orientation. The primary responsibility of the GNC officer is to ensure the hardware and software that perform these functions are working correctly. This control portion of the process consists of two modes: automatic and manual. In the automatic mode, the primary avionics software system allows the onboard computers to control the guidance and navigation of the space shuttle. In the manual mode the crew uses data from the GNC displays and hand controls for the guidance and navigation. The GNC officer ensures that the GNC system has the accuracy and capacity necessary to control the space shuttle in both modes and that it is being utilized correctly.

Once the space shuttle is in orbit, the Reaction Control System (RCS) is used for attitude control. Attitude is the orientation the space shuttle has relative to a frame of reference. The RCS jets control the attitude of the shuttle by affecting rotation around all three axes. Three terms, pitch, yaw, and roll, are used to describe the space shuttle's attitude. Moving the nose up and down is referred to as "pitch," moving the nose left and right is referred to as "yaw," and rotating the nose clockwise or counterclockwise is referred to as "roll" (Figure 1).

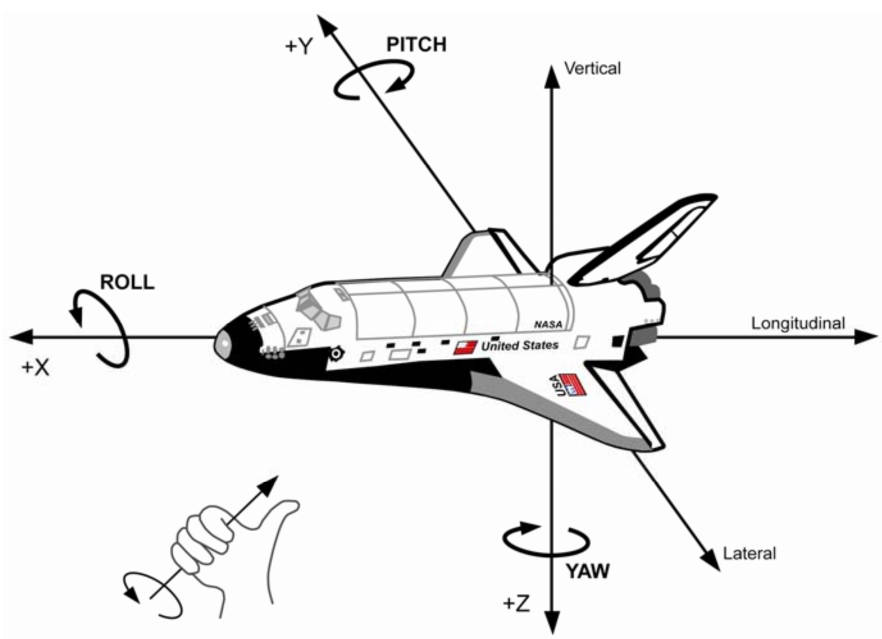


Figure 1: Diagram of the X, Y, and Z body Axes of the space shuttle. Rotation about the X-axis is called Roll, about the Y-axis is called Pitch, and about the Z-axis is called Yaw. The direction of the rotation follows the right hand rule, which states that the thumb of the right hand would be aligned with the positive axis and the direction of the rotation is positive in the direction of the fingers when curling around the axis. The arrows show positive rotation.

The RCS consists of thirty-eight primary jets that produce 3870 Newtons (870 lbs) of thrust each and six vernier jets that produce 107 Newtons (24 lbs) of thrust each. The smaller thrust of vernier jets allow for greater precision of movement. Fourteen primary and two vernier RCS jets are located in the nose of the vehicle and are called forward RCS jets (Figure 2). Twenty-four primary jets and four vernier jets are found on each of the two Orbital Maneuvering System (OMS) pods, located on both sides of the vertical tail and are called aft RCS jets (Figure 3).

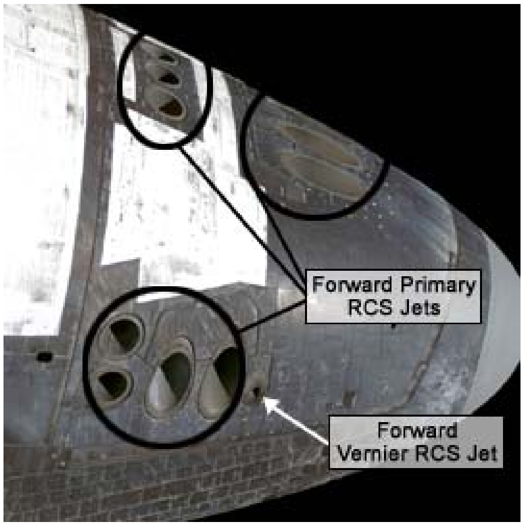


Figure 2: Forward primary and vernier RCS jets located on the nose of the space shuttle.

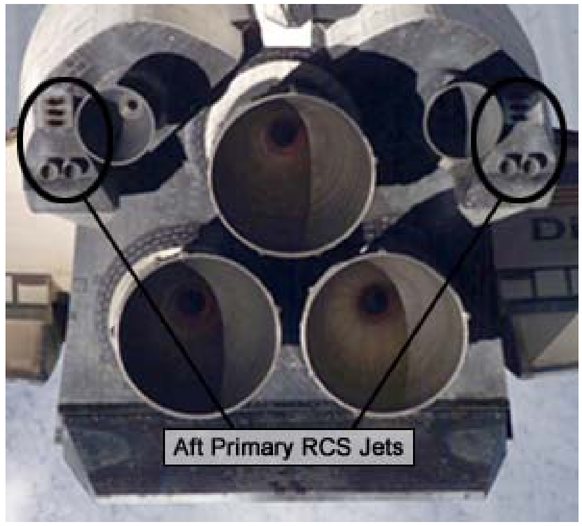


Figure 3: Aft primary RCS jets located on the tail of the space shuttle as part of the OMS pods.

Activities that must be performed by the space shuttle, such as deploying satellites and pointing mapping instruments and telescopes at the Earth and stars, require attitude adjustment. The GNC software and GNC officer control these adjustments by firing different RCS jets to achieve the correct yaw, pitch, and/or roll.

AP Course Topics

Newtonian Mechanics

- Circular Motion and Rotation
  - Uniform circular motion
  - Torque and rotational statics
  - Rotational kinematics and dynamics
  - Angular momentum and its conservation

NSES Science Standards

Physical Science

- Motions and forces
- Conservation of energy and increase in disorder

Science and Technology

- Abilities of technological design

History and Nature of Science

- Science as a human endeavor

Problem

It takes multiple RCS jets firing to perform any space shuttle maneuver. In some cases these jets are fired one at a time. Consider a space shuttle roll maneuver that is initiated by the aft right up primary RCS jet (Figure 4). The jet firing lasts for 0.0800 seconds and primarily affects the roll of the space shuttle but will also have a small affect on the pitch and yaw. For the purposes of this problem we will ignore the effects on the pitch and yaw and assume only the roll is affected.

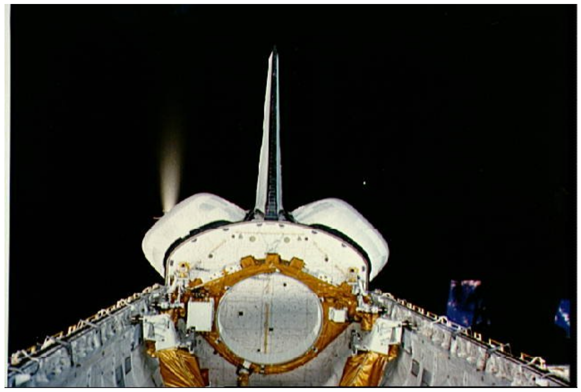


Figure 4: The aft right up primary RCS jet firing to help initiate a roll maneuver.

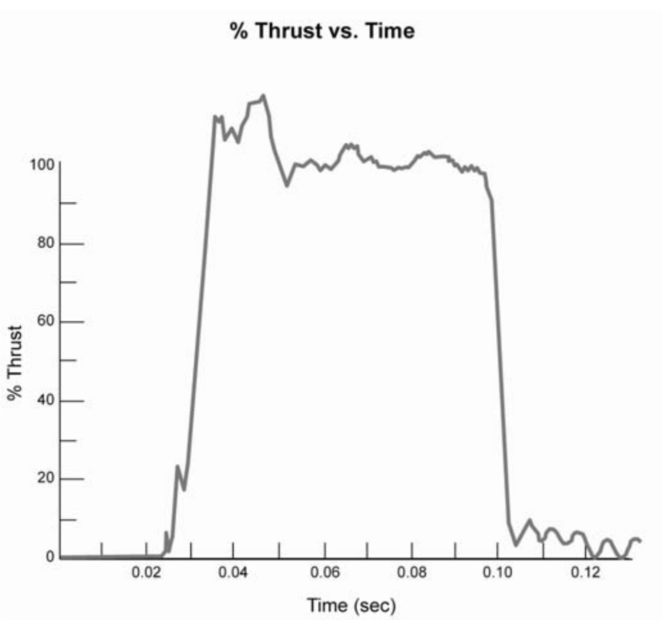


Figure 5: Typical thrust profile for the firing of one primary RCS jet. When the thrust is at 100%, the force is 3870 N (870 lbs).

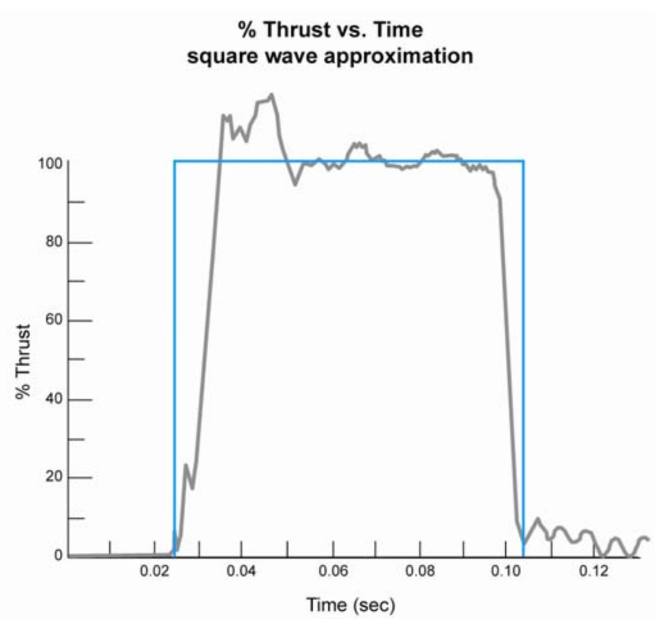


Figure 6: the square wave approximation of the thrust profile.

The % Thrust vs. Time profile of the firing is shown in Figure 5. The thrust produced by the RCS jets is achieved so quickly that the ramp-up and tail-off time is near instantaneous. This profile illustrates the point that the thruster opens about 0.025 seconds after commanded due to the time it takes for the jet selection logic and error checking to occur. Thrust of 100% is equal to the rated performance of 3870 N (870 lbs) of force in a vacuum. The oscillations after shut down are artifacts of the thrust ringing out and are not adding thrust. This profile, therefore, is very well represented by a square wave of 0.08 s duration (Figure 6).

Use correct significant figures and SI units throughout the problem.

A. Determine the magnitude of the impulse produced during the firing of the aft right up primary RCS jet (location shown in Figure 7).

B. Figure 4 shows a view of the aft right up primary RCS jet firing. Note that firing the RCS jet results in the blast being directed at a 90 degree angle with the space shuttle. If the moment arm of the jets is 3.25 m (see Figure 7), calculate the magnitude and direction of the torque produced by the RCS firing as if you were sitting on the flight deck looking through the forward cockpit windows.

C. Determine the magnitude of change in angular momentum that results from firing the jet. Recall that torque equals the rate of change of angular momentum $\left(\tau = \frac{dL}{dt}\right)$.

D. Explain why the impulse determined in question A does not equal the change in angular momentum calculated in question C.

E. The RCS jet firing contributes to a roll of the space shuttle. If the mass of the space shuttle on orbit is 107,602 kg, calculate the tangential velocity of the shuttle at the RCS jet location achieved by this firing.

F. Estimate the rotational inertia of the space shuttle due to the firing of the RCS jet.

G. Calculate the rotational kinetic energy acquired from the firing of the RCS jet.

H. Determine the magnitude of potential energy contained in the propellant used to initiate the roll.

I. Suppose that during the rotation the robotic arm (CanadArm 1) is extended to maximum length perpendicular to the axis of rotation (see Figure 7). Would the angular velocity increase, decrease, or remain the same? Justify your answer.

Space Shuttle Schematic

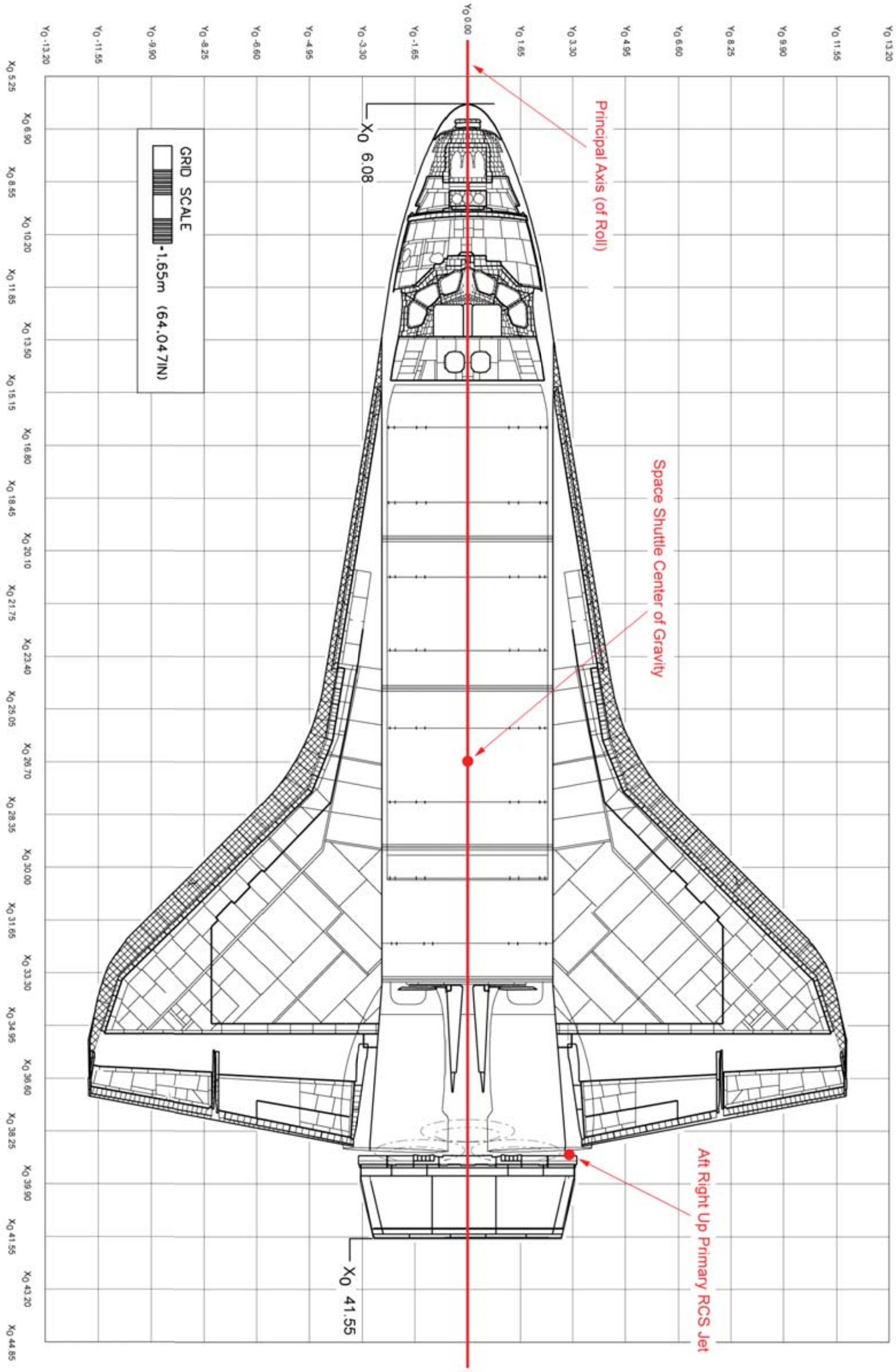


Figure 7: Space Shuttle Schematic. The origin of the graph is located at the highest part of the space shuttle system when it is on the launch pad, which is at the top of the External Tank.

Solution Key (One Approach)

Note: While all solutions can ultimately be arrived at using algebra, integration technique and understanding the cross-product support a more realistic and accurate application of physics.

A. Determine the magnitude of the impulse produced during the firing of the aft right up primary RCS jet (location shown on Figure 7).

$$F = 3870\ \text{N}$$
$$t_i = 0.025\ \text{s}$$
$$t_f = 0.105\ \text{s}$$

$$J = \int_{t_i}^{t_f} F dt$$
$$J = F \int_{0.025\ \text{s}}^{0.105\ \text{s}} dt$$
$$J = 3870\ \text{N} \cdot (0.105 - 0.025)\ \text{s}$$
$$J = 310\ \text{N} \cdot \text{s}$$

B. Figure 4 shows a view of the aft right up primary RCS jet firing. Note that firing the RCS jet results in the blast being directed at a 90 degree angle with the space shuttle. If the moment arm of the jets is 3.25 m (see Figure 7), calculate the magnitude and direction of the torque produced by the RCS firing as if you were sitting on the flight deck looking through the forward cockpit windows.

$$F = 3870\ \text{N}$$
$$r = 3.25\ \text{m}$$
$$\theta = 90^\circ$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$\vec{\tau} = -(3.25\ \text{m})(\sin 90^\circ)(3870\ \text{N})$$
$$\vec{\tau} = 12{,}600\ \text{N} \cdot \text{m, clockwise}$$

Note: a clockwise rotation is referred to as a “positive roll” (+roll).

C. Determine the magnitude of change in angular momentum that results from firing the jet. Recall that torque equals the rate of change of angular momentum $\left(\tau = \frac{dL}{dt}\right)$.

$$\tau = 12{,}600\ \text{N} \cdot \text{m}$$
$$t_i = 0.025\ \text{s}$$
$$t_f = 0.105\ \text{s}$$

$$\tau = \frac{dL}{dt}$$
$$dL = \tau\ dt$$
$$\int dL = \int_{0.025}^{0.105} \tau\ dt$$
$$\Delta L = \tau \int_{0.025}^{0.105} dt$$
$$\Delta L = (12{,}600\ \text{N} \cdot \text{m})(0.105\ \text{s} - 0.025\ \text{s})$$
$$\Delta L = 1000\ \text{N} \cdot \text{m} \cdot \text{s}$$

D. Explain why the impulse determined in question A does not equal the change in angular momentum calculated in question C.

The impulse curve in question A does indeed yield the change in momentum of the center of mass *if the impulse were delivered to the center of mass*. The RCS jet, however, produces a torque around an axis that extends through the center of mass. This can be verified by dividing the change in angular momentum by the length of the lever arm:

$$\Delta L = 1000 \text{ N} \cdot \text{m} \cdot \text{s}$$
$$r = 3.25 \text{ m}$$

$$\frac{\Delta L}{r} = \frac{1000 \text{ N} \cdot \text{m} \cdot \text{s}}{3.25 \text{ m}} = 310 \text{ N} \cdot \text{s}$$

E. The RCS jet firing contributes to a roll of the space shuttle. If the mass of the shuttle on orbit is 107,602 kg, calculate the tangential velocity of the shuttle at the RCS jet location achieved by this firing.

$$\Delta L = 1000 \text{ N} \cdot \text{m} \cdot \text{s}$$
$$r = 3.25 \text{ m}$$
$$m_{shuttle} = 107{,}602 \text{ kg}$$
$$L_i = 0 \text{ N} \cdot \text{m} \cdot \text{s}$$
$$v_i = 0 \tfrac{\text{m}}{\text{s}}$$

$$\Delta L = r \cdot \Delta p$$
$$L_f - L_i = r \cdot (mv_f - mv_i)$$
$$L_f - 0 = r \cdot (mv_v - m(0))$$
$$1000 \text{ N} \cdot \text{m} \cdot \text{s} = (3.25 \text{ m})(\sin 90°)(107{,}602 \text{ kg})v_f$$
$$v_f = 0.003 \tfrac{\text{m}}{\text{s}}$$

F. Estimate the rotational inertia of the space shuttle due to the firing of the RCS jet.

$$\Delta L = 1008 \text{ N} \cdot \text{m} \cdot \text{s}$$
$$v = 0.003 \tfrac{\text{m}}{\text{s}}$$
$$r = 3.25 \text{ m}$$

$$L = I\omega$$
$$v = r\omega$$
$$L = I\frac{v}{r}$$
$$1000 \text{ N} \cdot \text{m} \cdot \text{s} = I\frac{0.003 \frac{\text{m}}{\text{s}}}{3.25 \text{ m}}$$
$$I = 1{,}000{,}000 \text{ kg} \cdot \text{m}^2$$

G. Calculate the rotational kinetic energy acquired from the firing of the RCS jet.

$$I = 1{,}000{,}000 \text{ kg} \cdot \text{m}^2$$
$$v = 0.003 \tfrac{\text{m}}{\text{s}}$$
$$r = 3.25 \text{ m}$$

$$K = \frac{1}{2}I\omega^2$$
$$v = r\omega$$
$$K = \frac{1}{2}I\frac{v^2}{r^2}$$
$$K = \frac{1}{2}(1{,}000{,}000 \text{ kg} \cdot \text{m}^2) \cdot \frac{(0.003 \frac{\text{m}}{\text{s}})^2}{(3.25 \text{ m})^2}$$
$$K = 0.4 \text{ J}$$

Note to Educator: The kinetic energy associated with a roll is very small compared to the kinetic energy associated with pitch and yaw. The small kinetic energy of the roll is due to the very short moment arm and the 0.08 second impulse duration. Although the impulse duration of the

RCS jets that control the pitch and yaw remain the same the moment arms are significantly larger resulting in greater kinetic energies.

H. Determine the magnitude of potential energy contained in the propellant used to initiate the roll.

In the absence of all air resistance, energy is conserved. Therefore:

$$\Delta K = -\Delta U_{chemical}$$
$$\Delta U_{chemical} = 0.4 \text{ J}$$

I. Suppose that during the rotation the robotic arm (CanadArm 1) is extended to maximum length perpendicular to the axis of rotation (see figure 7). Would the angular velocity increase, decrease, or remain the same? Justify your answer.

The angular velocity would decrease.

In the absence of external forces angular momentum will remain constant. By extending the arm perpendicular to the axis of rotation the rotational inertia increases. To conserve angular momentum the angular velocity must therefore decrease.

Scoring Guide

Suggested 15 points total to be given.

Question		Distribution of points
A	*2 points*	1 point for correct bounds of integration 1 point for correct impulse
B	*2 points*	1 point for correct torque equation 1 point for correct magnitude and direction of torque
C	*2 points*	1 point for correct torque equation relating angular momentum 1 point for correct magnitude of change in angular momentum
D	*1 point*	1 point for explaining that the impulse is not being delivered to the center of mass but to the length of the arm that extends through the center of mass.
E	*2 points*	1 point for correct angular momentum/linear momentum equation 1 point for correct magnitude of final velocity
F	*2 points*	1 point for correct linear velocity/angular velocity conversion 1 point for correct magnitude of rotational inertia
G	*2 points*	1 point for correct rotational kinetic energy equation 1 point for correct magnitude of rotational kinetic energy
H	*1 point*	1 point for correct magnitude of potential energy consistent with calculated kinetic energy
I	*1 point*	1 point for identifying and justifying the decrease in angular velocity

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school AP Physics instructors.

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