Exploring Space Through MATH

Applications in Algebra 1

STUDENT EDITION

Ascent: 50 Seconds to MECO

Background

This problem is part of a series of problems that apply algebraic principles in NASA's human spaceflight.

The Space Shuttle Mission Control Center (MCC) and the International Space Station (ISS) Control Center use some of the most sophisticated technology and communication equipment in the world. Teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle and the ISS. They work together as a very knowledgeable and powerful team, spending many hours performing critical simulations as they prepare to support each mission and crew during normal operations and any unexpected events.

The space shuttle is a spacecraft designed to take people and cargo into Earth's orbit. The purpose of these space shuttle missions may include such things as satellite installations and repairs, experiments, transportation of large cargo, and drop off and pick up of astronauts on the ISS. In order to perform any of its missions, the space shuttle must first lift off from the Kennedy Space Center and make it into orbit. (Figure 1)

There are three main components of the space shuttle that enable it to launch into orbit. The main component is the Orbiter. It not only serves as the crew's home in space and is equipped to dock with the ISS, but it also contains maneuvering engines for finalizing the orbital trajectory. The External Tank (ET), the space shuttle's largest component, supplies the propellant to the Space Shuttle Main Engines (SSMEs). The third component is the Solid Rocket Boosters (SRBs). These are attached to the sides of the ET and provide the main thrust at launch. (Figure 2)

Figure 1: Space Shuttle Discovery at Liftoff

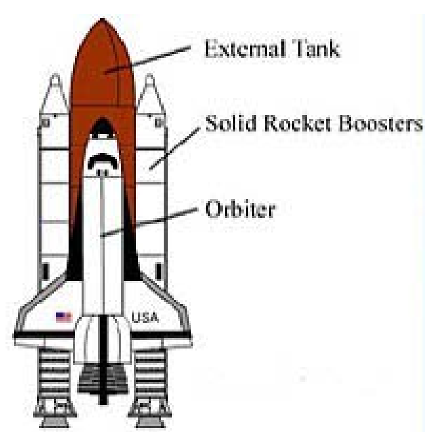


Figure 2: Main Components of the Space Shuttle

The ascent is a very dangerous and important phase of any space shuttle mission. This phase begins at liftoff and ends as the space shuttle reaches orbit. The main propulsion system (which consists of the ET and SSME) together with the SRBs supply the force needed to accelerate to the velocity of approximately 7.85 km/s that is required to attain orbit. This system is so powerful that within 1 minute the space shuttle breaks the sound barrier. As the velocity continues to increase, the stress on the space shuttle and the crew increases. Because of the immense power of its engines, the acceleration of the space shuttle must be kept below 3 *g*, or 0.03 km/s^2, to ensure the safety of the vehicle and the crew.

The space shuttle continues on its path (trajectory) until Main Engine Cut Off (MECO), soon after which it reaches orbit. It takes approximately 8 minutes and 30 seconds for the space shuttle to reach MECO, which occurs around 104 km (56 miles) above the surface of the Earth. The ET is dropped away to breakup and land in the ocean, and the orbiter performs a final burn approximately 30 minutes later to reach orbital altitude of around 320 km (200 miles).

The flight controllers who have the most responsibility during the space shuttle ascent are Booster, the Propulsion Officer (PROP), and the Flight Dynamics Officer (FDO). Booster is responsible for the safe operation of the ascent propulsion systems. Once the 8.5 minutes of ascent are complete, Booster's responsibility ends. PROP is responsible for monitoring the Orbital Maneuvering Systems (OMS) and the Reactions Control Systems (RCS). These systems supplement the SSMEs and SRBs during ascent and perform final orbit insertion burns after MECO. FDO is responsible for the planning and execution of the space shuttle's trajectory during ascent, orbit, rendezvous, and re-entry.

This problem highlights the analysis of the graphs of acceleration and velocity during the 8.5 minute ascent and focuses on the 50 seconds before MECO.

Instructional Objectives

- You will determine independent and dependent variables;
- You will identify domain and range;
- You will interpret graphs and tables;
- You will identify functions; and
- You will use tables to determine the function rule.

Exploring Graphs

Directions: Answer questions 1 – 13 in your group. Include units. Discuss answers to be sure everyone understands and agrees on the solutions.

Please answer the following questions about the *Graph 1: Acceleration*.

1. What variable is shown on the horizontal axis and what are the units?
2. Use an inequality to express the domain of the graph.
3. What variable is shown on the vertical axis and what are the units?
4. Use an inequality to express the approximate range of the graph..
5. What event occurs approximately 2 minutes into the flight of the space shuttle?
6. Describe what is happening to the acceleration of the space shuttle after SRB Separation?
7. At approximately 7.5 minutes into the flight what happens to the acceleration?
8. What happens to the acceleration at Main Engine Cut Off (MECO)? Explain why.

Please answer the following questions about the *Graph 2: Velocity.*

9. What variable is shown on the horizontal axis and what are the units?
10. Use an inequality to express the domain of the graph.
11. What variable is shown on the vertical axis and what are the units?
12. Use an inequality to express the range of the graph.
13. In the interval $0 \le t \le 510$ what appears to be happening to the velocity of the space shuttle?

Interpreting Graphs & Tables

Problem

The space shuttle has completed its mission. The shuttle navigators that support the flight controllers have compiled all of the data and placed it in graphic and tabular form. The flight controllers, FDO, Booster, and PROP, are now reviewing the space shuttle's ascent data collected by onboard computers and radars during launch. They review this data to ensure that the space shuttle performed as expected.

Directions: Answer questions 14 – 25 in your group. Include units. Discuss answers to be sure everyone understands and agrees on the solutions.

Interpreting Graphs

Refer to *Graph 1: Acceleration*.

14. Identify the independent and dependent variables with their units. Explain your choices.

15. *Graph 1: Acceleration* clearly defines different stages of the space shuttle launch. Using t, write an inequality to represent the domain for the Liftoff stage through Throttle Down stage?

16. What are the coordinates of the point at which the SRBs separate? Explain specifically what the coordinates represent?

17. Consider the interval from the Liftoff stage through Main Engine Cut Off (MECO).
    a. Using t for time in seconds after liftoff, write an inequality to represent the domain for this interval.

    b. Determine how much time elapses in minutes from liftoff to MECO.

18. On *Graph 1: Acceleration* refer to the segment from SRB Separation to MECO.
    a. For the domain of time, $120 \le t \le 450$, what is the range? Describe what happens to the acceleration during this period.

    b. For the domain of time, $460 \le t \le 510$, what appears to be the maximum acceleration? Why do you think the graph rises and falls repeatedly so that it almost appears horizontal?

Refer to *Graph 2: Velocity* where *t* is time and *v* is velocity.

19. In order to exit Earth's atmosphere the space shuttle must break the sound barrier and travel faster than the speed of sound. The speed of sound at sea level is 0.34 km/s. Approximately when does the shuttle reach the speed of sound?

20. Approximately how fast does the space shuttle travel after MECO (510 seconds)?

21. Is *Graph 2: Velocity* of the space shuttle ascent a function? Why or Why not?

Interpreting Tables

Fifty seconds before MECO the space shuttle's velocity continues to increase. Table 1 shows the velocities during that 50 second interval.

Note: This 50 second time interval before MECO occurs 460 seconds after the space shuttle is launched and can be seen in the highlighted section on Graph 2: Velocity*. Thus time 0 in Table 1 actually occurs 460 seconds after launch.*

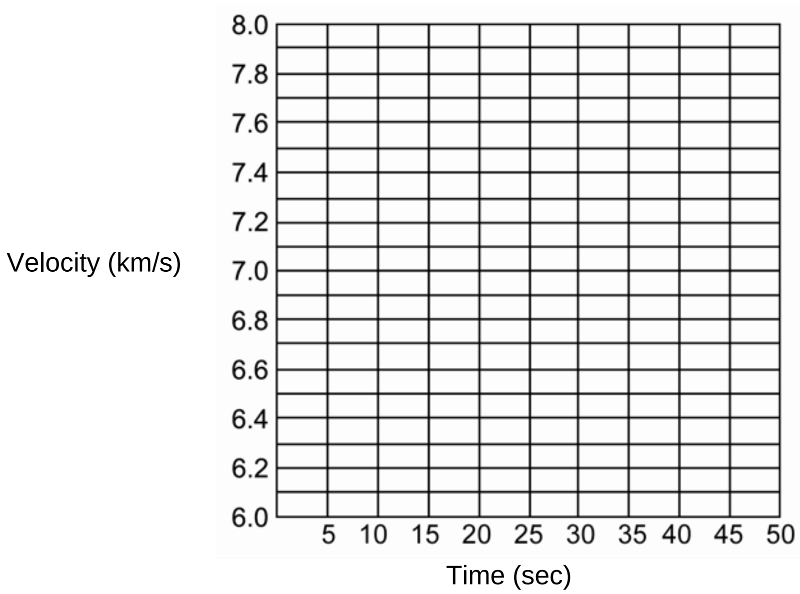
Table 1: Time vs. Velocity for STS-121

Time, *t* (s)	Velocity, *v* (km/s)
0	6.1
10	6.4
20	6.7
30	7.0
40	7.3
50	7.6

22. For each time interval in Table 1, find the change in velocity, *v*, and the change in time, *t.*
   a. What do you notice about the changes in velocity and time?
   b. What is the change in velocity for each interval? What is the change in time for each?
   c. Write the ratio of the change in *v* to the change in *t*.
   d. What is another name for the ratio in part c?
   e. What is the rate of change in velocity per second? Explain your answer.

23. Is the graph of the last 50 seconds before MECO a function? Why or Why not?

24. Using Table 1 as a reference, sketch the graph of Time vs. Velocity.



25. Use inequalities to represent the domain and range of the function in Table 1.

Directions: Answer question 26-29 in your group. Include units. Discuss answers to be sure everyone understands and agrees on the solutions.

26. Refer to *Graph 1: Acceleration*. Use an inequality to represent the range of the graph that corresponds to the domain between “Throttle Down” and “SRB Separation”.

27. Use the data from Table 1 or the graph you sketched in question 24 to determine an equation or function rule that models the table and graph of Time vs. Velocity. Discuss your reasoning with your group.

28. Using the calculator and the equation that you generated in question 27, determine the velocity of the space shuttle at 60 seconds and compare the value to the *Graph 2: Velocity*. Does the equation give a value that matches the graphed value? Can the equation be used after MECO. Explain why or why not.

29. Using a graphing calculator, enter the equation that you generated in question 27, adjust the viewing window, and graph it.
    a. How does the graph compare with the sketch in question 24?

    b. Access the **TABLE** screen and compare the velocity values with those in Table 1. Are the velocity values different? Explain your answer.

Directions: Using the Google Earth Tour (STS-119) on a computer, work with your partner to answer questions 30 – 34. Include units.

30. Open the Google Earth Tour for STS-119 and click on “Play me”. When the tour stops, click on the red dot for “MECO” to open the dialog box. What is the Mission Elapsed Time (MET) in minutes and seconds when MECO occurs? Calculate the time at 50 seconds before MECO?

31. Click on the box for “Velocity Placemarks” to expand the folder. Find the placemarks that fall within the domain of 50 seconds before MECO, and record the times and velocities (ft/s) in Table 2. Use the conversion factor of 1 ft = 0.0003048 km/s to convert the velocities (ft/s) in Column 2 to velocities (km/s), and record them in Column 3. Round answers to the nearest tenth.

Table 2: Google Earth Tour – Time vs. Velocity

MET (min:sec)	Velocity, *v* (ft/s)	Velocity, *v* (km/s)

32. What is the change in time for each interval? What is the change in velocity (km/s) for each interval?

33. Find the rate of change of the velocity? Round to the nearest hundredth.

34. During the 50 seconds before MECO, how does the rate of change of the velocity for STS-119 compare with the rate of change of the velocity for STS-121? Explain your answer.

Ascent: 50 Seconds to MECO – Acceleration and Velocity Graphs