Exploring Space Through ALGEBRA

EDUCATOR EDITION

Geometry and Algebra II

THE LUNAR LANDER – Ascending from the Moon

Instructional Objectives

Students will

- use trigonometric function rules to solve problems
- graph and analyze functions to determine a relationship between two variables

Prerequisites

Students should have a good knowledge of right triangle trigonometry and how to solve problems using trigonometric functions and inverse trigonometric functions. Students should also be able to manipulate and evaluate functions.

Background

This problem is part of a series of problems that apply Algebra and Geometry principles to U.S. Space Exploration policy.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

The vision for space exploration includes returning the space shuttle safely to flight, completing the International Space Station, developing a new exploration vehicle and all the systems needed for embarking on extended missions to the Moon, Mars, and beyond.

NASA is developing a new lunar lander, Altair, which will be capable of landing a new generation of explorers on the surface of the Moon by 2020. Altair finds its origins in Arabic and is derived from a phrase that means "the flying one." Altair is the name of the brightest star in the constellation Aquila and is the 12th brightest star in the night sky. In Latin, Aquila means "eagle," reminiscent of the historic Apollo lunar exploration module Neil Armstrong and Buzz Aldrin landed on the moon in 1969.

Similar in design to the Apollo Lunar Excursion Module (LEM), Altair will be much larger and will have the ability to carry four astronauts to the Moon's surface compared to the two-man Apollo LEM. The new lunar

Grade Level
9-12

Subject Area
Mathematics: Geometry and Algebra II

Key Concept
Application of trigonometric functions

Teacher Prep Time
15 minutes

Problem Duration
45-60 minutes

Technology
Graphing Calculator

Materials
Student Edition

Degree of Difficulty
Moderate to Difficult

Skill
Operations with trigonometric functions; manipulating and evaluating functions; graphing; calculator use

----

NCTM Principles and Standards
- Algebra
- Geometry
- Measurement
- Problem Solving
- Connections

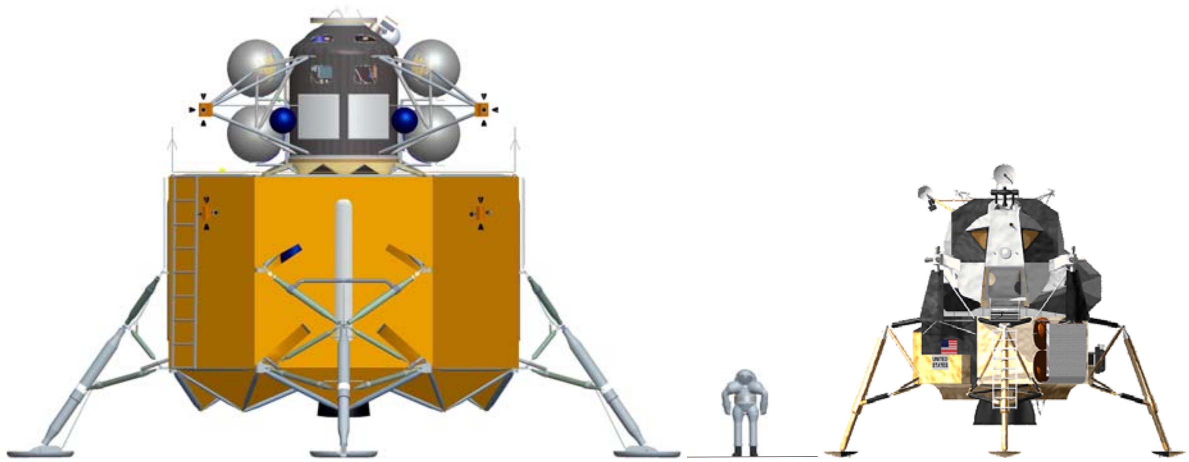


Figure 1: Comparison of the new lunar lander, Altair, (left) (NASA concept) and the Apollo LEM (not to scale)

lander will also have a much larger crew cabin volume, approximately 12 m^3 (approximately 424 ft^3), compared to the Apollo LEM, 6.65 m^3 (235 ft^3). Figure 1 shows a comparison of the Altair concept and the Apollo LEM.

Altair (See Figure 2) will consist of a descent stage, an ascent stage, and a large cargo volume that can be occupied by habitation modules or cargo. The descent stage provides the capability to get into lunar orbit and to perform a lunar landing. It also serves as the launch platform for the ascent stage and as a “flat bed truck” that will transport large cargo to the lunar surface. The ascent stage functions as the flight deck/crew cabin for landing on the lunar surface. It will provide life support for a limited number of days of surface stay, and will allow the crew to ascend back to lunar orbit.

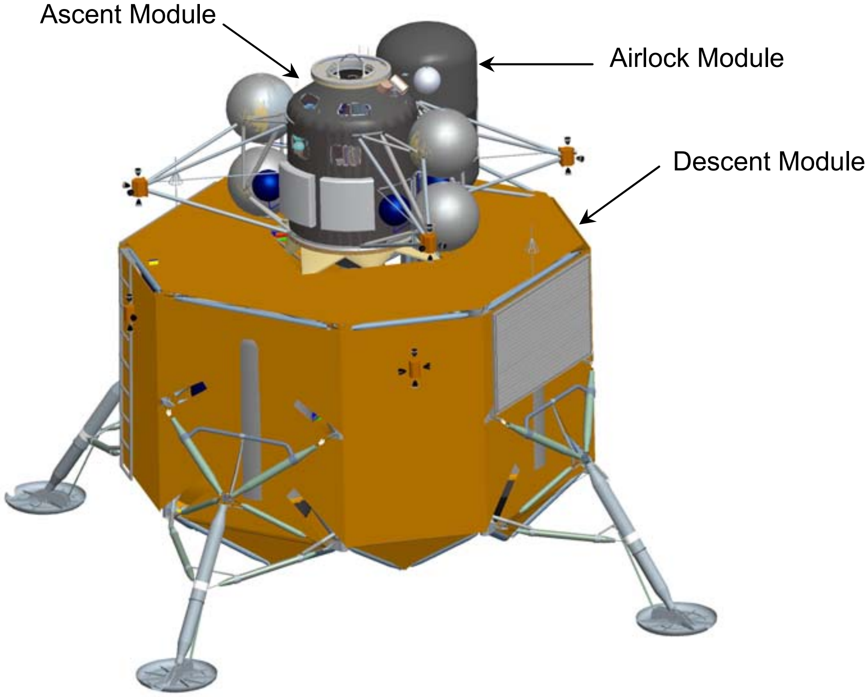


Figure 2: Altair lunar lander (NASA Concept)

During the Apollo era, astronauts could only stay on the lunar surface for a maximum of three days. This time, NASA plans to stay longer. By going to the Moon for extended periods of time, astronauts will search for resources and learn how to work safely in a harsh and unfamiliar environment – stepping stones to future exploration. As currently envisioned, four person crews will stay on the Moon for several days at a time. Altair will provide life support and a base for lunar surface exploration missions. These crews will incrementally build a lunar outpost with power, supplies, rovers and living quarters. Once the outpost is operational, the missions will be extended to two weeks, then two months and ultimately 180 days. Over the first decade of lunar habitation, space explorers will practice the techniques and skills needed for the eventual journey to Mars.

For more information about the lunar lander, Altair, and the U.S. Space Exploration policy, visit www.nasa.gov.

NCTM Principles and Standards

Algebra

- Understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases.
- Draw reasonable conclusions about a situation being modeled.

Geometry

- Use trigonometric relationships to determine lengths and angle measures.

Measurement

- Apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

Lesson Development

Students are asked to evaluate functions and use trigonometric ratios to find distances and angle measures. They are also asked to determine a function describing the angle of elevation in terms of height and a function describing the angle of elevation in terms of time. Students are then asked to graph the function of angle in terms of time, describe how the angle of elevation changes as time elapses and explain why.

Students will work in small groups or pairs to solve the problem. Students are encouraged to perform calculations individually and verify their answers with other members of their group. Students may be asked to demonstrate their answers to the last question.

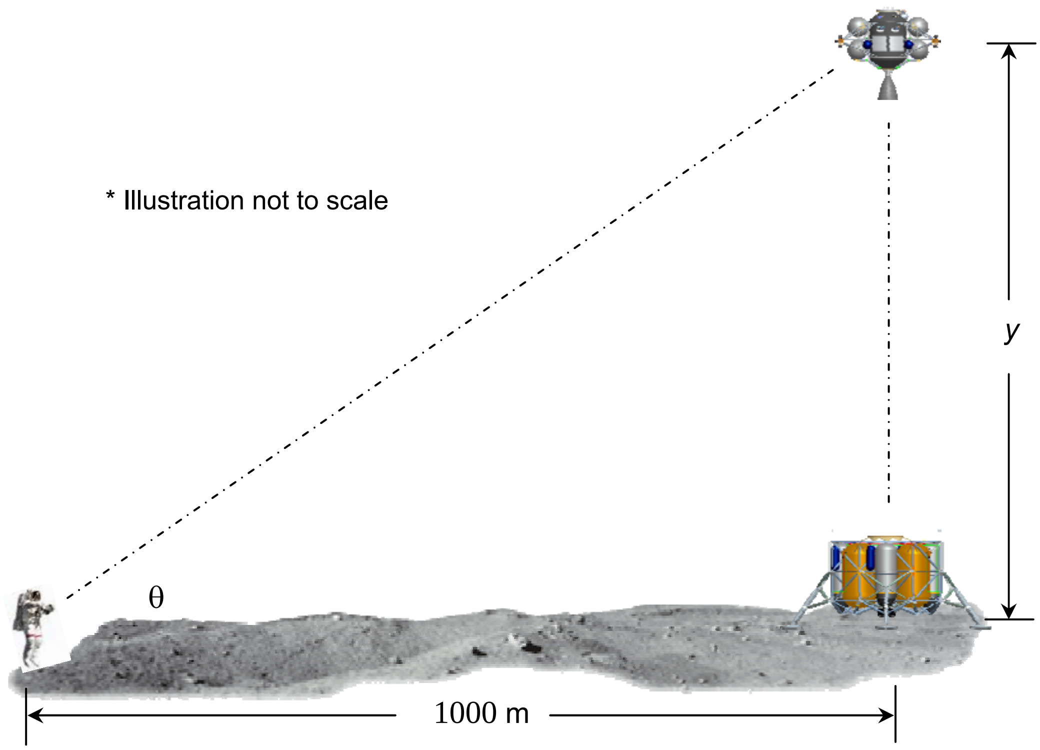


Figure 3: Problem Diagram

Problem

Suppose you are a NASA propulsion engineer working on Altair's ascent stage. Based on outside forces of the lunar environment acting on Altair as it ascends vertically, you have determined the equation of motion for the vehicle to be $\boldsymbol{y(t)=0.683t^2}$, where the height, ***y,*** is measured in meters, the time, ***t,*** is measured in seconds. The quantity, 0.683 m/s^2, is the launch acceleration required by the ascent stage to escape the Moon's gravity. Near a lunar outpost on the Moon, an astronaut with a camera at ground level and 1000 m from the base of Altair is filming the lift-off of the ascent of the vehicle. See Figure 3. Set your calculator mode to degrees before completing the following questions. Round your answers to the nearest whole number.

1. Find the height of the ascent stage vehicle 10 seconds after lift-off.
2. Find the angle of elevation of the camera 10 seconds after lift-off.
3. How much time has elapsed when the vehicle is 100 m above the surface? What is the angle of elevation of the camera at this time?
4. What is the height of the vehicle when the angle of elevation is 20°? How much time has elapsed at this point?
5. What is the angle of elevation when the ascent stage is 2000 m above the surface? When does this occur?
6. Write an equation to describe the angle of elevation of the camera, *θ*, in terms of *y*, the height of the vehicle.
7. Write an equation describing the angle of elevation, *θ*, in terms of *t*, the time elapsed since lift-off.
8. Graph the equation describing the angle of elevation, *θ*, in terms of *t* and the equation *y* = 90º on your graphing calculator. Set your window settings to show time up to 250 seconds and angle measure up to 100°. What is happening to the angle of elevation as time elapses? Does the angle ever reach 90°? Explain why this happens.

Wrap-Up

After students have completed the activity discuss with the class the reasons why the angle will never be equal to or greater than 90°. Discuss with the students the idea of the angle approaching 90° and getting so close that the astronaut would feel like he was holding the camera at a 90° angle.

Extensions

In the problem, the astronaut is standing 1000 m away from the ascent stage vehicle. In groups of three or four, have students discuss why 1000 m might be an appropriate distance. Have students discuss three different factors that would effect where the astronaut would stand. Some factors may include safety, the ability to get a good shot due to the angle, the dusty surface of the moon, and the zoom capabilities of the camera. Students should list positive and negative consequences that would occur if the astronaut chose a closer distance and if he/she chose a further distance. After students have discussed these ideas, have them write up a proposal for a new camera site. The proposal should include at least three different advantages for the new site. Have the groups present their proposals to the class.

Solution Key (one approach)

An astronaut, with camera at ground level, near a lunar outpost on the Moon is filming the liftoff of the ascent portion of Altair that is rising vertically according to the position equation $y(t)=0.683t^2$, where ***y*** is measured in meters and ***t*** is measured in seconds. The astronaut holding the camera is 1000 m from the base of Altair. See Figure 3. Set your calculator mode to degrees before completing the following questions. Round your answers to the nearest whole number.

1. Find the height of the ascent stage vehicle 10 seconds after lift-off.

$$y = 0.683t^2$$
$$y = 0.683(10)^2$$
$$y = 68 \text{ meters}$$

2. Find the angle of elevation of the camera 10 seconds after liftoff.

$$\tan\theta = \frac{68.3}{1000}$$
$$\theta = \tan^{-1}\left(\frac{68.3}{1000}\right)$$
$$\theta = 4°$$

3. How much time has elapsed when the vehicle is 100 m above the surface? What is the angle of elevation of the camera at this time?

$$\begin{aligned} y &= 0.683t^2 \\ 100 &= 0.683t^2 \\ 146.413 &= t^2 \\ t &= 12 \text{ seconds} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{100}{1000} \\ \theta &= \tan^{-1}(0.10) \\ \theta &= 6^\circ \end{aligned}$$

4. What is the height of the vehicle when the angle of elevation is 20°? How much time has elapsed at this point?

$$\begin{aligned} \tan 20^\circ &= \frac{y}{1000} \\ y &= 1000\tan 20^\circ \\ y &= 364 \text{ meters} \end{aligned}$$

$$\begin{aligned} y &= 0.683t^2 \\ 364 &= 0.683t^2 \\ 532.9 &= t^2 \\ t &= 23 \text{ seconds} \end{aligned}$$

5. What is the angle of elevation when the ascent stage is 2000 m above the surface? When does this occur?

$$\begin{aligned} \tan\theta &= \frac{2000}{1000} \\ \tan\theta &= 2 \\ \theta &= \tan^{-1}(2) \\ \theta &= 63^\circ \end{aligned}$$

$$\begin{aligned} y &= 0.683t^2 \\ 2000 &= 0.683t^2 \\ 2928.3 &= t^2 \\ t &= 54 \text{ seconds} \end{aligned}$$

6. Write an equation to describe the angle of elevation of the camera, θ, in terms of y, the height of the vehicle.

$$\begin{aligned} \tan\theta &= \frac{y}{1000} \\ \theta &= \tan^{-1}\left(\frac{y}{1000}\right) \end{aligned}$$

7. Write an equation describing the angle of elevation, θ, in terms of t, the time elapsed since lift-off.

$$\theta = \tan^{-1}\left(\frac{y}{1000}\right)$$

$$\theta = \tan^{-1}\left(\frac{0.683t^2}{1000}\right)$$

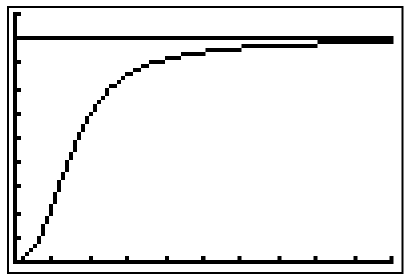
$$\theta = \tan^{-1}(0.000683t^2)$$

8. Graph the equation describing the angle of elevation, θ, in terms of t and the equation $y = 90º$ on your graphing calculator. Set your window settings to show time up to 250 seconds and angle measure up to 100°. What is happening to the angle of elevation as time elapses? Does the angle ever reach 90°? Explain why this happens.

The screen shots below show what the students would see on a TI-84 calculator.

Plot1 Plot2 Plot3
$\backslash Y_1 = \tan^{-1}(0.000683X^2)$
$\backslash Y_2 = 90$
$\backslash Y_3 =$
$\backslash Y_4 =$
$\backslash Y_5 =$
$\backslash Y_6 =$

WINDOW
Xmin=0
Xmax=250
Xscl=25
Ymin=0
Ymax=100
Yscl=10
Xres=1



By graphing and then using the TRACE option on the graph, one can see that the angle gets closer and closer to 90° as time increases. As time passes the vehicle is getting higher and higher. Since the astronaut is not directly below the vehicle, the angle cannot be 90°, but as it gets higher and higher it becomes very close to 90°.

Note: To demonstrate to the students that the angle approaches but never actually reaches 90°, have the students look at the table and scroll down for a while. They will see that they will never get to 90 unless they scroll down to 13000. If any student is persistent enough to scroll down that far, you can explain that the calculator eventually has to round off the number. A good setting for the table is to start at 0 and have it increase in increments of 10.

X	Y_1	Y_2
0	0	90
10	3.9072	90
20	15.28	90
30	31.579	90
40	47.539	90
50	59.645	90
60	67.868	90

X=0

X	Y_1	Y_2
70	73.364	90
80	77.114	90
90	79.754	90
100	81.67	90
110	83.101	90
120	84.194	90
130	85.049	90

X=70

X	Y_1	Y_2
210	88.098	90
220	88.267	90
230	88.415	90
240	88.544	90
250	88.658	90
260	88.759	90
270	88.849	90

X=270

Contributors

Thanks to the subject matter experts for their contributions in developing this problem:

NASA Experts

NASA Langley Research Center

David North
Space Mission Analysis Branch
Systems Analysis and Concepts Directorate

Problem Development

NASA Langley Research Center

Chris Giersch
Communications and Education Lead
Exploration and Flight Projects Directorate

NASA Johnson Space Center
Human Research Program Education and Outreach

Natalee Lloyd
Education Specialist, Secondary Mathematics

Monica Trevathan
Education Specialist, Instructional Technology

Martha Grigsby
Education Specialist, Secondary Mathematics

Traci Knight
Graphics Specialist

Virginia Beach City Public Schools

Josephine Letts
Mathematics Teacher
Ocean Lakes High School, VA

University of North Carolina Charlotte

Dr. David Pugalee
Research Associate
Center for Mathematics, Science & Technology Education

National Institute of Aerospace

Norman “Storm” Robinson, III
Education Specialist

Exploring Space Through Algebra

THE LUNAR LANDER – Ascending from the Moon

Feedback Form

Please take a minute to complete this feedback form. Your input will help improve this product and will help us create new, useful material.

Fax the completed form to: (281) 461-9350 – Attention: Martha Grigsby

Or type your responses in an email and send to: martha.grigsby@tietronix.com

Please select the appropriate response.

1. This problem was useful in my classroom. YES NO

2. The problem successfully accomplished the stated Instructional Objectives. YES NO

3. I will use this problem again. YES NO

4. Please provide suggestions for improvement of this problem and associated material:

____

____

____

5. Please provide suggestions for future Algebra problems, based on NASA topics, that you would like to see developed:

____

____

____

Thank you for your participation.

Please fax this completed form to Martha Grigsby at (281) 461-9350.