



Exploring Space Through ALGEBRA


EDUCATOR EDITION
**Algebra I
and Geometry**

Next Generation Spacecraft – Orion

Instructional Objectives

Students will

- decompose a larger geometric shape into smaller parts;
- apply the proper area formulas for various geometric shapes; and
- estimate the area of a complex geometric shape using decomposition methods.

Prerequisites

Prior to this activity, students should have had experiences decomposing geometric figures (breaking figures into smaller shapes) and applying area formulas. Students should be familiar with using calculators in evaluating formulas.

Background

This problem is part of a series of problems that apply Algebra and Geometry principles to U.S. Space Exploration policy.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

The vision for space exploration includes returning the space shuttle safely to flight, completing the International Space Station, developing a new exploration vehicle and all the systems needed for embarking on extended missions to the Moon, Mars, and beyond.

Orion is the vehicle NASA is developing to carry a new generation of explorers back to the Moon and later to Mars. Orion will succeed the space shuttle as NASA's primary vehicle for human space exploration. Figure 1 shows some components of the Orion spacecraft.

Grade Level
8-12

Subject Area
Mathematics: Algebra I,
Geometry

Key Concept
Decomposition of a figure
into small shapes to
estimate area

Teacher Prep Time
5 minutes

Problem Duration
45-60 minutes

Technology
TI-83 Plus family; TI-84
Plus family; TI Explorer;
other scientific calculator

Materials
- Student Edition
- Ruler or straight edge
- Colored pencils or
Highlighters
- Graph paper

Degree of Difficulty
Basic to Moderate

Skill
Operate with real numbers;
solve problems in a
geometric context; use
formulas to find areas

NCTM Principles and Standards
- Number and Operations
- Algebra
- Geometry
- Measurement
- Problem Solving
- Communication
- Connections
- Representation

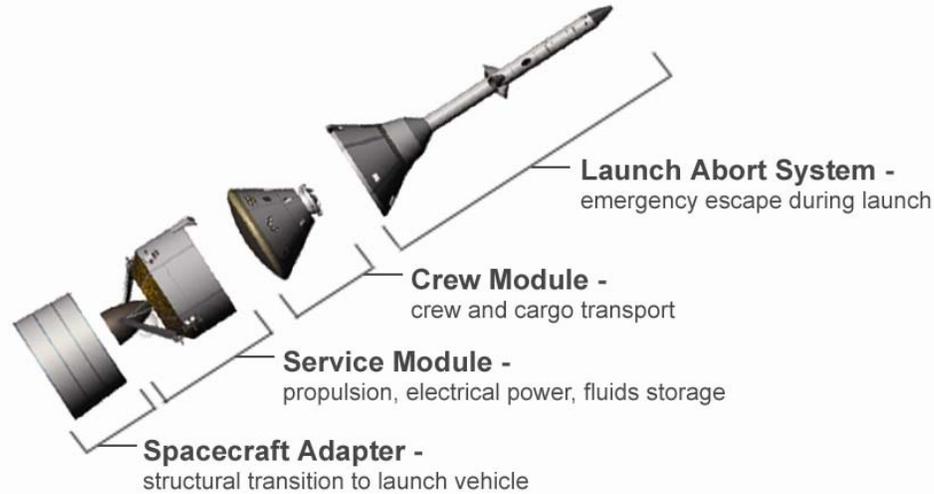


Figure 1: Components of the Orion spacecraft (NASA concept)

Orion will use an improved, larger blunt-body capsule, much like the shape of the Apollo capsule (Figure 2). With an outside diameter of 5 meters, the Orion crew module will have more than two and a half times the volume of an Apollo capsule.

During Orion's planning process, NASA studied several different kinds of entry vehicles and rockets. Although Apollo-era researchers were consulted, NASA did not set out to make the Orion spacecraft identical to the Apollo spacecraft. Ultimately, this design was found to meet the requirements while being the most effective within the safety goals.

For more information about Orion and the U.S. Space Exploration policy, visit www.nasa.gov.



NASA concept of the Orion crew module



Apollo capsule

Figure 2: Shape comparison of the Orion crew module and the Apollo capsule (not to scale)

NCTM Principles and Standards

Number and Operations

- Develop and use strategies to estimate the results of rational number computations and judge the reasonableness of the results.

**Algebra**

- Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations.

Geometry

- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

Measurement

- Understand both metric and customary systems of measurement.
- Understand relationships among units and convert from one unit to another within the same system.
- Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision.

Problem Solving

- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Use representations to model and interpret physical, social, and mathematical phenomena.

Problem

The Orion spacecraft will replace the space shuttle as NASA's spacecraft for human space exploration. The vehicle is designed to accommodate four to six astronauts traveling into space. This activity focuses on the Orion crew module, one of four functional modules of the Orion spacecraft. Students will find the areas of the largest vertical and horizontal cross-sections. This information will provide students a sense of the room within the crew module. Students will also be asked how many crew modules could fit in their classroom. This might be extended to larger areas such as the gymnasium or cafeteria.

Lesson Development

Students will work in pairs to find the areas of the largest vertical and horizontal cross-sections using a drawing of the Orion crew module. Students will record their work on the tables provided.



Encourage students to use different colored pencils or highlighters for each geometric shape to show how they are decomposing the figure into smaller parts. Using the largest vertical cross-section diagram of the crew module (Figure 4), students will likely see the top part of the figure as a trapezoid. The next area is rectangular, but students may select different segments for their length estimate. The bottom of the diagram might be decomposed into a triangle. Because the Orion crew module is symmetrical, students may double the areas along the way or do this as part of their final work.

The largest horizontal cross-section is a circle. Once students realize this to be the case, finding the area of the cross-section involves using the formula for area of a circle.

Students will use their final results to compare the horizontal and vertical cross-sections of the crew module to the floor space and height of their classroom. Errors may occur if students simply divide the area of the floor space of their classroom by the final area (horizontal) of the crew module, which is the area of a circle. Doing so fails to take into account the dimensions of the classroom and crew module. For example, some classrooms have a dimension of 10.6 meters by 7.6 meters (35 feet by 25 feet), with an area of (80.6 m²). If students simply divide the area, they may erroneously conclude that four Orion modules would fit into the space. However, the diameter of the Orion module (approximately 5 meters) would only allow two modules to fit in the space, provided that the ceiling is at least 3.3 meters high (just over 10 feet). A drawing of the footprint of the classroom and crew module would be helpful in answering this question.

Wrap-Up

Once the exercise is completed, have students form pairs or small groups to discuss and evaluate the various approaches taken and methods used to complete the activity. Doing so provides students with the opportunity to learn from others and to discuss the amount of error in each approach. As part of the discussion, students should include the various approaches used to estimate the areas, including the decomposition of the figures, and how symmetry was used in their approaches.

Extensions

Trace the outline of the crew module diagram onto graph paper. Estimate the area by counting grid squares that are included within the outline.

Students should pay attention to the dimensions of the squares on the graph paper and they will need to use this information in converting their final answers into square meters. This extension involves students in using proportions to estimate the area. Students will need to determine the scale factor in order to set up proportions that will allow them to find the actual estimated area. Students should compare their answers with the answers from the original activity and write a short discussion comparing the two outcomes.

Solution Key (One approach)

The Orion spacecraft will replace the space shuttle as NASA's spacecraft for human space exploration. This activity focuses on the Orion crew module, one of four functional modules of the Orion spacecraft. The vehicle is designed to accommodate four to six astronauts traveling into space.

1. To get a sense of the room inside the crew module, find the area, in square meters (m²), for the largest vertical cross-section (Figure 3; Figure 4). Show how you would decompose, or break the figure into smaller parts, to estimate the total vertical area. You may use a calculator. Record your information in the table provided (Table 1). Please round your answer to three decimal places.

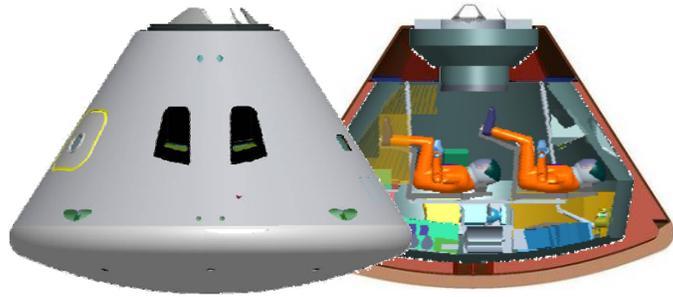


Figure 3: Vertical cross-section of the Orion crew module (NASA Concept)

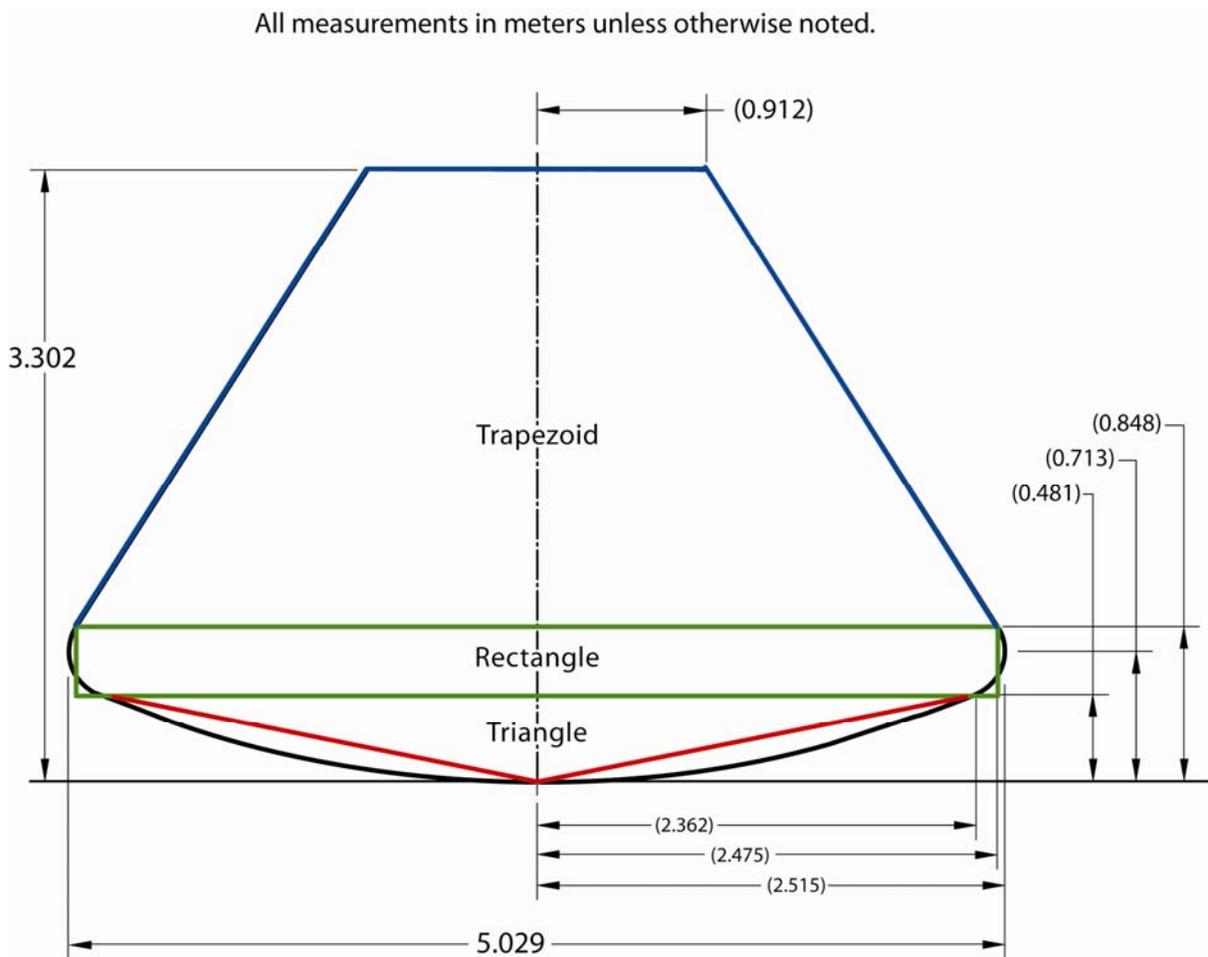


Figure 4: Largest vertical cross-section of the Orion crew module (This figure shows the student version with decomposition.)



Table 1: Vertical Cross-Section Area Data

| Figure | Area Formula | Area Formula with Values | Area (m ²) |
|-----------|-------------------------------|---|------------------------|
| Trapezoid | $A = \frac{1}{2}(b_1 + b_2)h$ | $A = \frac{1}{2} \times [(2 \times 2.475) + (2 \times 0.912)] \times 2.454$ | $A = 8.312$ |
| Rectangle | $A = lw$ | $A = (2 \times 2.475) \times 0.367$ | $A = 1.817$ |
| Triangle | $A = \frac{1}{2}bh$ | $A = \frac{1}{2} \times (2 \times 2.362) \times 0.481$ | $A = 1.136$ |

Total Area = Area of Trapezoid + Area of Rectangle + Area of Triangle

$$A = 11.265 \text{ m}^2$$

2. If the actual largest vertical cross-sectional area of the crew module is 11.665 m^2 , how far off was your estimate? Express your answer in terms of a percent (percent error). Please round your answer to the nearest percent.

$$\% \text{ error} = \left| \frac{\text{actual value} - \text{estimated value}}{\text{actual value}} \right| \cdot 100$$

$$\% \text{ error} = \left| \frac{11.665 - 11.265}{11.665} \right| \cdot 100$$

$$\% \text{ error} = 3\% \text{ under estimated}$$

3. Find the area, in square meters (m²), for the largest horizontal cross-section (Figure 5; Figure 6). Show how you would decompose, or break the figure into smaller parts, to estimate the total horizontal area. You may use a calculator to evaluate the formulas. Record your information in the table provided (Table 2). Please round your answer to three decimal places.

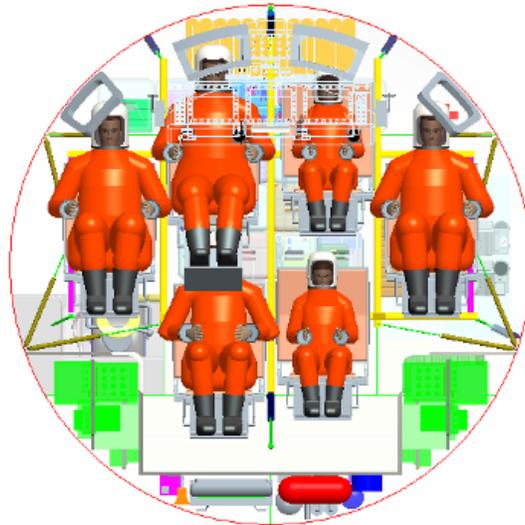


Figure 5: Horizontal cross-section of the Orion crew module (NASA Concept)

All measurements in meters unless otherwise noted.

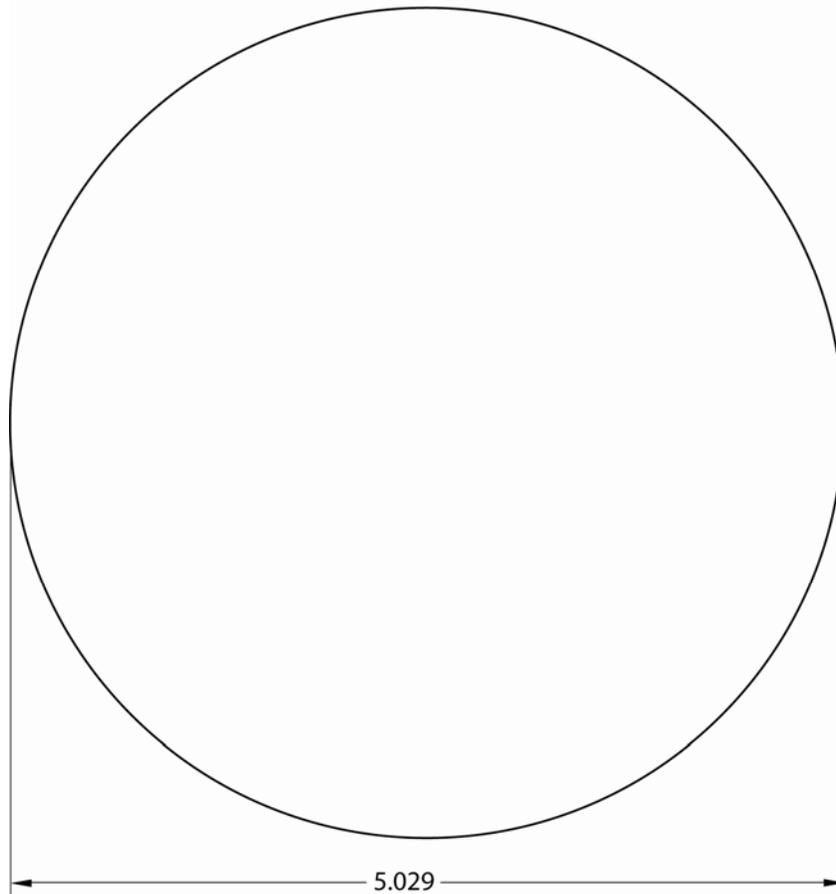


Figure 6: Largest horizontal cross-section of the Orion crew module



Table 2: Horizontal Cross-Section Area Data

| Figure | Area Formula | Area Formula with Values | Area (m ²) |
|--------|---------------|--------------------------|------------------------|
| Circle | $A = \pi r^2$ | $A = \pi \times 2.515^2$ | $A = 19.871$ |

$$\pi = 3.142$$

Total Area = Area of Circle

$$A = 19.871 \text{ m}^2$$

4. If lined up side by side, how many of the largest horizontal cross-sections of the Orion crew module (Figure 6) do you think would fit in your classroom? Explain your answer.

Answers will vary.

5. Would the largest vertical cross-section of the Orion crew module (Figure 4) fit in your classroom? Explain your answer.

Answers will vary.



Contributors

Thanks to the subject matter experts for their contributions in developing this problem:

NASA Experts

NASA Langley Research Center

Robert Dillman
Systems Engineer
Explorations Systems Engineering Branch
Systems Engineering Directorate

Barry Meredith
CEV Project Implementation Manager
Exploration and Flight Projects Directorate

Problem Development

NASA Langley Research Center

Chris Giersch
Communications and Education Lead
Exploration and Flight Projects Directorate

Sharon Bowers
Educator in Residence
Office of Strategic Communications and Education

NASA Johnson Space Center

Monica Trevathan
Education Project Lead
Human Research Program Education and Outreach

Traci Knight
Graphics Specialist
Human Research Program Education and Outreach

Virginia Beach City Public Schools

Josephine Letts
Mathematics Teacher
Ocean Lakes High School

University of North Carolina Charlotte

Dr. David Pugalee
Research Associate
Center for Mathematics, Science & Technology Education



Exploring Space through Algebra

Next Generation Spacecraft – Orion

Feedback Form

Please take a minute to complete this feedback form. Your input will help improve this product and will help us create new, useful material.

Fax the completed form to: (281) 461-9350 – Attention: Monica Trevathan

Or type your responses in an email and send to: Monica.Trevathan-1@nasa.gov

Please circle the appropriate response.

1. This problem was useful in my classroom. YES NO
2. The problem successfully accomplished the stated Instructional Objectives. YES NO
3. I will use this problem again. YES NO

4. Please provide suggestions for improvement of this problem and associated material:

5. Please provide suggestions for future Algebra problems, based on NASA topics, that you would like to see developed:

Thank you for your participation.

Please fax this completed form to Monica Trevathan at (281) 461-9350.