MATH AND SCIENCE @ WORK

AP* CALCULUS Student Edition

LUNAR SURFACE COMMUNICATIONS

Background

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

Outpost concepts are now being designed and studied by engineers, scientists, and sociologists to facilitate long-duration human missions to the surface of the Moon or other planetary bodies (Figure 1). Such outposts will include habitat modules, laboratory modules, power systems, transportation, life support systems, and protection from the environment.

These long-duration missions will also require robust and reliable communications. It will be important to maintain constant communications with Earth. Therefore, 24 hours per day/7 days per week coverage at the outpost could be a requirement. This will likely be accomplished by a combination of communication satellites in orbit around the planetary body and communication equipment on the surface.

The habitat (Figure 1) on the surface will need video downlink capability to Earth. In addition to the communication requirements between the planetary surface and Earth, it will also be important to maintain constant communications between surface crew members, regardless of their distance from the outpost.

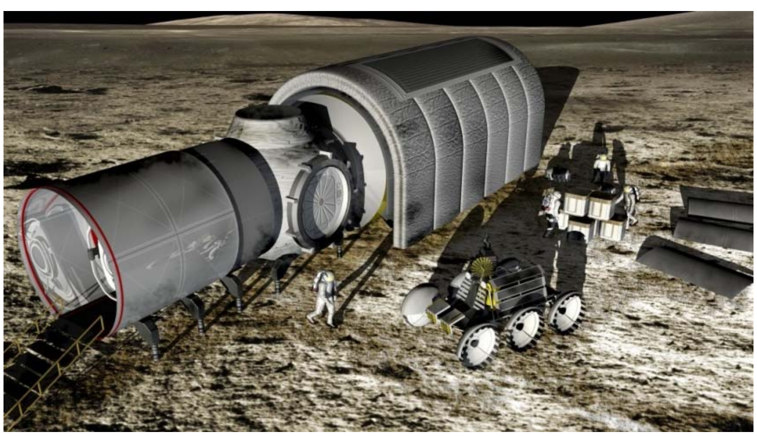

Figure 1: Habitat, airlock, and vehicles (NASA concept)

Surface to surface communications involves communicating between astronauts, rovers, robots, habitats, power stations, and science experiments, as well as communication within the habitats. For surface-based communication systems, there is a line of sight limitation on rover communication with the habitat. Astronauts must have either the habitat or the rover in their line of sight to maintain communications with Earth.

The communications system should be easily expandable. Future missions will not want to abandon existing equipment, but instead incorporate existing equipment into an expanding communications system.

These plans give NASA a huge head start in getting to Mars. We will already have rockets capable of transporting heavy cargo, as well as a versatile crew capsule. An outpost within a few days travel from Earth would give us needed practice of "living off the land" away from our home planet, before making the longer trek to Mars.

Problem

When relying on surface to surface communication understanding line of sight is critical. Consequently, an important measurement in planetary exploration is the distance to the horizon. This depends on the diameter of the planet and the height of the observer above the surface. Geometry can determine the height of a transmission antenna required to insure proper reception within a specified distance. Use the diagram in Figure 2 to answer the following questions.

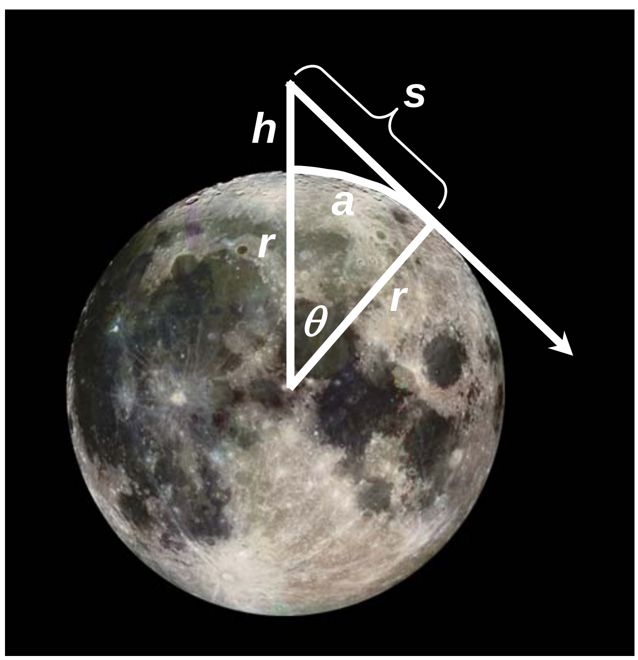

Figure 2: Problem Diagram of the Moon
NOTE: Diagram is exaggerated to show relationship and reference points.

A. If the radius of the Moon is given by *r*, and the height of the tower above the surface is given by *h*, use Figure 2 to derive the formula for the line of sight distance, *s*, to the horizon tangent point.

B. In terms of *r* and *h*, derive the formula for the arc length, *a*, which is the distance along the moon to the point of tangency.

C. On Earth, a radio station may have an antenna tower 50 meters (m) tall. What would be the reception distance *s*, to the nearest meter, if that same tower were on the Moon? The radius of the Moon is 1,738 kilometers (km).

D. Graph the equation for the line of sight distance from question A over the interval 0, 60. What happens as the antenna height increases? To the nearest meter, find the rate of change to the lunar line of sight with respect to the antenna height or *ds/dh* at *h* = 50 m. In practical terms, what does this mean?

E. Use local linear approximation to predict the distance for an antenna of 51 meters. How does this compare to the actual calculation using your equation from question A? Would local linear approximation be as accurate in predicting the distance for an antenna height of 11 meters? Explain your reasoning.

F. What is the rate of change of the distance, *a*, along the lunar surface to the lunar tower at the line of sight position when *h* = 50 m? Express your answer as a whole number.

G. Under what conditions do the line of sight formula (question D) and the arc length formula (question E) give significantly different answers? When would you use the arc length formula on the Moon or on some other solar system body?