MATH AND SCIENCE @ WORK

AP* CALCULUS Student Edition

SPACE SHUTTLE AUXILIARY POWER UNITS

Background

Since its conception in 1981, NASA has used the space shuttle for human transport, the construction of the International Space Station (ISS), and to research the effects of space on the human body. One of the keys to the success of the Space Shuttle Program is the Space Shuttle Mission Control Center (MCC). The Space Shuttle MCC at NASA Johnson Space Center uses some of the most sophisticated technology and communication equipment in the world to monitor and control the space shuttle flights.

Within the Space Shuttle MCC, teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle. They work together as a powerful team, spending many hours performing critical simulations as they prepare to support preflight, ascent, flight, and reentry of the space shuttle and the crew. The flight controllers provide the knowledge and expertise needed to support normal operations and any unexpected events.

One member of this team of experts is the Mechanical, Maintenance, Arm, and Crew Systems flight controller, whose call sign is MMACS (pronounced “Max”). One of the responsibilities of this position is to monitor the performance and fuel usage of the space shuttle’s Auxiliary Power Units (APUs). Three

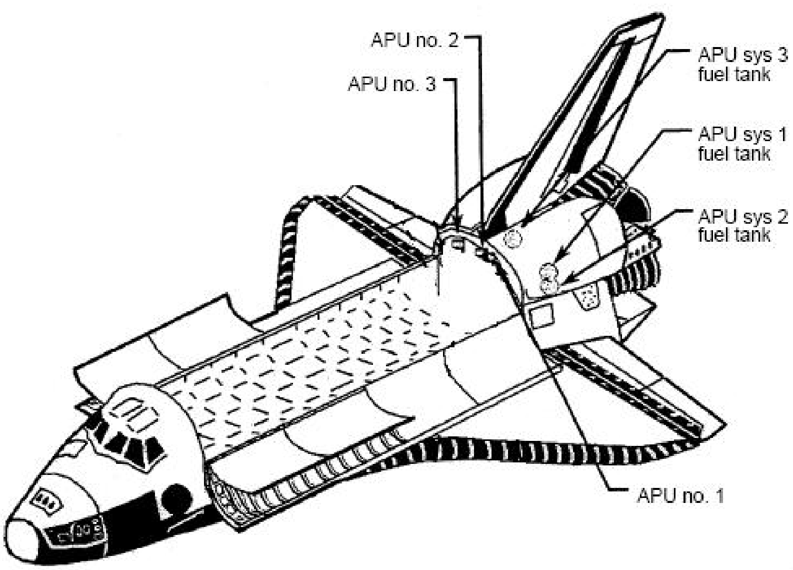

Figure 1: Space shuttle with APU and fuel tank locations

identical but independent hydraulic systems are used to assist in maneuvering the space shuttle, deploying the landing gear, and applying the brakes. Each of these hydraulic systems is powered by a separate APU. The APUs convert chemical energy into mechanical shaft power to drive the hydraulic main pumps. The chemical energy is in the form of hydrazine fuel (N_2H_4), which is stored in three independent tanks. The location of the APUs and fuel tanks can be seen in Figure 1.

The MMACS flight controller monitors the fuel usage of the APUs by looking at plots of fuel tank pressure versus time, as well as fuel quantity versus time. The fuel quantity is not directly measured, but is calculated using the fuel tank pressure and a number of constants.

Problem

When the fuel tank valves are opened and the Auxiliary Power Unit (APU) is started, the fuel, hydrazine (N_2H_4), is forced out of the tank using gaseous nitrogen (GN_2) at high pressures. A schematic of a fuel tank and APU can be seen in Figure 2. As the APU burns fuel, the mass of fuel in the tank decreases, while the mass of nitrogen remains constant. The pressure of the fuel tank is directly measured and used to calculate the quantity of fuel.

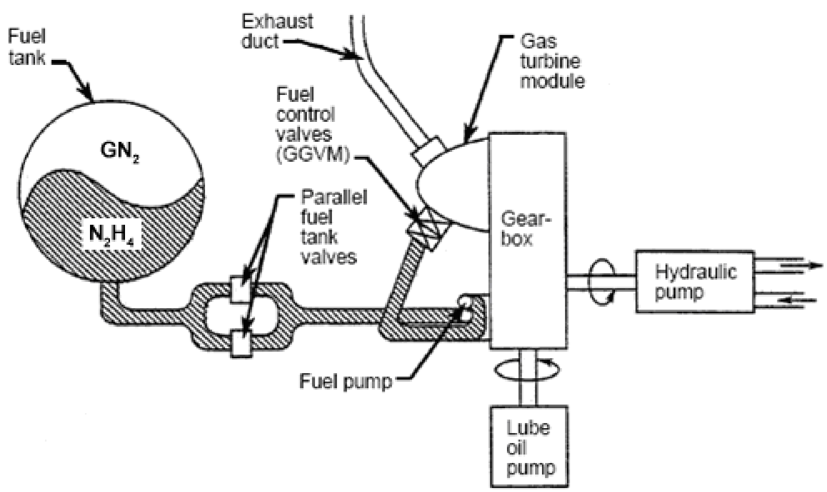

Figure 2: Schematic of a fuel tank and APU

The list of variables and constants is provided in Table 1 and will prove beneficial in solving the following questions.

Table 1: Variables and constants used throughout the problem

V_{N_2} = Volume of Nitrogen (N_2)	changing variable
$m_{N_2H_4}$ = Mass of Hydrazine (N_2H_4)	changing variable
P = Pressure (psi)	changing variable
V_{tank} = Volume of Tank	constant = 11,494 in^3
m_{N_2} = Mass of Nitrogen (N_2)	constant = 2.688 lbs
Rho = density of Hydrazine (N_2H_4)	constant = 0.0363 $\frac{lbs}{in^3}$
T = Temperature (Rankin)	constant = 530 ^oR
R = Universal Gas	constant = 661.8 $\frac{in \cdot lbs}{lbs \cdot {}^oR}$

A. Assume you are the MMACS flight controller and your control panel shows the following fuel tank pressure vs. time plot (figure 3).

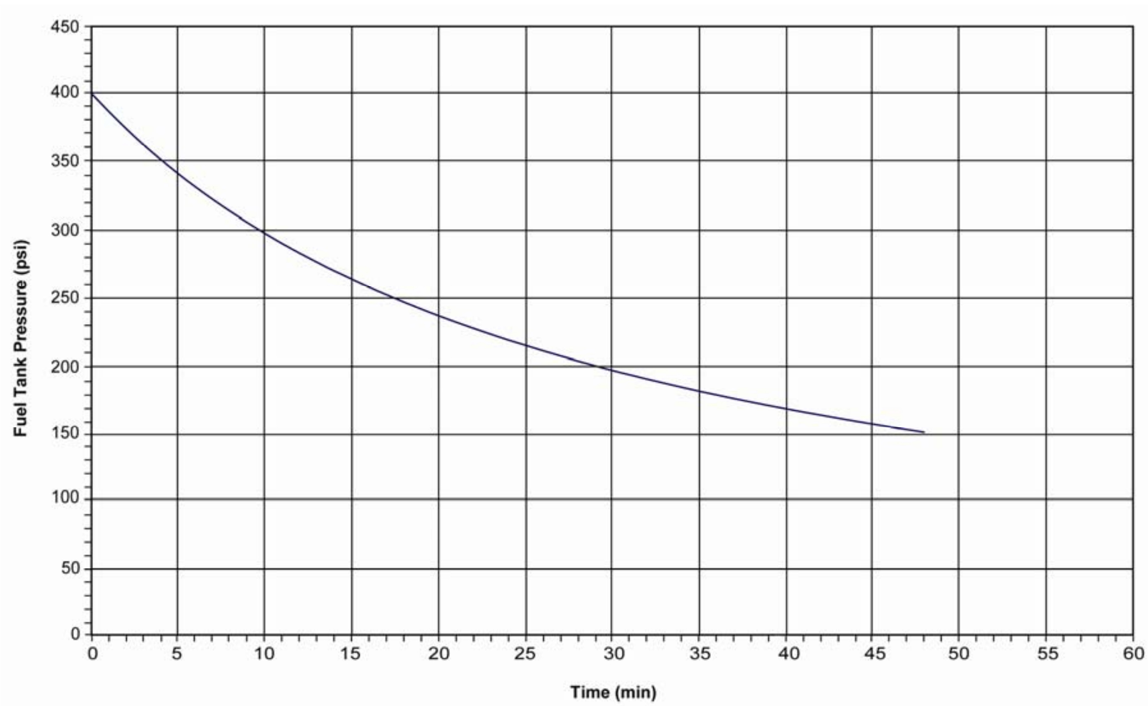

Figure 3: Fuel Tank Pressure vs. Time

I. Use the graph to explain what is happening to the fuel tank pressure over time.

II. Find an appropriate interval around 1 minute and estimate the average rate of change of the fuel tank pressure. What is the average rate of change of the pressure around 15 minutes? Around 35 minutes?

III. Explain the meaning of the average rates of change just found.

IV. How would your estimated average rates of change compare to the instantaneous rates of change or $\frac{dP}{dt}$ at each point? Explain your reasoning.

B. Fuel mass is a combination of the fuel's density, fuel pressure at a specific time, and the volume of the tank. It is computed using a formula that relates the quantities in the following three equations.

$V_{N_2} = \frac{m_{N_2} \cdot R \cdot T}{P}$ (ideal gas law for V_{N_2})

$V_{tank} = V_{N_2} + V_{N_2H_4}$ (tank volume balance)

$m_{N_2H_4} = V_{N_2H_4} \cdot Rho$ (the mass fuel density equation for incompressible fluid)

Use algebra to derive the equation for the mass of hydrazine:

$$m_{N_2H_4} = Rho \cdot V_{tank} - \frac{Rho \cdot m_{N_2} \cdot R \cdot T}{P}$$

C. Use the equation derived in question B to help answer the questions below.

I. Complete the table below by calculating the mass of hydrazine in the fuel tank. Use Figure 3 to find pressure at a given time.

Time (min)	Pressure (psi)	$m_{N_2H_4}$ (lbs)
1		
15		
35		

II. Use your values to sketch a linear graph of Fuel Quantity ($m_{N_2H_4}$) vs. Time and estimate the rate of change of the line. If a graphing calculator is available, use the capabilities of the calculator to plot the points and to find the slope.

D. Differentiate the equation for mass of hydrazine, $m_{N_2H_4} = Rho \cdot V_{tank} - \frac{Rho \cdot m_{N_2} \cdot R \cdot T}{P}$, with respect to time, *t*, to determine the fuel usage rate. Remember the only variables that are changing are mass of hydrazine, $m_{N_2H_4}$, and pressure, *P*. Explain the practical meaning of the derivative with units.

E. The solutions found in questions A – D will help you answer the following.

I. Complete the following table using the rate of change formula from question D to calculate the rate of change of hydrazine ($m_{N_2H_4}$). Use Figure 3 and your answers from question A when completing the table.

Time (min)	Pressure (psi)	$\frac{dP}{dt}$ (psi/min)	$\frac{dm_{N_2H_4}}{dt}$ (lbs/min)
1			
15			
35			

II. How does this calculated rate of change of hydrazine $\left(\frac{dm_{N_2H_4}}{dt}\right)$ compare to the slope of the graph of Fuel Quantity ($m_{N_2H_4}$) vs. Time found in question C, part II?

F. As the MMACS flight controller, you notice an unusual trend in the fuel tank pressure vs. time plot indicating a possible leak. Since it is difficult to troubleshoot a leak while an APU is in operation, Mission Control will have the crew on board the space shuttle shut one APU down. Figure 4 represents the data from the APU.

I. Based on the graph (Figure 4), how would you determine if there is a leak in the fuel tank?

II. If there is a leak, how fast is the leak rate in lbs/min at t = 50 minutes?

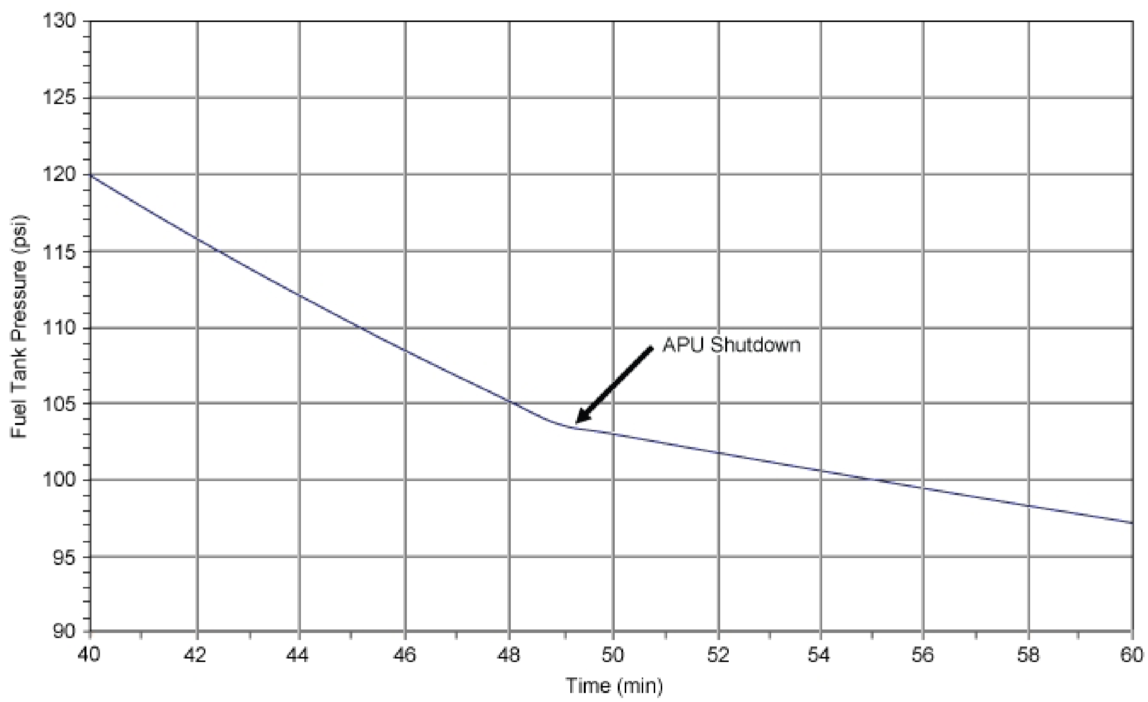

Figure 4: Fuel Tank Pressure vs. Time