



MATH AND SCIENCE @ WORK

AP* CALCULUS Student Edition



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SPACE SHUTTLE ASCENT

Background

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries, and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

Since its first flight in 1981, the space shuttle has been used to extend research, repair satellites, and help with building the International Space Station, or ISS. However, by 2010 NASA plans to retire the space shuttle in favor of a new Crew Exploration Vehicle, or CEV. Until then, space exploration depends on the continued success of space shuttle missions. Critical to any space shuttle mission is the ascent into space.



Figure 1: Space Shuttle Discovery at lift-off during STS 121.



Figure 2: Onlookers view the launch of STS 121.

The ascent phase begins at liftoff and ends at insertion into a circular or elliptical orbit around the Earth. To reach the minimum altitude required to orbit the Earth, the space shuttle must accelerate from zero to 8,000 meters per second (almost 18,000 miles per hour) in eight and a half minutes. It takes a very unique vehicle to accomplish this.

There are three main components of the space shuttle that enable the launch into orbit. The orbiter is the main component. It not only serves as the crew's home in space and is equipped to dock with the International Space Station, but contains maneuvering engines for finalizing orbit. The external tank, the largest component of the space shuttle, supplies the propellant to the orbiter's three main maneuvering engines. The two solid rocket boosters, the third component, provide the main thrust at launch and are attached to the sides of the external tank (Figure 1). The components of the space



shuttle experience changes in position, velocity and acceleration during the ascent into space. These changes can be seen when one takes a closer look at the entire ascent process (Figure 3).

The ascent process begins with the liftoff from the launch pad. Propellant is being burned from the Solid Rocket Boosters, or SRB, and the external tank, or ET, causing the space shuttle to accelerate very quickly. This high-rate of acceleration as the space shuttle launches through the Earth's atmosphere causes a rapid increase in dynamic pressure, known as Q in aeronautics. The structure of the space shuttle can only withstand a certain level of dynamic pressure (critical Q) before it suffers damage. Before this critical level is reached, the engines of the space shuttle are throttled down to about 70%. At about one minute after launch the dynamic pressure reaches its maximum level (max Q). The air density then drops rapidly due to the thinning atmosphere and the space shuttle can be throttled to full power without fear of structural damage.

At about 2 minutes after launch, the atmosphere is so thin that the dynamic pressure drops down to zero. The SRB, having used their propellant, are commanded by the space shuttle's onboard computer to separate from the external tank. The jettison of the booster rockets marks the end of the first ascent stage and the beginning of the second.

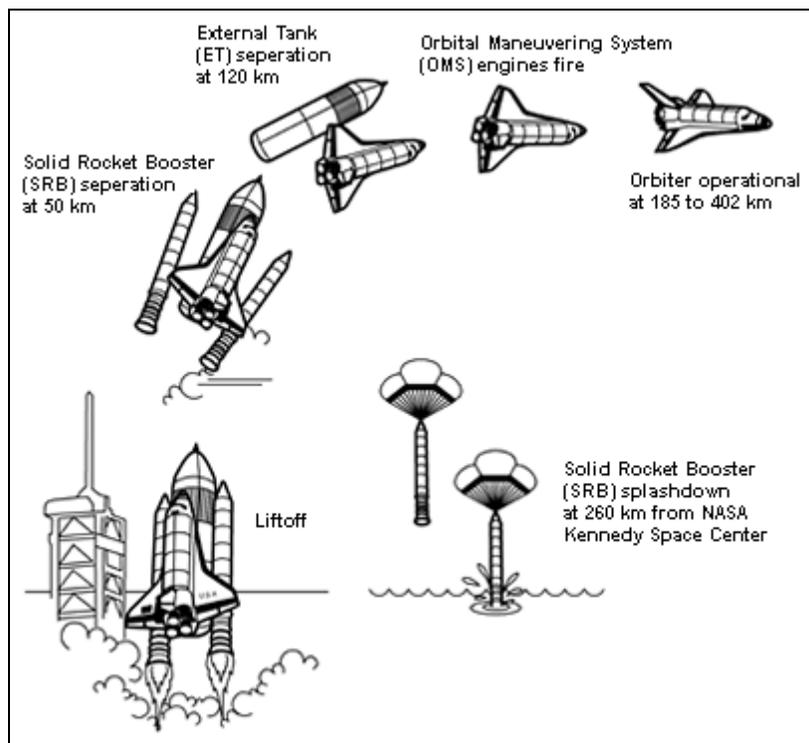


Figure 3: Space Shuttle Ascent Process

The second stage of ascent lasts about six and a half minutes. The space shuttle gains more altitude above Earth and the speed increases to the nearly 7,850 m/s (17,500 mph) required to achieve orbit. The main engines are commanded by the onboard computer to reduce power, ensuring that acceleration of the space shuttle does not exceed 29.4 m/s^2 (3 g). Within thirty seconds the space shuttle reaches Main Engine Cut-Off, or MECO. For the next eleven seconds, the space shuttle coasts through space. At nine minutes, the command to jettison the nearly empty external tank is given by the onboard computer leaving the space shuttle's maneuvering engines to control any movement needed to achieve final orbit.

As we look to the future of space exploration, the ascent stage will remain a critical part of any successful mission.



Problem

Before each mission, projected data is compiled to assist in the launch of the space shuttle to ensure safety and success during the ascent. To complete this data, flight design specialists take into consideration a multitude of factors such as the weight of the space shuttle, propellant used, mass of payload being carried to space, and mass of payload returning. They must also factor in atmospheric density, which is changing throughout the year. After running multiple scenarios through computer modeling, information is compiled in a table showing exactly what should happen each second of the ascent. This information helps the NASA engineers and flight controllers to quickly evaluate whether or not the space shuttle is operating as expected during the launch phase.

Table 1: STS-121 Ascent Data

Time (s)	Altitude (m)	Velocity (m/s)	Acceleration (m/s ²)
0	-8	0	2.45
20	1244	139	18.62
40	5377	298	16.37
60	11,617	433	19.40
80	19,872	685	24.50
100	31,412	1026	24.01
120	44,726	1279	8.72
140	57,396	1373	9.70
160	67,893	1490	10.19
180	77,485	1634	10.68
200	85,662	1800	11.17
220	92,481	1986	11.86
240	98,004	2191	12.45
260	102,301	2417	13.23
280	105,321	2651	13.92
300	107,449	2915	14.90
320	108,619	3203	15.97
340	108,942	3516	17.15
360	108,543	3860	18.62
380	107,690	4216	20.29
400	106,539	4630	22.34
420	105,142	5092	24.89
440	103,775	5612	28.03
460	102,807	6184	29.01
480	102,552	6760	29.30
500	103,297	7327	29.01
520	105,069	7581	0.10

Note: Notice from the table that the altitude is negative at liftoff. Zero altitude can be described as a specific distance from the center of the Earth. Since the Earth is not perfectly spherical the location of the launch just happens to be below this specified point. Also, because this is a calculated number, some degree of error may be present.



On July 4, 2006 Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121, to begin a rendezvous with the ISS. Data for mission STS-121 showing the altitude, velocity, and acceleration of the space shuttle every 20 seconds from liftoff to MECO, is displayed in Table 1.

In the following questions you will use the data from Table 1 to determine a position function, a velocity function and an acceleration function. *Position* is defined as a location in a coordinate system. In this problem, the altitude of the space shuttle in relation to time after launch is the space shuttle's vertical position. *Velocity* is defined as the rate of change of the vertical position. It describes how fast the space shuttle is moving. It is measured in meters per second (m/s). *Acceleration* is defined as the rate of change of velocity. It describes how the velocity of the space shuttle is changing and is measured in meters/second² (m/s²).

Because of the power and speed of lift off, precision is of utmost importance. Keep this in mind as you work with the data in this problem.

Note: To save time and errors the data files for the table to be uploaded to a TI calculator with instructions have been provided on the website.

- A. Enter the data from Table 1 into your graphing calculator. Create a scatterplot of the space shuttle's altitude in relation to time. Remember to adjust your viewing window. Use the regression functions on your calculator to determine which type of function best fits the data (i.e. linear, quadratic, cubic, quartic, exponential, power, etc.).

Note: When using the regression functions it is sometimes necessary to specify which lists you are using and is very useful to specify where you want the equation placed. For example the command $\text{QuadReg}(L1, L2, Y_1)$ will give the quadratic regression equation using L1 as the independent variable and L2 as the dependent variable and will automatically place this equation in Y_1 . You will find the Y variables by typing the VARS key, selecting Y-VARS, and then FUNCTION.

- B. Write the function describing the space shuttle's altitude? Let h stand for altitude and t stand for time. In writing the equation, you will need to decide what decimal place to round the coefficients to. An alternative to this is to write the coefficients using significant figures. In that case, you will need to decide how many significant figures to use. Explain your reasoning for your decision. (In your calculator, keep the original equation in Y_1 to be use for following questions.)
- C. Using your graphing calculator and the nDeriv command, set Y_2 , the altitude function, to be the first derivative and apply the first derivative test to Y_2 , to determine the relative extrema on the interval $[0, 520]$. Remember to adjust your viewing window. Compare the extrema with the data in table 1. How closely does your function represent the data in table 1? What do the extrema represent about the altitude of the space shuttle?
- D. Determine the points of inflection of the altitude function. Keep your function needed to find the points of inflection in Y_3 . Describe the concavity of the altitude function.



- E. Analyze the altitude function along the interval $[0, 160]$. What is happening to the space shuttle when the concavity changes?
- F. Use your graphing calculator to make a scatterplot of the velocity versus time from Table 1. Explain what is happening to the velocity of the space shuttle over the interval $[0, 520]$. Is there one function that best fits the data? Why or why not? Be careful to select the correct lists when finding the regressions. What is your velocity function? Let v stand for velocity and t stand for time. Keep this equation in Y_4 on your graphing calculator.
- G. Create a scatterplot of acceleration vs. time. What happens to the space shuttle's acceleration between $[0, 100]$? Why does the space shuttle's acceleration increase in a quadratic fashion along the interval $[120, 460]$? What is happening to the acceleration on the interval $[480, 500]$ and why? Why does the acceleration drop drastically along the interval $[500, 520]$? What happens to the space shuttle during these time intervals?
- H. Is there one regression function that best fits the acceleration data? Why or why not? What is the acceleration function that describes the interval $[120, 460]$? Let a stand for acceleration and t stand for time. Express the coefficients as three significant figures. Keep this equation in Y_5 on your graphing calculator.
- I. Compare the graphs in Y_2 (the first derivative of the altitude function) and in Y_4 (the velocity function found by doing the regression of the data). Do this by turning all other equations off. Do not clear the other equations. Remember to adjust your viewing window. How do these graphs compare? Explain the differences in what they are representing.
- J. Now compare the graphs in Y_3 (the second derivative of the altitude function) and in Y_5 (the acceleration function found by doing the regression of the data). How do these graphs compare? Explain the differences in what they are representing.