ELECTRONIC ANNEX EA2: ESTIMATES OF COOLING TIMES FOR MARTIAN METEORITE ALH84001 AFTER EJECTION FROM MARS

Consider the cooling curve for the ALH84001 meteorite starting at an initial ejection temperature T_0, and radiatively cooling to ambient in a vacuum at a distance D from the Sun. If we assume the Sun to be a perfect spherical black body from the Stephan-Boltzmann Law the total power it emits, P_{Sun}, is given by the expression:

$$P_{Sun} = (4\pi R_{Sun}^2) \cdot \sigma T_{Sun}^4 \tag{1}$$

where R_{Sun} is the radius of the Sun, T_{Sun} is the surface temperature, and σ is the Stephan-Boltzmann constant (~ 5.6704×10^{-8} W·m^{-2}·K^{-4}). If we assume ALH84001 to also be a perfect spherical blackbody of radius R_{ALH} then the fraction of the Sun's radiant power it would intercept at a distance D will be:

$$P_{ALH}^{Adsorb} = \left(\frac{\pi R_{ALH}^2}{4\pi D^2}\right) \cdot P_{Sun} = \left(\frac{R_{ALH}^2 R_{Sun}^2}{D^2}\right) \cdot \sigma T_{Sun}^4 \tag{2}$$

It will also be radiating energy according to its surface temperature which from the Stephan-Boltzmann Law is given by the expression:

$$P_{ALH}^{Emit} = (4\pi R_{ALH}^2) \cdot \sigma T_{ALH}^4 \tag{3}$$

At thermal equilibrium the energy absorbed by the meteorite from the Sun and the energy emitted will be equivalent (i.e., $P_{ALH}^{Adsorb} = P_{ALH}^{Emit}$), solving for T_{ALH} we have:

$$T_{ALH} = \sqrt{\frac{R_{Sun}}{D}} \cdot T_{Sun} \tag{4}$$

Taking the surface temperature and radius of the Sun as T_{sun} ~ 5960 K and R_{sun} ~ 6.96×10^8 m respectively, and assuming the distance D to be equivalent to the mean orbital radius of Mars (D ~ 2.27×10^{11} m), the equilibrium temperature for the meteorite has a value of T_{ALH} ~ 233 K. This represents the ambient temperature to which ALH84001 will asymptotically approach on cooling.

The shape of ALH84001 can be approximated as a scalene ellipsoid with equatorial radii of a = 0.085 m and b = 0.048 m, and a polar radius of c = 0.033 m. The volume of such a scalene ellipsoid is given by the expression:

$$V = \frac{4}{3}\pi abc \approx 5.50 \times 10^{4}\ m^{3} \tag{5}$$

The ablation depth for ALH84001 is estimated to be on the order of 0.03 m (Nishiizumi et al., 1994) so that the pre-atmospheric entry size can be estimated at a_{pre} = 0.115 m, b_{pre} = 0.078 m, and c_{pre} = 0.063 m and so the pre-atmospheric volume, V_{Pre}, is given by the expression:

$$V_{pre} = \frac{4}{3}\pi a_{pre} b_{pre} c_{pre} \approx 2.37 \times 10^{-3}\ m^{3} \tag{6}$$

The mass of ALH84001, M, at collection was 1.940 kg, so the estimated pre-atmospheric entry mass M_{Pre} would simply be given by:

$$M_{pre} = \frac{V_{pre} V}{M} \approx 8.360\ kg \tag{7}$$

The surface area for the scalene ellipsoid representing ALH84001 before atmospheric entry can be approximated using the Knud Thomsen formulism[1] (where p ~ 1.6075) as:

$$S_{pre} \approx 4\pi \left(\frac{a_{pre}^{p} b_{pre}^{p} + a_{pre}^{p} b_{pre}^{p} + b_{pre}^{p} c_{pre}^{p}}{3} \right)^{\frac{1}{p}} \approx 0.0900\ m^{3} \tag{8}$$

The chemical composition of ALH84001 can be approximated to that of its major phase, orthopyroxene (Mittlefehldt, 1994), to give a formula of $Fe_{55}Ca_{7}Mg_{139}Si_{200}O_{600}$. This gives a pre-atmospheric entry composition equivalent to 114.5 moles of O_2, 76.2 moles of Si, 53.0 moles of Mg, 20.9 moles Fe, and 2.5 moles of Ca, from the Avogadro constant this corresponds to a total

[1] http://home.att.net/~numericana/answer/ellipsoid.htm#thomsen

of N ~ 2.30×10^{26} atoms. From the values of S_{Pre} and N we can determine a simple expression for the rate of radiative emission from the surface of the meteorite. If we assume infinite thermal conductivity so that the internal temperature of the meteorite is always equal to its surface temperature, and that this temperature is characterized by pure translational kinetic energy according to the classical equipartition theorem, the rate of radiative emission becomes:

$$dt = \frac{3Nk}{2\epsilon\sigma S_{pre} T_{hot}^4} \cdot dT \tag{9}$$

where ε is the emissivity (assumed to be 1), k is the Boltzmann constant (1.381×10^{-23} J·K), N is the number of atoms in the body and S_{pre} is the radiative surface area and T_0 is the initial temperature of the meteorite. Integrating with respect to T the cooling time $t_{cooling}$ for the meteorite to cool from an initial temperature T_1 down to a temperature T_1 is given by:

$$\begin{aligned} t_{cooling} &= \frac{-3Nk}{2\epsilon\sigma S_{pre}} \int_{T_0}^{T_1} \frac{1}{T^4} \cdot dT = \frac{Nk}{2\epsilon\sigma S_{pre}} \left[\frac{1}{T_1^3} - \frac{1}{T_0^3}\right] \\ &\approx 3.11 \times 10^{11} \cdot \left[\frac{1}{T_1^3} - \frac{1}{T_0^3}\right] \end{aligned} \tag{10}$$

Using our previously derived values of N ~ 2.30×10^{26} atoms and S_{pre} ~ 0.0900 m^2 for ALH84001 we can plot the cooling curve and time to cool to calculated ambient temperature of 233 K from an initial peak temperature T_0 ~ 1000 K as shown in Fig. EA-2-1.

It should be realized that this cooling rate will be shorter than the real cooling time since the internal rate of heat conduction is obviously finite so that the rate of heat transfer from the interior would limit the rate of radiative loss from the surface. A more accurate solution would involve analytical solving the parabolic partial differential diffusion equation describing the temperature field for the interior of the meteorite and then combining this with the Stefan-Boltzmann radiative boundary condition at the meteorite surface. Nevertheless the cooling times calculated using this '*back-of-the-envelope*' will provide a reasonable order-of-magnitude estimation for relatively small bodies such as a meteorite where the ratio of the volume to surface area is not too large.

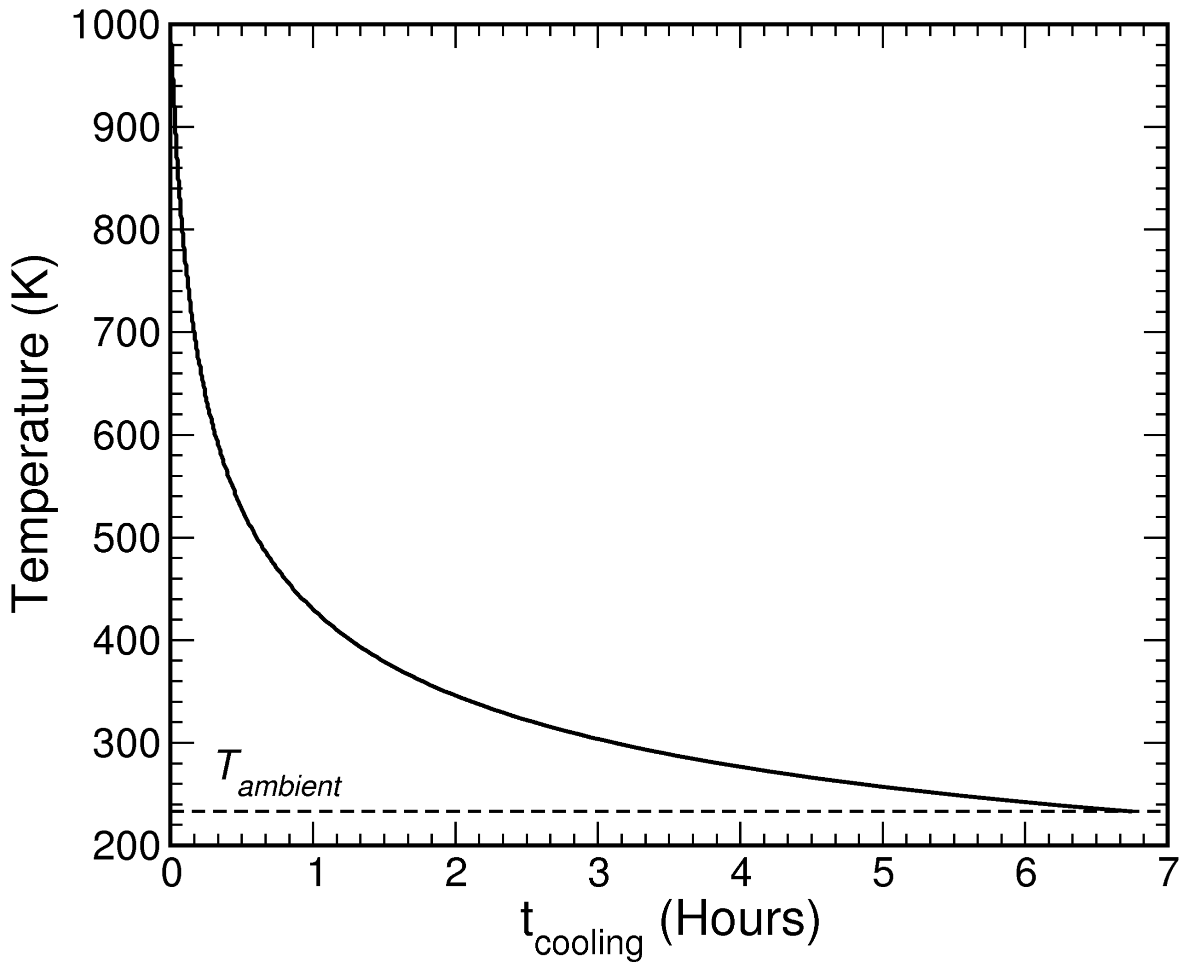

Fig. EA-2-1. Approximate radiative cooling curve for a black body with a composition and pre-atmospheric entry mass equivalent to that estimated for the ALH84001 meteorite. The initial starting temperature is 1000 K and the ambient temperature to which the body asymptotically cools is 233 K corresponding to the equilibrium temperature for a black body at a distance from the Sun equal to the mean orbital radius of Mars. For simplicity the body is assumed to have infinite thermal conductivity so that the bulk and surface temperatures always remain equivalent.

REFERENCES

Mittlefehldt, D. W. (1994) ALH84001, a cumulate orthopyroxenite member of the Martian meteorite clan. *Meteoritics* **29**, 214-221.

Nishiizumi, K., Caffee, M. W., and Finkel, R. C. (1994) Exposure histories of ALH 84001 and ALHA 77005. *Meteoritics* **29**, 511.