Application of Graph Theory to Requirements Traceability

A methodology for visualization of large requirements sets

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L-3 Communications

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Traceability is key to both requirements development and requirements verification. Each project has unique approaches to traceability and verification.
## Motivation for a Visualization Methodology

Studying characteristics of information flow in large Requirements sets

<table>
<thead>
<tr>
<th>Requirement</th>
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<tbody>
<tr>
<td>The flight system shall support the DOR tone capability in the SDST, including wideband DOR tones at X-band.</td>
<td>The flight software shall command the transponder as defined by the transponder documentation.</td>
<td>The flight software shall configure the transponder telemetry inputs in accordance with the active FS side whenever the transponder is powered ON. Reference transponder ICD for selection table.</td>
</tr>
<tr>
<td>The flight system shall accommodate PCM / PSK / PM modulation for the X-band downlink.</td>
<td>The flight software shall provide the capability to command an “active” telecom side which determines the “active” transponder in use and the uplink channel.</td>
<td>The flight software shall only perform the necessary SOT initializations if a commanded “active” telecom side is different from the currently “active” telecom side.</td>
</tr>
<tr>
<td>The flight software shall provide the capability to enable or disable the X-Band exciter for the active transponder.</td>
<td>The flight software shall provide the capability to enable or disable c mode for the active transponder.</td>
<td>The flight software, upon initial application of transponder power ON, shall provide for a configurable default state. Subsequent power ON transitions will default to last commanded state.</td>
</tr>
<tr>
<td>The flight software shall provide the capability to enable or disable X-Band Ranging for the active transponder.</td>
<td>The flight software shall provide the capability to set the Ranging Mode to BASEBAND or EXTERNAL for the active transponder.</td>
<td>The flight software shall provide the capability to command the X-Band Subcarrier for the active transponder to one of the following frequencies: 281.25 KHz squarewave, 281.25 KHz sinewave, 25 KHz squarewave, or 25 KHz sinewave.</td>
</tr>
<tr>
<td>The flight software shall provide the capability to set the X-Band Squarewave Telemetry Modulation Index to one of 128 discrete values (0x00 to 0x7F) for the active SDST.</td>
<td>The flight software shall provide the capability to command the X-Band Sinewave Telemetry Modulation Index to one of 16 discrete values (0x0 to 0xF) for the active SDST.</td>
<td>The flight software shall provide the capability to configure the X-Band telemtry modulation mode to SUBCARRIER or BPSK for the active transponder.</td>
</tr>
</tbody>
</table>

Quickly communicate regarding patterns involving hundreds or thousands of requirements
Graph Theory History

Leonhard Euler: The seven bridges problem
Publication in 1736 as the first description of graph theory, and is generally regarded as the origin of topology

Vanermonde: The knights tour problem

Cauchy and L’Hullier: Relationships between faces, edges, and vertices of convex polyhedrons

Study of pair-wise relationships between objects
Graphs are the parent family to a variety of topologies:
directed graphs
trees – Cayley and differential calculus
coloring problem
What is a graph?

- Graph theory is the study of mathematical structures used to model relationships between objects in finite collections.
- A graph is composed of nodes and edges
- Graphs can be classified as undirected, directed, tree, planar, etc depending upon the nature of the connections.

The Seven Bridges Problem
Four nodes, seven edges
Graphs all around us

- PERT Chart
  - Directed graph
  - Acyclic (no loops)
Flow Charts as Graphs

- Directed graph
- Sometimes cyclic
Graphs of Requirements Sets
Getting to the good stuff soon now...

Types of Graphs
- Simple graph – nodes and edges
- Directed graph – nodes and edges with direction (digraph)
- Acyclic graph – no cycles (loops)
- Connected graph – every node is reachable from any other node
- Tree – connected acyclic graph
- Forest – acyclic graph but unconnected

In the general case, requirements traceability forms an acyclic digraph, or forest
- Generally no single top-level node
- Generally not connected
- Almost always acyclic
- Directed

In the following examples of real system requirements graphs, the graphs are drawn as digraphs with the arrow pointing from the parent to the child. Untraced requirements are shown with red borders. We use boxes to denote the nodes simply because they fit the numbers better. These examples show a subnet of the full requirements net for clarity.
Device Traceability Topology

Requirements fan out dramatically from L-4 to L-5

Note traceability between ICD and Requirements

Device ICD Requirements are traced to both L-4 and L-5 requirements. Two L-4 requirements are untraced to L-5, one is probably incorrect trace to the ICD.
Traceability Patterns

• Large fan-out from parent to child suggest a large change in level of abstraction.

• One-to-one suggests under-specified lower-level requirements.

• Hour-glass traces seem to indicate serious problems in the intermediate requirements document; traceability event-horizon. May indicate verification difficulties.
Requirements fan out dramatically from L-4 to L-5.

A more complex topology
The flight software shall command the instrument as defined by the instrument documentation.

Typical ICD philosophy – descriptive and untraced to requirements.
Graphs as Traceability Diagnostics

- Histograms of connection counts:
  - Statistics of connection counts may suggest decomposition problems
  - Distributions are typically exponential (Internet, Kevin Bacon – movie graph)

Conjecture: Exponent may be relatable to the overall degree of abstraction change between linked requirements: High values mean small change
A subgraph of the Hollywood graph.
Automation of the Graphing Process

PowerPoint is NOT the best tool for analysis

Automatic graph generation from A matrix and specification of groups
Numerous applications available

- Graphviz http://www.graphviz.org/
- Jgraph – www.jgraph.com
- Guess - http://graphexploration.cond.org/
How Connected is a Graph?

Separability of subnets -> modularity of requirements to limit propagation of change
Expressing Graphs as Mathematical Structures - Vocabulary

Vertex: Endpoint (or connection point or node)
Edge: Connection between vertices
Incidence List: Array of pairs (tuples if directed) of vertices or connections
Adjacency List: List of pairs of vertices as a list (2x n array)
Incidence Matrix: Vertices by Edges matrix where each entry contains the endpoint data (1 = incident, 0 = not incident)
Adjacency matrix (A): N by N matrix where N = the number of vertices in the graph. Entries are either 0 if not connected, 1 if connected. If there is an edge from vertex k to vertex j then A(j,k)=1
Degree: Matrix of connection counts on the diagonal (D)
Laplacian matrix: L=D-A, where D= the diagonal degree matrix

⚠️ Danger: Math Ahead
Connectivity and Graphing

Here comes the math

The smallest nonzero eigenvalue of the Laplacian matrix is called the Fiedler value (or spectral gap).

### Graphs and Their Fiedler Values

<table>
<thead>
<tr>
<th>Graph</th>
<th>Fiedler Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path</td>
<td>$1/n^{**2}$</td>
</tr>
<tr>
<td>Grid</td>
<td>$1/n$</td>
</tr>
<tr>
<td>3D Grid</td>
<td>$n^{**2/3}$</td>
</tr>
<tr>
<td>Expander</td>
<td>1</td>
</tr>
<tr>
<td>Binary tree</td>
<td>$1/n$</td>
</tr>
<tr>
<td>dumbell</td>
<td>$1/n$</td>
</tr>
</tbody>
</table>

Small values of the Fiedler number mean the graph is easier to cut into two subnets. If the number is large, then every cut of the graph must cut many edges.

Conjecture: Would a large Fiedler number for a requirements graph indicate a system that was difficult to partition into subnets, thus difficult to change?
A Simple Graph and Spectral Analysis

Laplacian (D-A)

1 -1
-1 2 -1
-1 4 -1 -1 -1
-1 1

A Matrix

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0 1 1 1 1</td>
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<td>0</td>
<td>1 0</td>
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<td>0</td>
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Lx=λx where λ is an eigenvalue. And x is a non-null eigenvector. Because L is symmetric the eigenvalues are all real.

λ={0, 0.486, 1, 1, 2.428, 5.086}

Fiedler number = 0.486 implying somewhere between an expander (1) and a tree form (1/6).
Summary

• Graphs can be useful visualization tools for large requirements sets
  – Big picture viewpoint
  – Patterns easily recognized
  – Multi-level tracing
  – Identification of subnets

• Potential for analysis
  – Relationship between connection histogram and requirement decomposition
  – Ability to quantify interconnectedness by spectral analysis