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ESTIMATION OF MODAL PARAMETERS USING A WAVELET-BASED APPROACH

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Abstract

Modal stability parameters are extracted directly from aeroservoelastic flight test data by decomposition of accelerometer response signals into time-frequency atoms. Logarithmic sweeps and sinusoidal pulses are used to generate DAST closed loop excitation data. Novel wavelets constructed to extract modal damping and frequency explicitly from the data are introduced. The so-called Haley and Laplace wavelets are used to track time-varying modal damping and frequency in a matching pursuit algorithm. Estimation of the trend to aeroservoelastic instability is demonstrated successfully from analysis of the DAST data.

Nomenclature

ARW-1	first aeroelastic research wing
$C(s,u)$	continuous wavelet transform
DAST	drone for aerodynamic and structural testing
f	frequency (Hz)
$f(t)$	signal vector
$\mathcal{F}$	Fourier transform
g	gravitational acceleration
g	window function
g_γ	time-frequency atom
i	$\sqrt{-1}$
$K_{(\gamma,\phi)}$	normalization constant for `tfa`'s
ny	lateral acceleration
nz	normal acceleration
R	residual of projection in matching pursuit
Re	real part
s	scale
`tfa`	time-frequency atom
sps	samples per second
u	time shift
Symbols	
γ	space of parameters (s,ξ,u)
γ^-	space of parameters $(s,-\xi,u)$
ζ	damping ratio
ξ	modulation frequency of `tfa`
ϕ	radian phase angle
Ψ	analyzing wavelet
ω	radian frequency

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Introduction

Envelope expansion of new or modified aircraft often necessitates flight stability testing to verify safety margins to prevent against aeroservoelastic instability. Flight data is acquired for stability estimation and system identification to compare with analytical predictions as a function of flight parameters (Mach, dynamic pressure, α). Any anomalies are regarded with care to guarantee safety of flight. Excitation systems are often essential to resolve stability trends from noisy measurements since atmospheric turbulence is generally insufficient to determine modal characteristics.

Time-frequency representations of time varying systems offer an alternative to traditional time and frequency domain stability tracking algorithms[2]. While damping estimates can be unreliable stability predictors[3], damping and frequency trends are useful for noting changes in system dynamics as functions of flight condition.

This paper introduces novel wavelets for modal time-frequency and time-damping estimation directly from the data, without use of intermediate model identification schemes. Stability parameters are extracted directly from the data by correlating properly designed wavelets with the system responses. Revisiting DAST[4] ARW-1 (figure 1) (drone for aerodynamic and structural testing with the first aeroelastic research wing) data from flight testing performed in 1980, during which a catastrophic closed loop aeroservoelastic instability was encountered, modal frequency and damping are estimated as a function of time until instability. It is shown how wavelet processing can reliably identify time varying system stability trends.

Stability Analysis using Time-Frequency Atoms

Basis Functions

Fourier analysis maps a signal onto an amplitude-frequency plane by decomposing the signal into a weighted sum of infinite sinusoids. Its usefulness depends on properties of time invariance, linearity, and stationarity of the signal. Errors are also introduced from time truncation and windowing schemes. Signals that have time varying, transient, nonlinear, or nonstationary characteristics demand analysis functions which do not have the restrictions of Fourier methods.

Wavelets are basis functions of finite duration which decompose a signal into scaled and time-shifted versions of the analyzing function. The continuous wavelet transform is defined as[8]

$$C(s,u) = \int_{-\infty}^{\infty} f(t)\Psi(s,u,t)dt$$

where u is the time translation, s the scale, and Ψ the analyzing wavelet. Gabor wavelets are Gaussian-windowed sinusoids (figure 2) described as

$$\Psi(s,u,t) = e^{-\frac{1}{2}(\frac{t-u}{s})^2}\cos(t)$$

Conventional wavelet analyses map signals onto the scale-time or frequency-time plane since scale and frequency are inversely proportional to each other. This restriction inhibits the extraction of modal parameters from flight data. Hence, time-frequency atoms (`tfa`'s)[5] are introduced which are well localized in time, and frequency is independent of scale. The `tfa`'s decompose a signal based on its local structure in terms of time, frequency, and scale independently. General families of `tfa`'s are generated by scaling, translating, and modulating a window function $g(t)$ (see Appendix). Gabor wavelet window functions

$$g\left(\frac{t-u}{s}\right) = e^{-\frac{1}{2}(\frac{t-u}{s})^2}$$

produce the following `tfa`'s over the parameter space of $\gamma = (s,\xi,u)$ (ξ is frequency)

$$g_\gamma(t) = (\pi s^2)^{-\frac{1}{4}} e^{-\frac{1}{2}(\frac{t-u}{s})^2} e^{j\xi t}$$

The parameters (s,u,ξ) are independently adjusted based on local signal properties.

A Haley wavelet is presented here as the double-sided exponentially damped sinusoid of figure 3 expressed as

$$\Psi(s,u,t) = e^{-|\frac{t-u}{s}|}\cos(t)$$

and generating the following `tfa`'s over $\gamma = (s,\xi,u)$ (see Appendix),

$$g_\gamma(t) = \frac{1}{\sqrt{s}} e^{-|\frac{t-u}{s}|} e^{j\xi t}.$$

This wavelet is designed to independently extract modal damping and damped vibration frequency from a set of data, both as functions of time. Therefore, signals are mapped to the frequency-time *and* damping-time planes. Viscous damping ratio, ζ, is estimated empirically by the following expression

$$\zeta = \frac{10\pi T}{s\xi}$$

where T is the sampling interval of the discrete time signal. Intuitively, as the scale, s, of the wavelet lengthens, damping ratio is reduced. For damping ratios typical of structural systems, parameter ξ estimates $\omega_n \approx \omega_d$, or natural frequency ω_n as an equivalent approximation of damped frequency ω_d.

Finally, a Laplace wavelet is introduced as a right-sided version of the Haley wavelet, or a representation of an impulse response (figure 4) and is expressed as

$$\Psi(s,u,t) = \begin{cases} \sqrt{\frac{2}{s}} e^{-|\frac{t-u}{s}|}\cos(t) & t-u \geq 0 \\ 0 & t-u < 0 \end{cases}$$

Matching Pursuit

The matching pursuit algorithm[5] decomposes a signal into a linear expansion of `tfa`'s that are from a redundant dictionary of functions. The wavelet bases are chosen according to desired features to be detected, or parameters to be extracted, from a signal. With a parametric dictionary of specific time-frequency atoms, matching pursuit will define an adaptive scale-frequency-time transform. The algorithm proceeds as follows (see Appendix):

1. Choose initial `tfa` corresponding to largest coefficient of $C(s, u)$ decomposition
2. Optimize over $\gamma = (s, \xi, u)$ by maximizing inner product of the signal with `tfa` dictionary
3. Extract optimal `tfa` from the signal
4. Repeat optimal extraction procedure until decomposition residual is below specified tolerance

Once the signal is decomposed to a desired degree, reconstruct a signal from the extracted dictionary elements. The energy distribution of the reconstructed signal in the frequency-time domain is generated using a Wigner distribution[5].

Simulation Tests

Results from the analysis of simulated data is presented to test the accuracy of the frequency and damping values from application of matching pursuits using the Haley and Laplace wavelets. Various types of simulated data are used in the tests. Haley decompositions are tested using the following signals of 100 sec each:

1. One double-sided exponentially damped sinusoid
2. Three separated double-sided exponentially damped sinusoids
3. One double-sided exponentially damped sinusoid with 60% noise
4. Three separated double-sided exponentially damped sinusoids with 60% noise

Laplace decompositions are also tested using the above, replacing double-sided with single-sided sinusoids. From the results of table 1, the capability of these `tfa`'s to extract the true values is very encouraging. Values of estimated frequency and damping match the simulated 'true' values quite well. For clean data, the estimated values are essentially exact. Noisy data results introduce a variation in some estimates, but the true value is always in the range of variation.

Analysis of DAST Data with Wavelets

The objective of the NASA DAST program was to pursue investigations in transonic and supersonic regimes using a series of aeroelastic research wings on drones[6]. These drones were equipped with a flutter suppression system to enable flight beyond the open-loop stability boundary. Wingtip accelerometer response data at 500 sps was acquired for logarithmic chirps and sinusoidal doublets of a single cycle 20 Hz sine wave into the aileron control surface.

This paper will present analysis of right wingtip accelerometer data in response to an aileron chirp input and a series of doublets just before the drone encountered an aeroservoelastic instability[4] with the flutter suppression system engaged. Several types of wavelet `tfa`'s are used in the analysis to process the short time data measurements of the transient modal responses. Each analysis (figures 5 through 8) produces plots of the original signal (a. top), the residual signal after the matching pursuit decomposition-extraction procedure (b), reconstruction of the original signal from the decomposition (c), frequency estimates as a function of time (d), and damping estimates as a function of time, if applicable (e).

Gabor, Haley, and Laplace wavelets are utilized with the matching pursuit algorithm to extract modal natural frequencies from the flight data. Figure 5 presents an example of the information obtained from this procedure using a Gabor `tfa` . The top plot, figure 5a, shows the original accelerometer response from the chirp input to the aileron. Figure 5b presents the noise remaining in this signal after extracting the `tfa`'s which result in the reconstructed signal of figure 5c.

The bottom plot, figure 5d, presents the estimated values of the modal natural frequencies. Each circle on this plot indicates a frequency for which the wavelet had a high correlation with the measured data. Circles representing the highest energy extraction from the signal by the `tfa` correspond to modal

responses. The increasing frequency as a function of time for the wavelet correlations agrees with the logarithmically increasing frequency in the chirp signal and demonstrates the additional time information that cannot be obtained from traditional Fourier analysis.

Modal frequencies similar to figure 5d are computed using the Haley and Laplace wavelet `tfa`'s . These frequencies agree closely with the Gabor wavelet analysis; however, the Haley and Laplace `tfa`'s additionally extract modal damping. For this reason, the remaining discussion will consider only the Haley and Laplace wavelet analysis.

The matching pursuit algorithm is able to extract information about multiple modal responses by correlating wavelets of different frequency and damping (related to wavelet scale) with the measured data. Modal estimates are separated for clarity of presentation with figure 6d-e displaying the estimated Wing Torsion mode and figure 7d-e displaying the estimated Wing Bending modal estimates.

Frequency estimates of the Wing Torsion mode in figure 6d remain nearly constant at 25.5 Hz throughout the data set. The damping value of $\zeta \approx 0.075$ also shows little variation throughout the series of doublets. These values agree with the results of previous post-flight estimates[1, 7].

The Wing Bending mode is of particular interest since this mode went unstable and resulted in the instability which destroyed the drone. The information obtained by the matching pursuit analysis with the Haley wavelet is given in figure 7. The estimated natural frequency for this mode shows some variation at different test points throughout the data set but remains centered around 20 Hz. Estimated damping, however, shows significant variation. The `tfa` analysis tracks modal damping decreasing steadily from $\zeta \approx 0.02$ to near zero after 40 seconds of data.

Similary, the Laplace `tfa` analysis given in figure 8 indicates the impending instability. The estimated damping in figure 8e shows the same decrease in damping from $\zeta \approx 0.02$ to near zero after 40 seconds of data.

The Haley and Laplace wavelet analysis agree with the true behaviour of the DAST drone. Each wavelet indicates the drone to be on the verge of instability for the Wing Bending mode after 40 seconds of data. In reality, this mode did go unstable immediately after the 40 seconds during which this data set was recorded.

Algorithms are currently being developed which implement this matching pursuit strategy with reduced computational effort. The large amount of energy remaining in the signals after the correlated wavelets are extracted in figures 6b, 7b, and 8b is due to terminating the analysis before all the energy could be extracted by the wavelets. The process was stopped once the main features of the data were determined.

Conclusions

An aeroservoelastic system was used to test a method for time-frequency estimation of modal parameters using wavelets. Time-frequency-scale data processing allows improved visualization and understanding of the signal information content. Tuned basis functions for specific applications assist in identifying structure (shape, amplitude, frequency, duration, and timing of events) in the data.

Wavelets are exploited to parameterize the data such that stability trends can be detected accurately in a time-varying flight test scenario. In this appication, time-frequency atoms are used in a matching pursuit algorithm to extract modal frequency and damping as functions of time. DAST data of the final seconds before a closed loop instability are analyzed to validate this approach as a stability indicator and modal tracker. Future work will enhance this capability for application in an on-line test environment.

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Appendix

Time-frequency Atoms

Time-frequency atoms are functions well localized in both time and frequency which can decompose signals adaptively according to particular signal structures. General families of time-frequency atoms can be generated by scaling, translating and modulating a window function $g(t) \in \mathbf{L}^2(\mathbf{R})$. Assume $g(t)$ is real, continuously differentiable, $\| g(t) \| = 1$, $\int_{-\infty}^{\infty} g(t) \neq 0$, and $g(0) \neq 0$. For any scale $s > 0$, frequency modulation ξ, and time translation u, denote $\gamma = (s, \xi, u) \in \Gamma = \mathbf{R}^+ \times \mathbf{R}^2$ and define

$$g_\gamma(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s} \right) e^{i\xi t}. \qquad (1)$$

The factor $\frac{1}{\sqrt{s}}$ normalizes $\| g(t) \| = 1$. Generally $g(t)$ is even, centered at u with most of its energy concentrated near u, and determined by size proportional to s. The Fourier transform of $g(t)$ is

$$\hat{g}_\gamma(\omega) = \sqrt{s}\hat{g}(s(\omega - \xi))e^{-i(\omega-\xi)u}$$

For even $g(t)$, $\hat{g}(\omega)$ is even and centered at frequency $\omega = \xi$. Its energy is concentrated near center and modulation frequency ξ.

To analyze signal structures of varying sizes, it is necessary to use time-frequency atoms of different scales. A wavelet transform decomposes signals over different scales, but the resulting atoms are restricted by the relation $\xi \propto \frac{1}{s}$. For signals that include scaling and highly oscillatory structures, the appropriate constraints on the scale and modulation parameters of the time-frequency atoms g_γ cannot be defined *apriori*. Hence, an adaptive selection of the elements of the dictionary $\mathcal{D} = (g_{\gamma_n})_{\gamma \in \Gamma}$ is needed depending on the local properties of signal $f(t)$. Finite linear expansions of time-frequency atoms are dense in $\mathbf{L}^2(\mathbf{R})$, so the dictionary $\mathcal{D}$ is complete.

Matching Pursuit Algorithm

To compute a linear expansion of a signal $f \in \mathbf{L}^2(\mathbf{R})$ from the set $\mathcal{D}$, successive approximations of f are performed with orthogonal projections on the vectors in $\mathcal{D}$ to best match dominant patterns in the data. The following algorithm, called *matching pursuit*, performs the adaptive decomposition.

A signal vector f can be decomposed into ($<,>$ denotes inner product)

$$f = < f, g_{\gamma_o} > g_{\gamma_o} + Rf$$

where Rf is the residual vector after projecting f onto vector g_{γ_o}. Since g_{γ_o} is orthogonal to Rf,

$$\| f \|^2 = | < f, g_{\gamma_o} > |^2 + \| Rf \|^2.$$

Minimize $\| Rf \|$ by choosing $g_{\gamma_o} \in \mathcal{D}$ such that $| < f, g_{\gamma_o} > |$ is maximum, or at least find a vector g_{γ_o} that is sub-optimal in the sense that

$$| < f, g_{\gamma_o} > | \geq \lambda \sup_{\gamma \in \Gamma} | < f, g_\gamma > |$$

where $0 \leq \lambda \leq 1$ is an optimality parameter. Now, by induction, suppose $R^0 f = f$ so at the nth step the residue $R^n f$ is matched

$$| < R^n f, g_{\gamma_n} > | \geq \lambda \sup_{\gamma \in \Gamma} | < R^n f, g_\gamma > |.$$

Now decompose $R^n f$ into

$$R^n f = < R^n f, g_{\gamma_n} > g_{\gamma_n} + R^{n+1} f$$

defining the residue at step $n+1$. Since $R^{n+1} f$ is orthogonal to g_{γ_n},

$$\| R^n f \|^2 = | < R^n f, g_{\gamma_n} > |^2 + \| R^{n+1} f \|^2.$$

and the decomposition up to the mth stage results in

$$\begin{aligned} f &= \sum_{n=0}^{m-1} (R^n f - R^{n+1} f) + R^m f \\ &= \sum_{n=0}^{m-1} < R^n f, g_{\gamma_n} > g_{\gamma_n} + R^m f. \end{aligned}$$

Therefore, the original signal f is decomposed into a sum of the dictionary elements chosen to best match its residues. Although this decomposition is nonlinear, energy conservation is preserved since

$$\| f \|^2 = \sum_{n=0}^{m-1} | < R^n f, g_{\gamma_n} > |^2 + \| R^m f \|^2.$$

Gabor Atoms

The scale modulated form of the complex Gabor atom of equation 1 can be written ($\xi = 2\pi f_c$, where f_c is the center modulation frequency in Hertz)

$$\begin{aligned} g_\gamma(t) &= \frac{C_o}{\sqrt{s}} e^{-\frac{1}{2}(\frac{t-u}{s})^2} e^{i\xi t} \\ g_{\gamma^-}(t) &= \frac{C_o}{\sqrt{s}} e^{-\frac{1}{2}(\frac{t-u}{s})^2} e^{-i\xi t} \end{aligned}$$

where $\gamma = (s, \xi, u)$ and $\gamma^- = (s, -\xi, u)$, respectively. The C_o to get $\| g_\gamma(t) \| = 1$ is $C_o = \pi^{-\frac{1}{4}}$, so

$$g_\gamma(t) = (\pi s^2)^{-\frac{1}{4}} e^{-\frac{1}{2}(\frac{t-u}{s})^2} e^{i\xi t}.$$

When a signal $f(t)$ is real, dictionaries of real time-frequency atoms must be used to get a decomposition with real expansion coefficients. For any $\gamma = (s, \xi, u)$, with $\xi \neq 0$ and any phase $\phi \in [0, 2\pi)$, define

$$g_{(\gamma,\phi)} = \frac{K_{(\gamma,\phi)}}{\sqrt{s}} g\left(\frac{t-u}{s}\right) \cos(\xi t + \phi).$$

The phase ϕ now appears explicitly rather than implicitly as in the complex representation of equation 1. It is the phase angle of the inner product $< R^n f, g_\gamma >$ for each term of the decomposition. In the real Gabor case,

$$g_{(\gamma,\phi)} = \frac{K_{(\gamma,\phi)}}{\sqrt{s}} e^{-\frac{1}{2}(\frac{t-u}{s})^2} \cos(\xi t + \phi).$$

where real atoms are related to complex atoms by

$$g_{(\gamma,\phi)} = \frac{K_{(\gamma,\phi)}}{2}(e^{i\phi} g_\gamma(t) + e^{-i\phi} g_{\gamma^-}(t))$$

Real matching pursuits are not equivalent to complex decompositions because $g_\gamma(t)$and $g_{\gamma^-}(t)$are not orthogonal. The constant $K_{(\gamma,\phi)}$to maintain $\| g_{(\gamma,\phi)} \| = 1$ for real atomic decompositions becomes

$$K_{(\gamma,\phi)} = \sqrt{\frac{2}{1 + \mathrm{Re}(e^{i2\phi} < g_\gamma(t), g_{\gamma^-}(t) >)}} \quad (2)$$

and for real Gabor atoms[5]

$$K_{(\gamma,\phi)} = \sqrt{\frac{2}{1 + \mathrm{Re}(e^{i2(\phi+\xi u)} e^{-(s\xi)^2})}}.$$

Haley Atoms

In the so-called Haley atomic decomposition introduced in this paper, the window function is $g(t) = C_o e^{-|t|}$, and $C_o = 1$ to get $\| g_\gamma \| = 1$, so that

$$g_\gamma(t) = \frac{1}{\sqrt{s}} e^{-|\frac{t-u}{s}|} e^{i\xi t}.$$

Now equation 2 still holds, but the inner product is different. Specifically, by changing variables from t to $-p$,

$$\begin{aligned} < g_\gamma(t), g_{\gamma^-}(t) > &= \frac{1}{s}\int_{-\infty}^{\infty} e^{-2|\frac{t-u}{s}|} e^{i2\xi t} \mathrm{d}t \\ &= \frac{1}{s}\int_{-\infty}^{\infty} e^{-2|\frac{p+u}{s}|} e^{-i2\xi p} \mathrm{d}p \end{aligned}$$

which is also $\frac{1}{s}$ multiplied by the Fourier transform of $e^{-2|\frac{p+u}{s}|}$ with ω replaced by 2ξ. Since $\mathcal{F}[e^{-2t}] = \frac{4}{4+\omega^2}$, from the Fourier scaling and time-shift properties,

$$\begin{aligned} \mathcal{F}[x(at)] &= \frac{1}{a} X\left(\frac{f}{a}\right) \\ \mathcal{F}[e^{-2|\frac{t}{s}|}] &= s\left[\frac{4}{4+(s\omega)^2}\right] = X(\omega) \end{aligned}$$

and

$$\begin{aligned} \mathcal{F}[x(t-t_o)] &= e^{-i2\pi f t_o} X(f) \\ &= e^{-i\omega t_o} X(f) \\ \mathcal{F}[e^{-2|\frac{t+u}{s}|}] &= e^{i\omega u} X(\omega) \\ &= \frac{4 s e^{i\omega u}}{4 + (s\omega)^2}. \end{aligned}$$

Multiplying this result by $\frac{1}{s}$ and replacing ω by 2ξ yields

$$\begin{aligned} < g_\gamma(t), g_{\gamma^-}(t) > &= \frac{1}{s}\left[\frac{4 s e^{i2\xi u}}{4+4\xi^2 s^2}\right] \\ &= \frac{e^{i2\xi u}}{1 + (s\xi)^2} \end{aligned}$$

for the expression in equation 2. Hence, for the Haley atom,

$$K_{(\gamma,\phi)} = \sqrt{\frac{2}{1 + \mathrm{Re}\left(\frac{e^{i2(\phi+\xi u)}}{1+(s\xi)^2}\right)}}$$

Laplace Atoms

Laplace atoms are a trivial modification of Haley atoms by a factor of $\frac{1}{2}$, resulting in $\frac{C_o^2}{2} = 1$ to get $\| g_\gamma \| = 1$, so that

$$g_\gamma(t) = \sqrt{\frac{2}{s}} e^{-|\frac{t-u}{s}|} e^{i\xi t}.$$

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Signal	1	2	3	4
f_{true}	8	8,4,20	8	8,4,20
ζ_{true}	.05	.005,.0075,.00375	.005	.005,.0075,.0037
Haley Wavelet Results				
f_{est}	8	8,4,20	8	8,1-4,20
ζ_{est}	.05	.006,.008,.004	.005	.005,.0075,.0037
Laplace Wavelet Results				
f_{est}	8	9-10,4-6,20	8	8-10,4,20
ζ_{est}	.05	.005,.0075,.00375	.005-.2	.005-.2,.0075,.003

Table 1: Results of Matching Pursuit Tests using Simulated Data - f is frequency (Hz), ζ is damping ratio

Mode	Calculated	Measured
Symmetric Mode		
Wing 1st Bending	9.1	9.6
Fuselage 1st Bending	16.5	16.2
Wing Bending-torsion	29.6	29.1
Antisymmetric Mode		
Fuselage 1st Bending	12.3	13.5
Wing 1st Bending	21.7	19.3
Stabilizer 1st Bending	30.0	27.0

Table 2: DAST ARW-1 Calculated and GVT-measured Elastic Modal Frequencies (Hz)

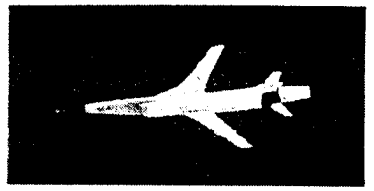

Figure 1: DAST ARW-1

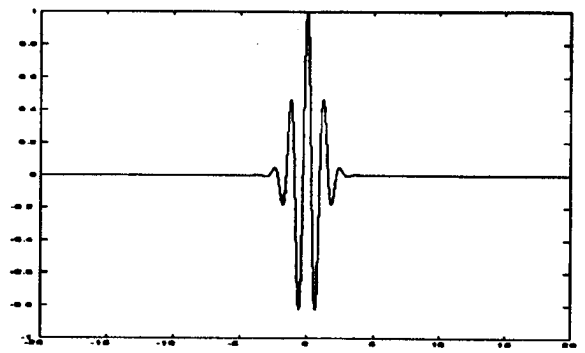

Figure 2: Gabor wavelet

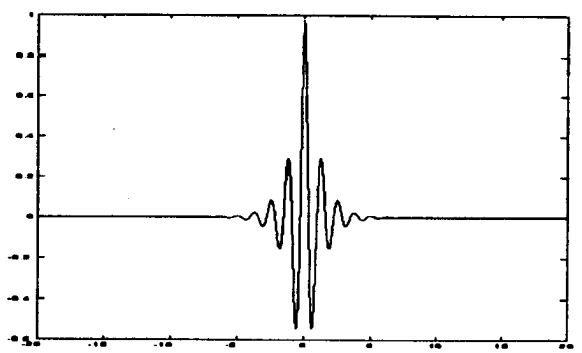

Figure 3: Haley wavelet

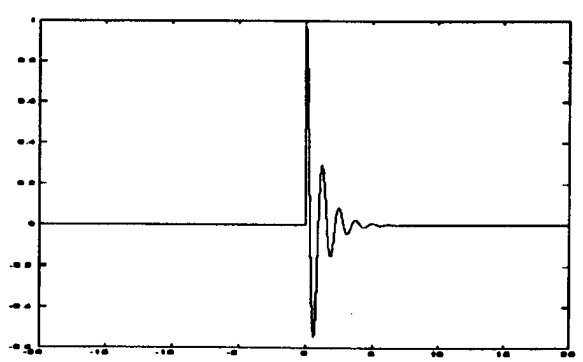

Figure 4: Laplace wavelet

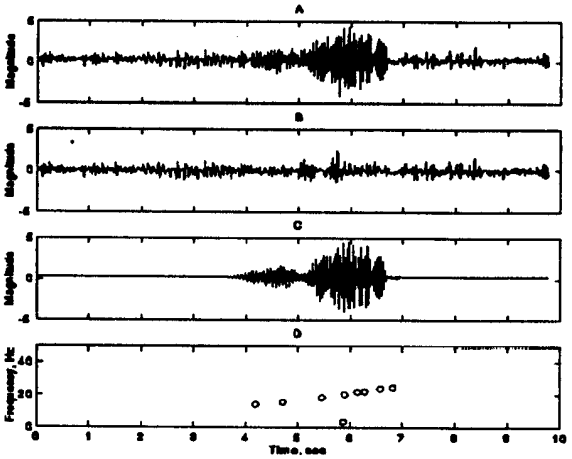

Figure 5: Gabor analysis of DAST chirp data

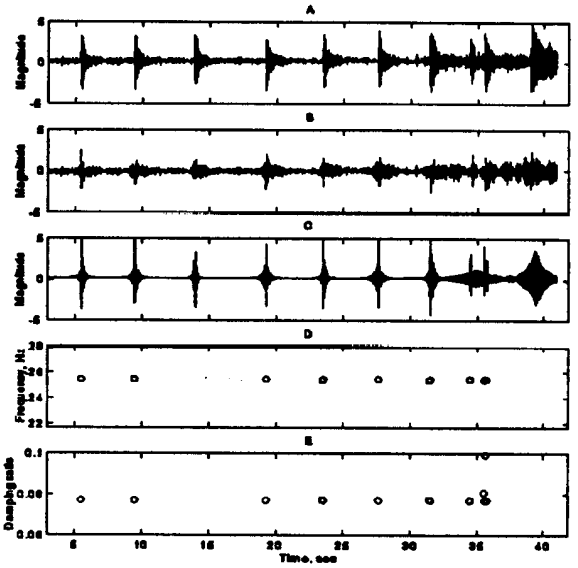

Figure 6: Haley analysis of DAST - Wing Torsion

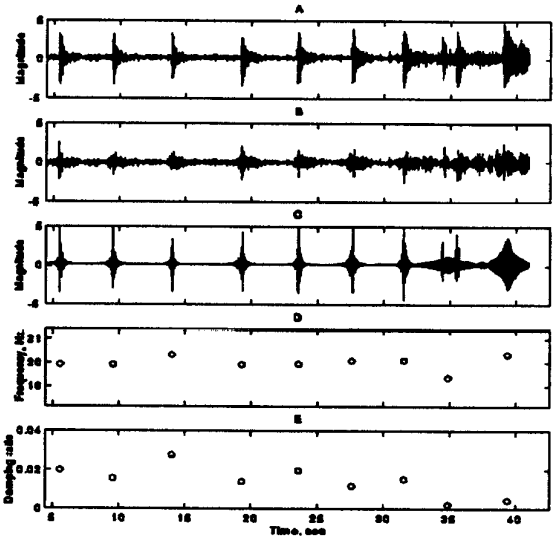

Figure 7: Haley analysis of DAST - Wing Bending

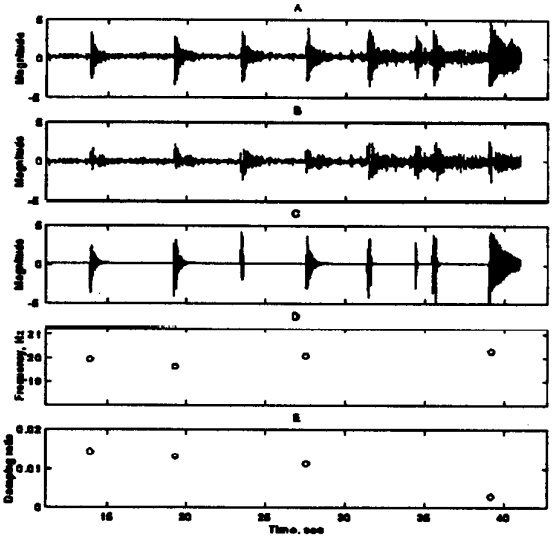

Figure 8: Laplace analysis of DAST - Wing Bending

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13. ABSTRACT (Maximum 200 words)

Modal stability parameters are extracted directly from aeroservoelastic flight test data by decomposition of accelerometer response signals into time-frequency atoms. Logarithmic sweeps and sinusoidal pulses are used to generate DAST closed loop excitation data. Novel wavelets constructed to extract modal damping and frequency explicitly from the data are introduced. The so-called Haley and Laplace wavelets are used to track time-varying modal damping and frequency in a matching pursuit algorithm. Estimation of the trend to aeroservoelastic instability is demonstrated successfully from analysis of the DAST data.

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