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Effects of Simplifying Assumptions on Optimal Trajectory Estimation for a High-Performance Aircraft

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INTRODUCTION

When analyzing the performance of an aircraft, certain simplifying assumptions, which decrease the complexity of the problem, can often be made. The degree of accuracy required in the solution may determine the extent to which these simplifying assumptions are incorporated. A complex model may yield more accurate results if it describes the real situation more thoroughly. However, a complex model usually involves more computation time, makes the analysis more difficult, and often requires more information to do the analysis. Therefore, to choose the simplifying assumptions intelligently, it is important to know what effects the assumptions may have on the calculated performance of a vehicle. This paper examines several simplifying assumptions, compares the effects of simplified models to those of the more complex ones, and draws conclusions about the impact of these assumptions on flight envelope generation and optimal trajectory calculation. Models which affect an aircraft are analyzed, but the implications of simplifying the model of the aircraft itself are not studied. The examples in this paper are atmospheric models, gravitational models, different models for equations of motion, and constraint conditions.

The results are calculated using the energy state approximation (Bryson, Desai, and Hoffman, 1969). In the energy state approximation, the aircraft is modeled as a point mass in the vertical plane, and specific energy is used as a state variable. The results are for a high-performance aircraft in minimum time-to-energy trajectories.

NOMENCLATURE

a	speed of sound, $\frac{\text{ft}}{\text{sec}}$
D	drag, lb
E_s	specific energy, ft
f	function, or forces, lb
G	gravitational constant, $\frac{\text{ft}^3}{\text{sec}^2\, lb}$
g	acceleration due to gravity, $\frac{\text{ft}}{\text{sec}^2}$
h	geopotential altitude, ft
J	cost function
L	lift, lb
L_m	temperature lapse rate, $\frac{R}{\text{ft}}$
M	Mach number
M_e	mass of the Earth, lb
M_0	mean molecular weight
m	mass of the vehicle, slug
P	pressure, $\frac{\text{lb}}{\text{ft}^2}$
PLA	power lever angle
P_s	specific power, $\frac{\text{ft}}{\text{sec}}$
$\bar{q}$	dynamic pressure, $\frac{\text{lb}}{\text{ft}^2}$
R	range, ft
R_e	radius of the Earth, ft

R^*	universal gas constant, $\frac{\text{ft lb}}{\text{(lb-mol)}\,^\circ\text{R}}$
T	thrust, lb, or temperature, °R
t	time, sec
V	velocity, $\frac{\text{ft}}{\text{sec}^2}$
w	fuel weight, lb
$\dot{w}$	fuel flow, $\frac{\text{lb}}{\text{hr}}$
z	geometric altitude, ft
α	angle of attack, rad
γ	flightpath angle, rad, or ratio of specific heats
ρ	density of air, $\frac{\text{slug}}{\text{ft}^3}$
$\dot{}$	first derivative

Subscripts

ave	average
b	base
idl	idle
h	in the horizontal plane
lim	limit
max	maximum
min	minimum
ref	reference
v	in the vertical plane
x	x-body axis of the vehicle
z	z-body axis of the vehicle

ENERGY STATE APPROXIMATION

Background

The trajectories discussed in this paper are calculated using the energy state approximation theory (Bryson, Desai, and Hoffman, 1969). This theory models the aircraft as a point-mass and assumes a flat, nonrotating Earth, which is reflected in the equations of motion used. The general equations of motion for a point-mass in the vertical plane are derived by Miele (1962). They can be given by, first, summing the forces in the direction of the velocity vector

$$T_x \cos\alpha - T_z \sin\alpha - D - mg\sin\gamma = m\dot{V} \quad (1)$$

where T is thrust, α is angle of attack, D is drag, mg is the vehicle's weight, γ is the flightpath angle, m is the vehicle's mass, and $\dot{V}$ is the first derivative of velocity. By summing the forces perpendicular to the velocity vector

$$T_x \sin\alpha + T_z \cos\alpha + L - mg\cos\gamma = mV\dot{\gamma} \quad (2)$$

where L is lift, V is velocity, and $\dot{\gamma}$ is the first derivative of the flightpath angle. These equations assume that the vehicle thrust vector is fixed with respect to the vehicle body axes and acts in both the x and z body axes of the vehicle.

In the energy state approximation, the total energy per unit mass, or specific energy, (E_s) is considered a state variable and is derived from the energy equation (Bryson, Desai, and Hoffman, 1969)

$$E = mgz + \frac{1}{2}mV^2 \tag{3}$$

where E is energy, and z is geometric altitude. This is divided by the vehicle weight to give the specific energy

$$E_s = z + \frac{V^2}{2g} \tag{4}$$

When this quantity is differentiated with respect to time, the equation for specific power (P_s) is given

$$P_s = \dot{z} + \frac{V\dot{V}}{g} \tag{5}$$

An expression for $\dot{V}$ can be derived from equation 1 resulting in

$$\dot{V} = \frac{T_x \cos\alpha - T_z \sin\alpha - D - mg\sin\gamma}{m} \tag{6}$$

An expression for $\dot{z}$ can be derived from the definition of γ

$$\gamma = \sin^{-1}\frac{\dot{z}}{V} \tag{7}$$

so that

$$\dot{z} = V\sin\gamma \tag{8}$$

Substituting equations 6 and 8 into equation 5 results in:

$$P_s = V\sin\gamma + V\left(\frac{T_x \cos\alpha - T_z \sin\alpha - D - mg\sin\gamma}{mg}\right) \tag{9}$$

By subtracting out the $V\sin\gamma$ terms this equation gives an expression for P_s.

$$P_s = V\left(\frac{T_x \cos\alpha - T_z \sin\alpha - D}{mg}\right) \tag{10}$$

Equation 10 gives P_s as a function of velocity, thrust, angle of attack, and drag. Drag, however, is a function of Mach number, altitude, and angle of attack. Thrust is a function of power lever angle (PLA). Therefore, it can be seen that P_s is a function of altitude, Mach number, angle of attack, and PLA and is independent of flightpath angle.

Given the expression for P_s, it is possible to derive the cost function for a minimum time-to-energy path. The minimum time-to-energy cost function is defined as (Erzberger, Barman, and McLean, 1975)

$$J = \int_0^t dt \tag{11}$$

where J is a cost function and t is time.

From equation 5, specific power is defined as

$$P_s = \frac{dE_s}{dt} \tag{12}$$

Substituting equation 12 into equation 11 gives

$$J = \int_0^{E_s} \frac{dE_s}{P_s} \tag{13}$$

To find the minimum for equation 13, it is sufficient to minimize the quantity inside the integral $\left(\frac{dE_s}{P_s}\right)$. Since the energy state approximation assumes that cost functions are computed along curves of constant specific energy, it is sufficient to minimize the quantity $\frac{1}{P_s}$. This is the same as maximizing P_s. The cost function for a minimum time-to-energy trajectory, then, can be given by:

$$\text{maximize } J = P_s \tag{14}$$

Flight Envelope Calculations

The flight envelope can be calculated by solving the general equations of motion (eqs. 1 and 2) for trim in straight and level flight. For this case, $\gamma = 0$, $\dot{V} = 0$, and $\dot{\gamma} = 0$. Therefore, the general equations of motion for a point mass in the vertical plane are reduced to:

$$T_x \cos\alpha - T_z \sin\alpha - D = 0 \tag{15}$$

$$T_x \sin\alpha + T_z \cos\alpha + L - mg = 0 \tag{16}$$

For actual implementation in a numerical nonlinear equation solver, the above equations can be written as:

$$f_h(\alpha, \text{PLA}, z, M) = \frac{T_x \cos\alpha - T_z \sin\alpha}{D} - 1 \tag{17}$$

$$f_v(\alpha, \text{PLA}, z, M) = \frac{L + T_x \sin\alpha - T_z \cos\alpha}{mg} - 1 \tag{18}$$

The trim equations of motion, then, are defined by setting $f_v(\alpha, \text{PLA}, z, M) = 0$ and $f_h(\alpha, \text{PLA}, z, M) = 0$. Note that these equations make no attempt to force the pitching moment equation to zero. While this is consistent with the assumption of the vehicle as a point mass, the effects of the vehicle's longitudinal surfaces are neglected.

The flight envelope is calculated in four parts (fig. 1): the angle-of-attack boundary, the thrust boundary, the Mach boundary, and the dynamic pressure boundary. The angle-of-attack boundary is defined by setting the angle of attack equal to the maximum angle of attack of the vehicle. An altitude is picked, and the trim equations of motion are solved for that altitude. This gives

$$f_h(\alpha_{lim}, \text{PLA}, z_{ref}, M) = 0 \tag{19}$$

and

$$f_v(\alpha_{lim}, \text{PLA}, z_{ref}, M) = 0 \tag{20}$$

This gives two equations and two unknown variables, which can be solved using a numerical nonlinear equation solver.

The thrust boundary is calculated using the same method. The PLA is set to its limiting value and a Mach number is chosen for the calculations. The two trim equations of motion are then solved for the corresponding angle of attack and altitude:

$$f_h(\alpha, \mathrm{PLA}_{lim}, z, M_{ref}) = 0 \tag{21}$$

and

$$f_v(\alpha, \mathrm{PLA}_{lim}, z, M_{ref}) = 0 \tag{22}$$

The Mach boundary is given by setting the Mach number equal to the vehicle's Mach limit.

The dynamic pressure boundary is calculated by solving the equation relating dynamic pressure and velocity, using the dynamic pressure limit of the vehicle

$$V_{ref} = \sqrt{\frac{2\,\bar{q}_{lim}}{\rho}} \tag{23}$$

where $\bar{q}$ is the dynamic pressure and ρ is the density of air. The Mach number is found from the resulting velocity, and the trim equations of motion are solved using a reference altitude

$$f_h(\alpha, \mathrm{PLA}, z_{ref}, M_{ref}) = 0 \tag{24}$$

and

$$f_v(\alpha, \mathrm{PLA}, z_{ref}, M_{ref}) = 0 \tag{25}$$

Trajectory Calculations

To calculate trajectories, the cost function is maximized along curves of constant specific energy within the flight envelope of the vehicle. For the numerical results in this paper, this was done in the following way:

1. An array of specific energy curves encompassing the vehicle's flight envelope was determined (fig. 1).
2. A point along the first curve, representing the lowest value of specific energy, was chosen, yielding an altitude and a corresponding Mach number.
3. This altitude and Mach number point was entered into an optimization routine that varied angle of attack and PLA to determine a maximum value for the cost function, specific power, at that point.
4. A different altitude and Mach number point on the first curve was then chosen, and the optimization routine calculated a maximum value for the cost function at the new point.
5. These two maximum values for the cost function were compared and the higher of the two was stored.
6. The maximum cost function value was calculated at other points along the first energy curve and the highest value found along the curve was stored. This value identified the optimum point on that energy curve.
7. The optimum point was then calculated along the remaining energy curves within the flight envelope. The altitude and Mach number path connecting these points defined the optimal trajectory. An optimal trajectory is shown in figure 1.

Parameters such as the time, range, and fuel used along the trajectory can be calculated between energy curves. This is not the same as the analysis at each point along an energy curve, which assumes steady, horizontal flight. Information for the total trajectory can be gained by summing the time, range, and fuel used between each energy curve.

The time it takes to go from one energy curve to another is given by

$$\Delta t = \frac{\Delta E_s}{P_{s_{ave}}} \tag{26}$$

Range (R) is then calculated based on the time

$$\Delta R = V \cos \gamma \Delta t \tag{27}$$

where γ is given in equation 7. Finally, fuel used is a product of the fuel flow ($\dot{w}$) multiplied by the time increment

$$\Delta w = \dot{w} \Delta t \tag{28}$$

where w is fuel weight.

Aircraft Model

As stated previously, the energy state approximation models the aircraft as a point mass in the vertical plane. This paper does not assess the assumptions made about the aircraft, but concentrates instead on assumptions made about outside parameters which will affect the trajectory calculations. These trajectory calculations were based on a mathematical model of a typical modern, supersonic fighter aircraft. This model supplied lift, drag, thrust, and fuel flow as a function of Mach number, altitude, angle of attack, and PLA throughout the flight envelope.

MODELING ASSUMPTIONS

Atmospheric Models

Models of the atmosphere are used to determine altitude-dependent parameters such as air density, pressure, temperature, and speed of sound. This paper looks at the 1962 standard atmosphere (NASA/U.S. Air Force/U.S. Weather Bureau, 1962), as well as models for a standard day, cold day, and hot day for Edwards Air Force Base, California (Johnson, 1975). The equations used to determine atmospheric-dependent parameters are given here.

The general equation for temperature is (NASA/U.S. Air Force/U.S. Weather Bureau, 1962)

$$T = T_b + L_m(h - h_b) \tag{29}$$

Here, L_m is the gradient of the temperature with geopotential altitude, h_b is the geopotential altitude at the base of a particular layer by a specific value of L_m, and T_b is the value of T at altitude h_b. Temperature profiles for the four day types are given in figure 2.

The general equation for pressure (P) is dependent on the value of the gradient (L_m). If $L_m \neq 0$, then P is given by

$$\frac{P}{P_b} = \left(\frac{T_b}{T_b + L_m H}\right)^{\frac{gM_0}{R^* L_m}} \tag{30}$$

and if $L_m = 0$, P is given by

$$\frac{P}{P_b} = \exp\left(-\frac{gM_0 H}{R^* T_b}\right) \tag{31}$$

Here, $H = h - h_b$, M_0 is the mean molecular weight, and R^* is the universal gas constant.

Density is calculated from the pressure and temperature:

$$\rho = \frac{M_0 P}{R^* T} \tag{32}$$

and, finally, the speed of sound is calculated from the temperature:

$$a = \sqrt{\frac{\gamma R^* T}{M_0}} \tag{33}$$

Gravitational Model Equations

Acceleration due to gravity can be calculated several ways. The two ways that make sense for a model that assumes a flat, nonrotating Earth are: 1) calculating gravity as a function of distance from the surface of the Earth, and 2) giving gravity as a constant value, assuming no variation with altitude.

The acceleration due to gravity as a function of distance from the Earth is (NASA/U.S. Air Force/U.S. Weather Bureau, 1962)

$$g = \frac{GM_e}{(R_e + z)^2} \tag{34}$$

where G is a gravitational constant, M_e is the mass of the Earth, and R_e is the radius of the Earth.

A simpler means of determining the acceleration due to gravity is to assume that it does not vary with distance from the Earth. The value for g, then, is given by the constant

$$g = 32.174 \frac{\text{ft}}{\text{sec}^2} \tag{35}$$

Equations of Motion

The general equations of motion for a point mass in the vertical plane are given by equations 1 and 2. Several assumptions can be made which affect the flight envelope and trajectory calculations in two distinct ways. First, assumptions for the equations of motion will affect the calculations of the flight envelope. Second, the equations of motion are used in calculating specific power, which is used as the cost function for calculating minimum time-to-energy trajectories.

To calculate the flight envelope, the aircraft is assumed to have straight-and-level flight at trim conditions, giving equations 15 and 16:

$$T_x \cos\alpha - T_z \sin\alpha - D = 0$$

$$T_x \sin\alpha + T_z \cos\alpha + L - mg = 0$$

These give the most general method of modeling the trim equations of motion used in calculating the vehicle's flight envelope.

If the thrust vector is assumed to be aligned with the x-body axis of the vehicle, then there is no thrust along the z-body axis, and the equations simplify to

$$T\cos\alpha - D = 0 \tag{36}$$

$$T\sin\alpha + L - mg = 0 \tag{37}$$

To further simplify the equations, it can be assumed that the angle of attack is small, in which case $\cos\alpha \approx 1$ and $\sin\alpha \approx 0$. This gives

$$T - D = 0 \tag{38}$$

$$L - mg = 0 \tag{39}$$

These same assumptions can be used in calculating specific power. The most general equation for specific power is given by equation 10:

$$P_s = V\left(\frac{T_x \cos\alpha - T_z \sin\alpha - D}{mg}\right)$$

If it is assumed that the thrust lies along the body axis of the vehicle, then T_z is zero and the equation for specific power becomes:

$$P_s = V\left(\frac{T\cos\alpha - D}{mg}\right) \tag{40}$$

Finally, assuming that the angle of attack is small gives:

$$P_s = V\left(\frac{T - D}{mg}\right) \tag{41}$$

Constraint Conditions

Constraint conditions limit the possible solutions to an optimization problem. The more constraints put on the problem, the smaller the range of solutions becomes. Because constraint conditions are applied to the optimization, the choice of the constraint conditions affects only the optimal trajectory and has no affect on the flight envelope.

The simplest case in this paper is an unconstrained trajectory. This means that there are no constraints on the cost function, and only necessary limits are put on angle of attack and PLA to keep them within realistic values. The maximization problem for optimizing unconstrained trajectory is

$$\begin{array}{ll} \text{maximize} & J(\alpha, \text{PLA}, h, M) \\ \text{subject to} & \alpha_{min} \leq \alpha \leq \alpha_{max} \\ & \text{PLA}_{idl} \leq \text{PLA} \leq \text{PLA}_{max} \end{array}$$

The quantity $J(\alpha, \text{PLA}, h, M)$ is the cost function being optimized, specific power.

Because the energy state approximation assumes that the aircraft is in straight and level flight at each point, this can be reflected in the constraint conditions. To do this, a constraint equation that balances the equations of motion in the vertical direction is added. The maximization problem for optimizing trajectory constrained to level flight is:

$$\begin{array}{ll} \text{maximize} & J(\alpha, \text{PLA}, h, M) \\ \text{subject to} & f_v(\alpha, \text{PLA}, h, M) = 0 \\ & \alpha_{min} \leq \alpha \leq \alpha_{max} \\ & \text{PLA}_{idl} \leq \text{PLA} \leq \text{PLA}_{max} \end{array}$$

Finally, the trajectory can be constrained to realistic flightpath angles. This can also be thought of as constraining the velocity in the vertical direction, so that the rate of climb ($\dot{z}$) can never exceed the total velocity. This can be seen from the definition of the flightpath angle

$$\gamma = \sin^{-1}\frac{\dot{z}}{V}$$

The maximization problem for optimizing a trajectory constrained to realistic flightpath angles is

$$\begin{aligned} \text{maximize} \quad & J(\alpha, \text{PLA}, h, M) \\ \text{subject to} \quad & f_v(\alpha, \text{PLA}, h, M) = 0 \\ & -\tfrac{\pi}{2} \leq \gamma \leq \tfrac{\pi}{2} \\ & \alpha_{min} \leq \alpha \leq \alpha_{max} \\ & \text{PLA}_{idl} \leq \text{PLA} \leq \text{PLA}_{max} \end{aligned}$$

RESULTS

Effects of Atmospheric Models

The effects of the atmospheric models on the trajectory and flight envelope are shown as a result of aerodynamic effects only; the effects of atmosphere on engine performance were not considered. The results show that the effects on aerodynamics are significant. Figure 3 shows trajectories for the four different atmospheric models analyzed: standard day, Edwards cold day, Edwards standard day, and Edwards hot day. The flight envelope was calculated for standard day and Edwards cold day only.

The choice of atmospheric model had an effect on the optimal trajectory, especially in the high energy region, (fig. 3). At the higher energies, the maximum P_s occurs at a higher altitude for the warmer days. The differences are due to the differences in densities at those altitudes between the atmospheric models. The temperature affects the density, which affects the calculated lift and drag. This is also reflected in the flight envelope, which shows the effects of atmospheric model on the dynamic pressure boundary, which is a function of the density of the air, as well as the thrust boundary at high energies.

Figures 4(a), (b), and (c) show the effects of the different atmosphere models on the performance of the vehicle. In agreement with the data shown on figure 3, figure 4(a) shows that the vehicle must climb higher on warmer days in order to maximize its cost function. These higher altitude trajectories take more time and range (fig. 4(b)) and use more fuel (fig. 4(c)) than the lower altitude trajectories to achieve the same energy. They are, therefore, less efficient. If a large degree of accuracy is required in the calculations, the atmosphere must be modeled as close to the actual conditions as possible.

Effects of Gravitational Models

The effect of the two different gravitational models on flight envelope and trajectory is shown in figure 5. The two gravitational models are the altitude dependent model, based on a simple Newtonian force equation (eq. 34), and the simpler model in which gravity is assumed constant (eq. 35). Figure 5 shows that the flight envelope and optimum trajectory are overlaid for the two gravity models.

Effects of Equations of Motion

Figure 6 shows the effects of the differences in the equations of motion models on the flight envelope and optimal trajectory. For the most general equations, the angle between the thrust vector and the body axis was increased until a noticeable effect was seen on the trajectory calculations. This angle was determined to be approximately 6°. At angles up to 6°, there was only a small effect on the flight envelope and trajectory. The only difference is in the angle-of-attack boundary of the flight envelope. The case with the thrust aligned with body axis overlays the case with the 6° angle between thrust and body axis, but both are slightly different from the zero angle-of-attack case.

Figures 7(a) and (b) show that simplifying the equations of motion had little effect on the calculated performance of the vehicle. It took a thrust misalignment angle of 6° to show an effect on performance, in which case a slight increase in range and fuel used were needed to achieve the same energy state as the simpler cases.

Effects of Constraint Conditions on Trajectory Calculations

Constraint conditions had some effect on the trajectory, as figure 8(a) shows. For the constrained to level flight and the flightpath constrained cases, a condition of the constraints was that the optimization pick a point where the forces in the vertical direction are balanced. Because of this, limiting the flightpath to realistic angles had the same effect as constraining the trajectory to level flight. The unconstrained case, however, shows that a different optimal trajectory was picked.

There is no significant difference in aircraft performance between the constrained to level flight case and the flightpath constrained case. However, both differed from the unconstrained case. The constraint conditions caused a less favorable but more realistic trajectory to be chosen. Figures 8(b) and (c) show that the constrained cases take more range and fuel to achieve the same energy state than the unconstrained case does.

CONCLUSIONS

This paper examined the effects of various modeling assumptions on flight envelope generation and optimal trajectory estimation. The parameters investigated included atmospheric models, gravitational models, equations of motion, and constraint conditions.

The different types of atmospheric models showed a definite effect on the trajectory, especially at high energies. Of the four examined, the colder days had better performance characteristics. The warmer day models took more time and distance to achieve the same energy state. If a large degree of accuracy is required in calculations, the atmosphere should be modeled as close to the actual conditions as possible.

The two gravitational models examined were gravity as a function of altitude and a constant valued gravity force. They showed no difference in effect on flight envelope or trajectory calculations.

Three different sets of equations of motion were examined. The assumption of zero angle of attack showed a noticeable but small effect on the angle-of-attack boundary of the flight envelope, but showed no effect on the trajectory. For small angles between the thrust vector and the body axes, simplifying the equations of motion in the vertical plane had very little effect on calculated performance.

The different constraint conditions had an effect on the trajectory. Constraining the trajectory to level flight limited the possible points to be chosen for the optimal path, and so made a difference in the optimal trajectory. The further constraint of limiting the flightpath to reasonable limits had no effect compared to the constrained to level flight case.

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Edwards, California, January 3, 1990

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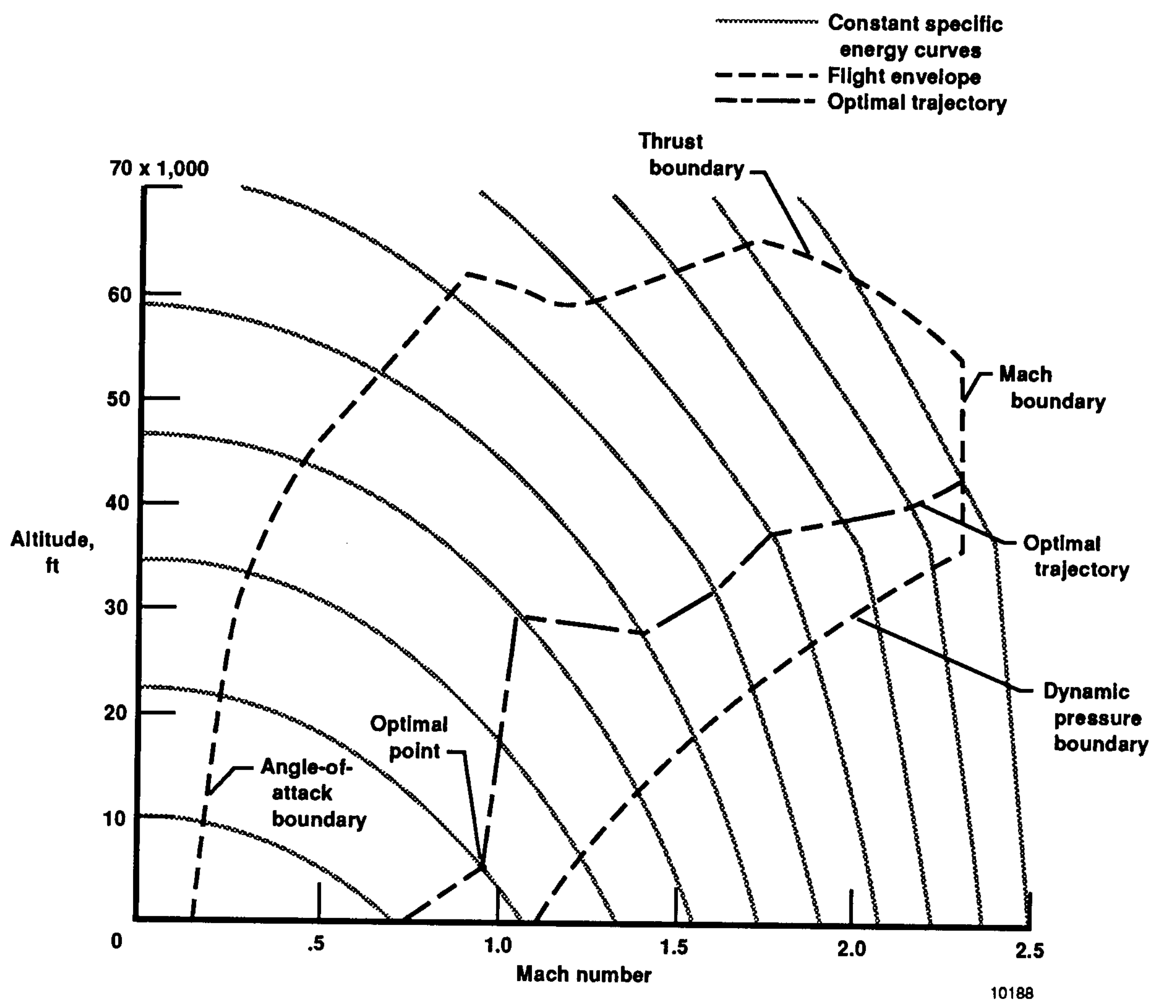


Figure 1. Sample flight envelope and optimal trajectory for a high-performance vehicle.

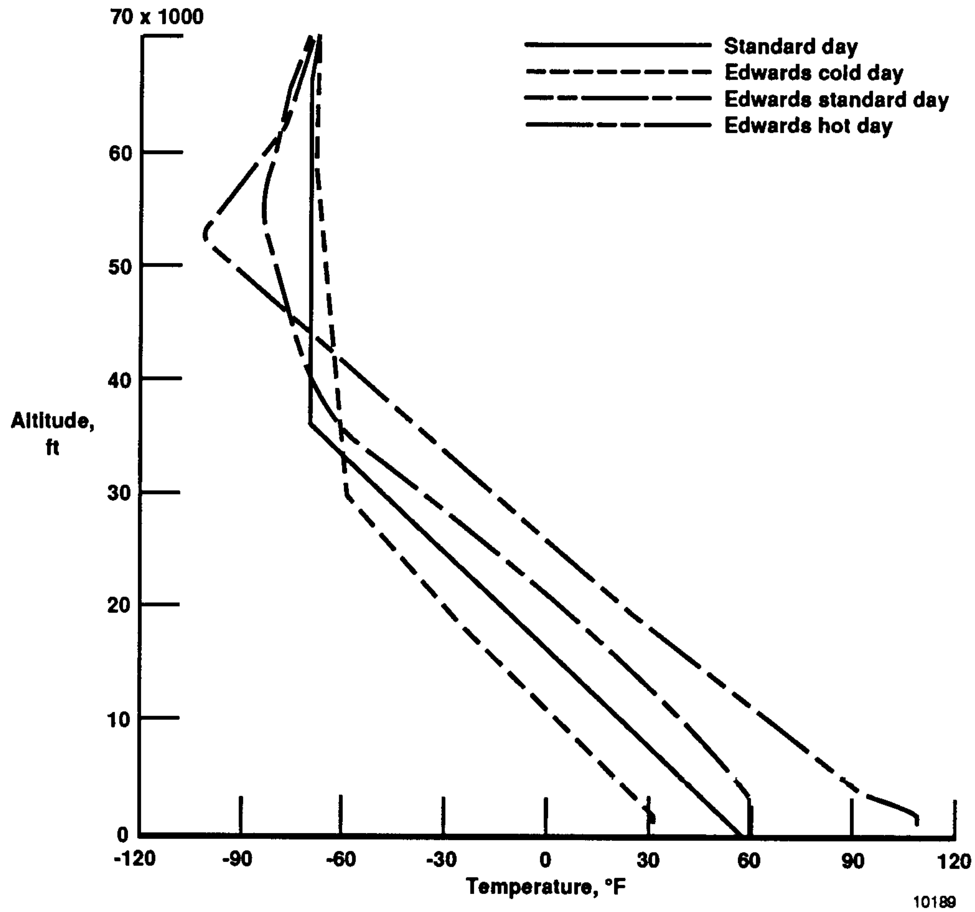


Figure 2. Temperature profiles for different day types.

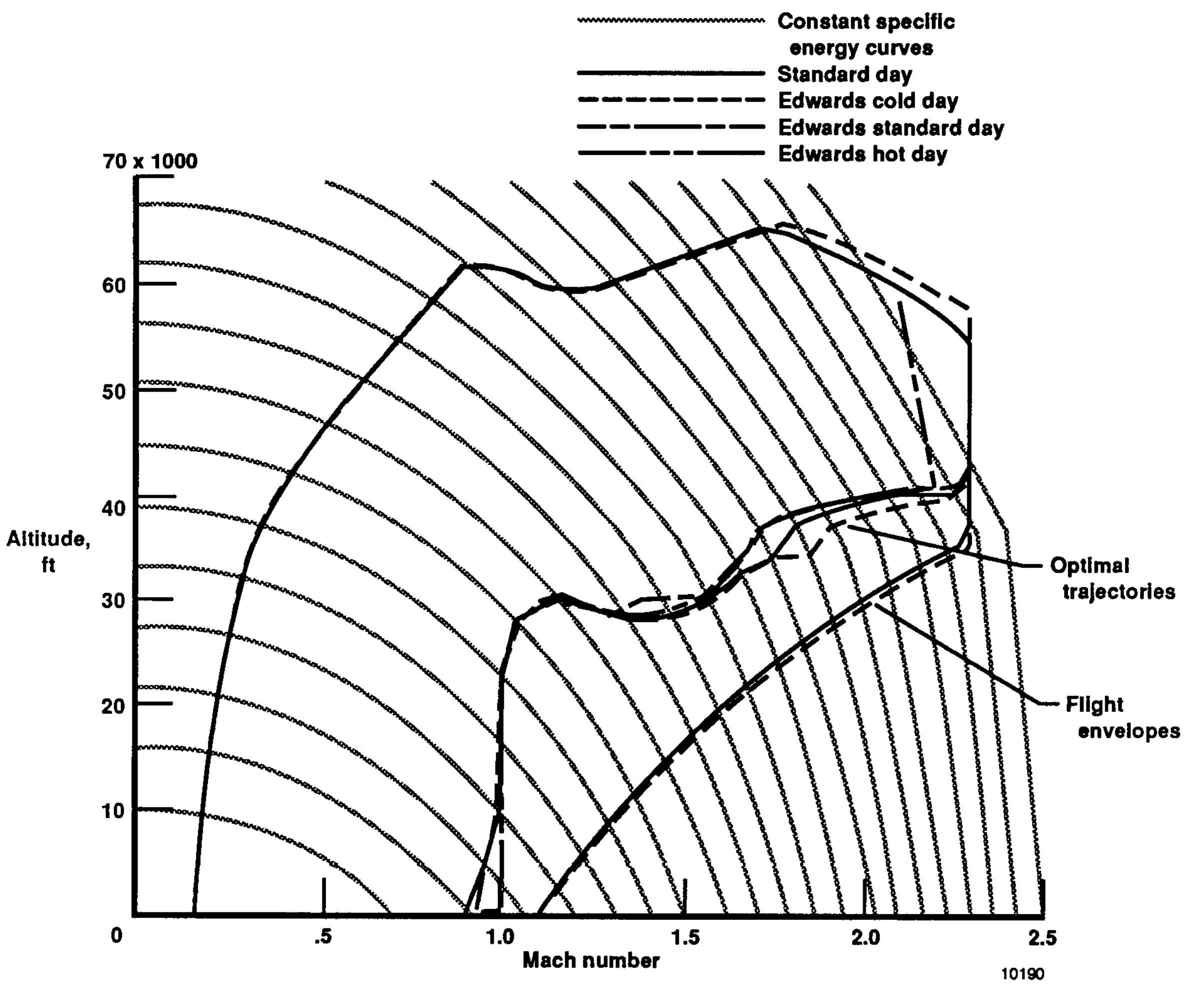


Figure 3. Effect of atmospheric model on flight envelope and trajectory. Flight envelope calculated for standard day and Edwards cold day only.

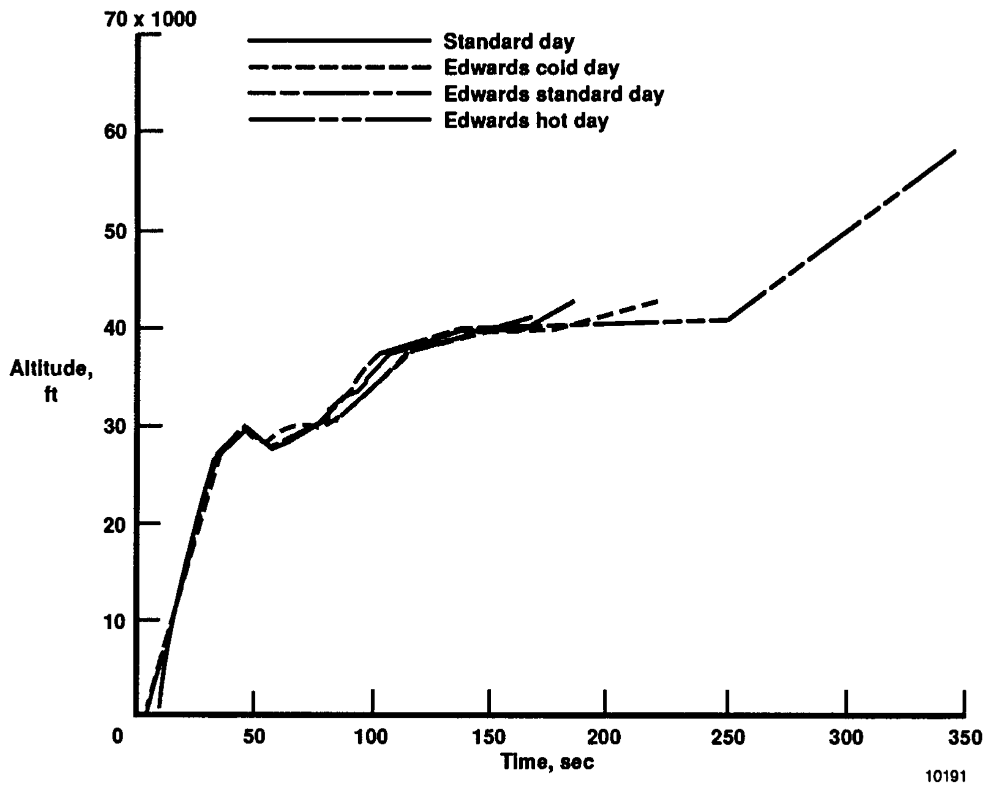
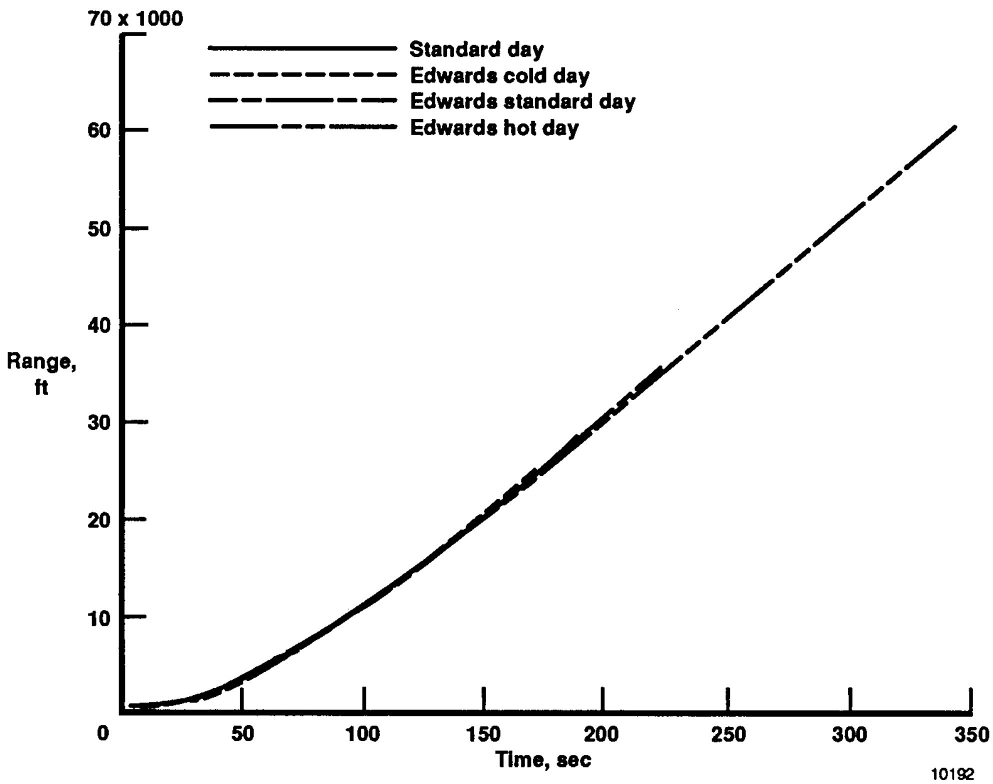
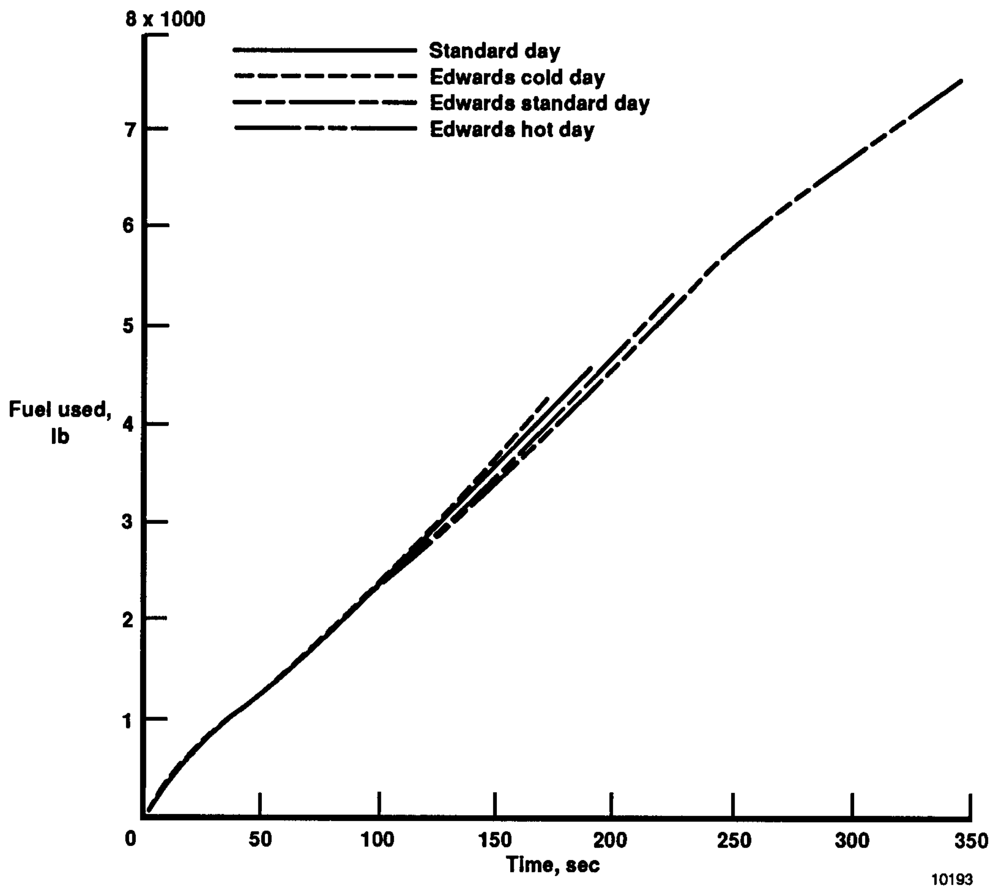


Figure 4. Effects of atmospheric model.
(a) Altitude as a function of time.



(b) Range as a function of time.

Figure 4. Continued.



(c) Fuel used as a function of time.

Figure 4. Concluded.

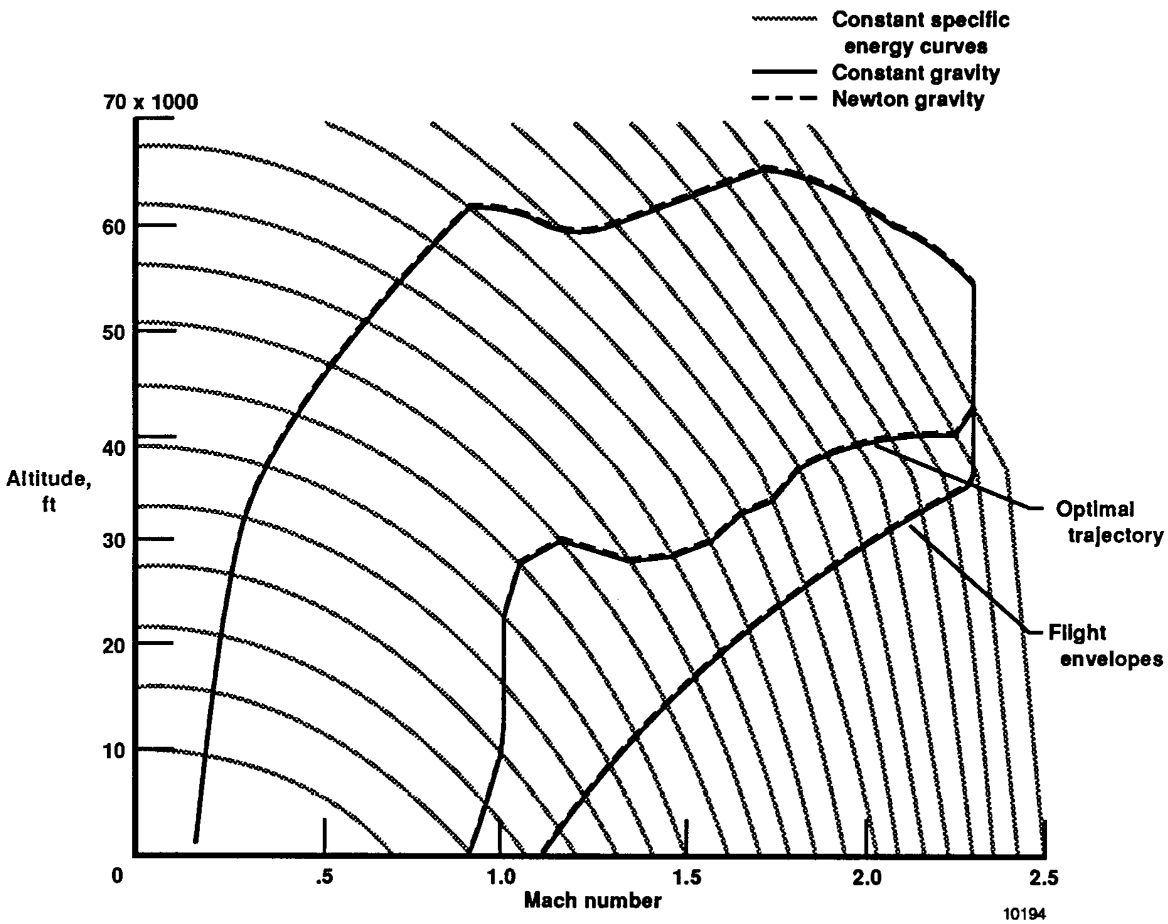


Figure 5. Effect of gravitational model on flight envelope and trajectory.

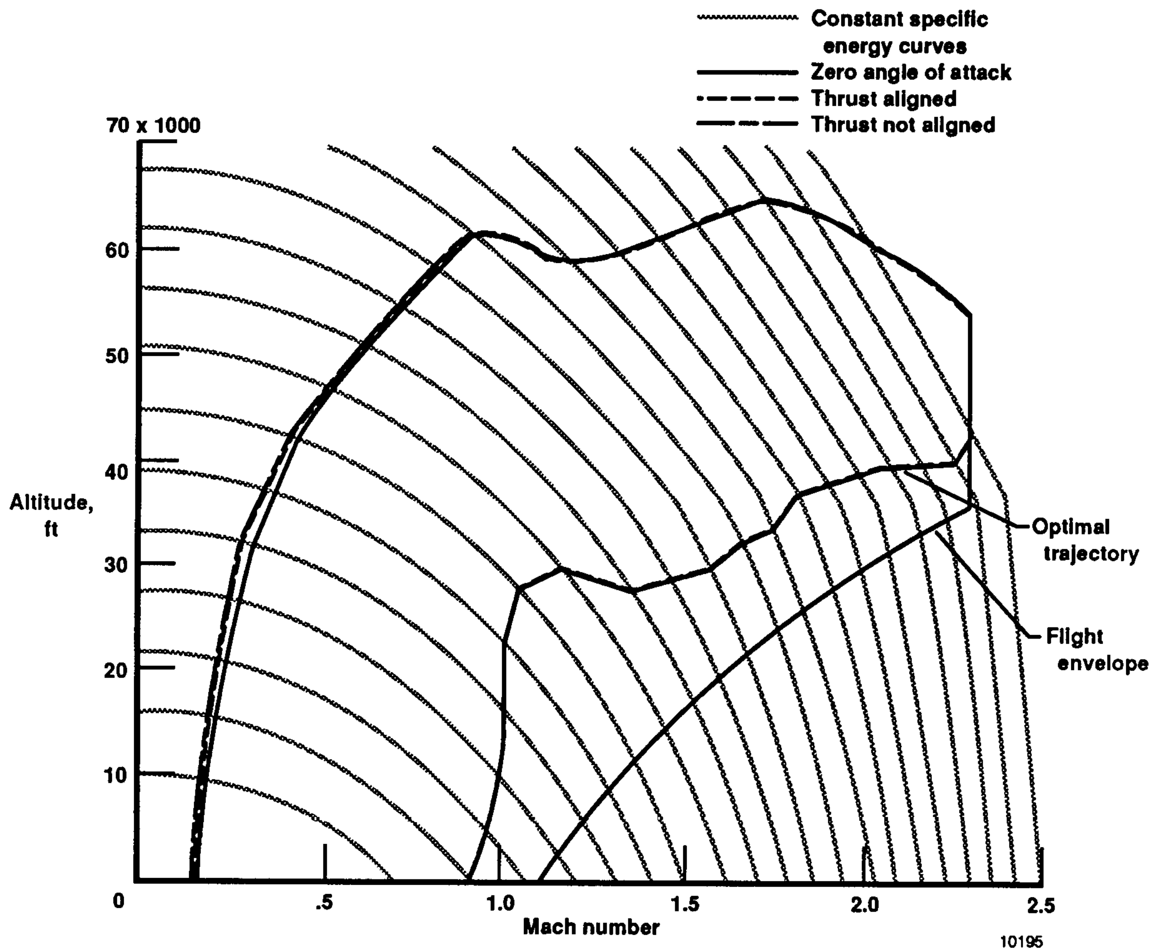
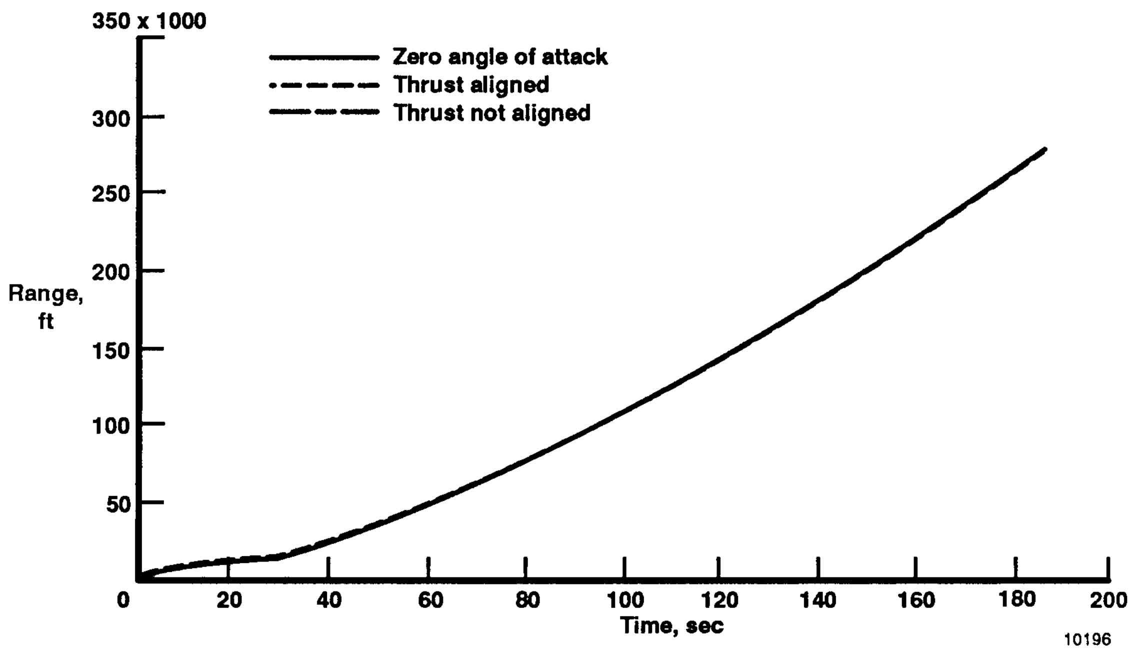
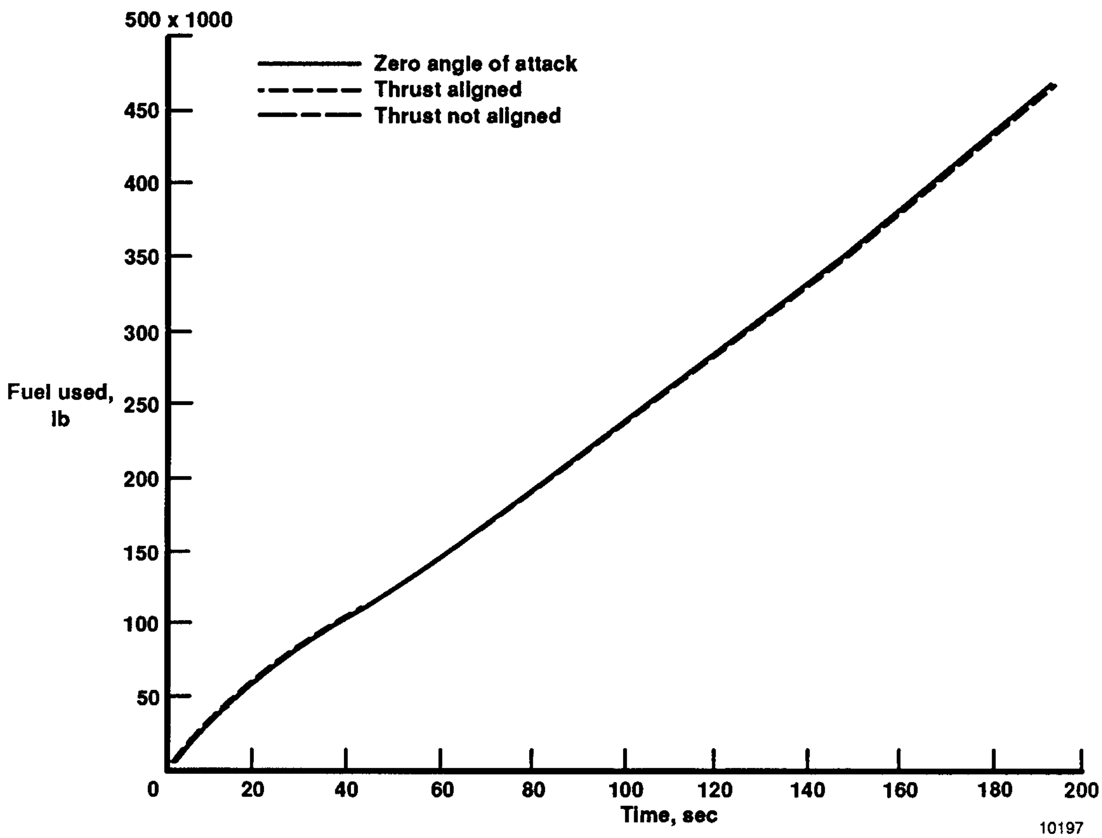


Figure 6. Effect of equations of motion on flight envelope and trajectory.

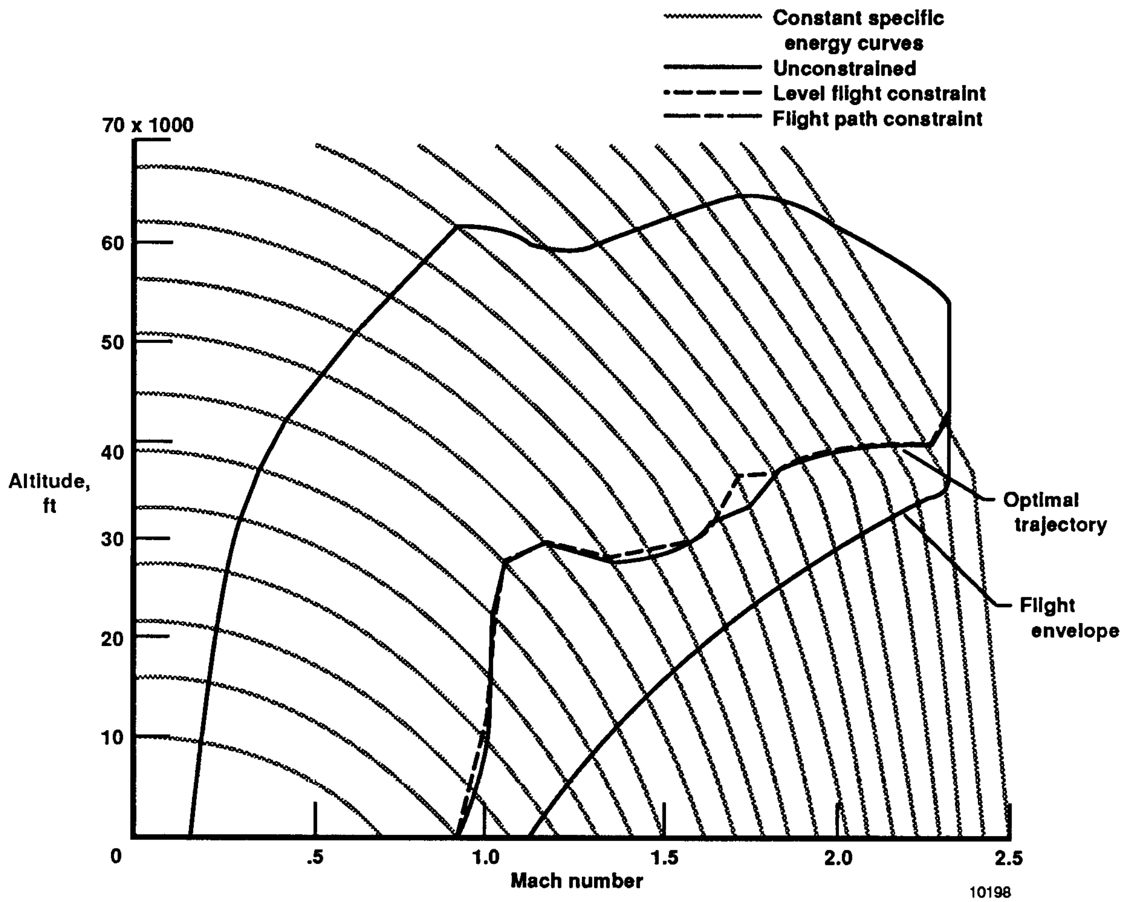


(a) Range as a function of time.
Figure 7. Effects of equations of motion.



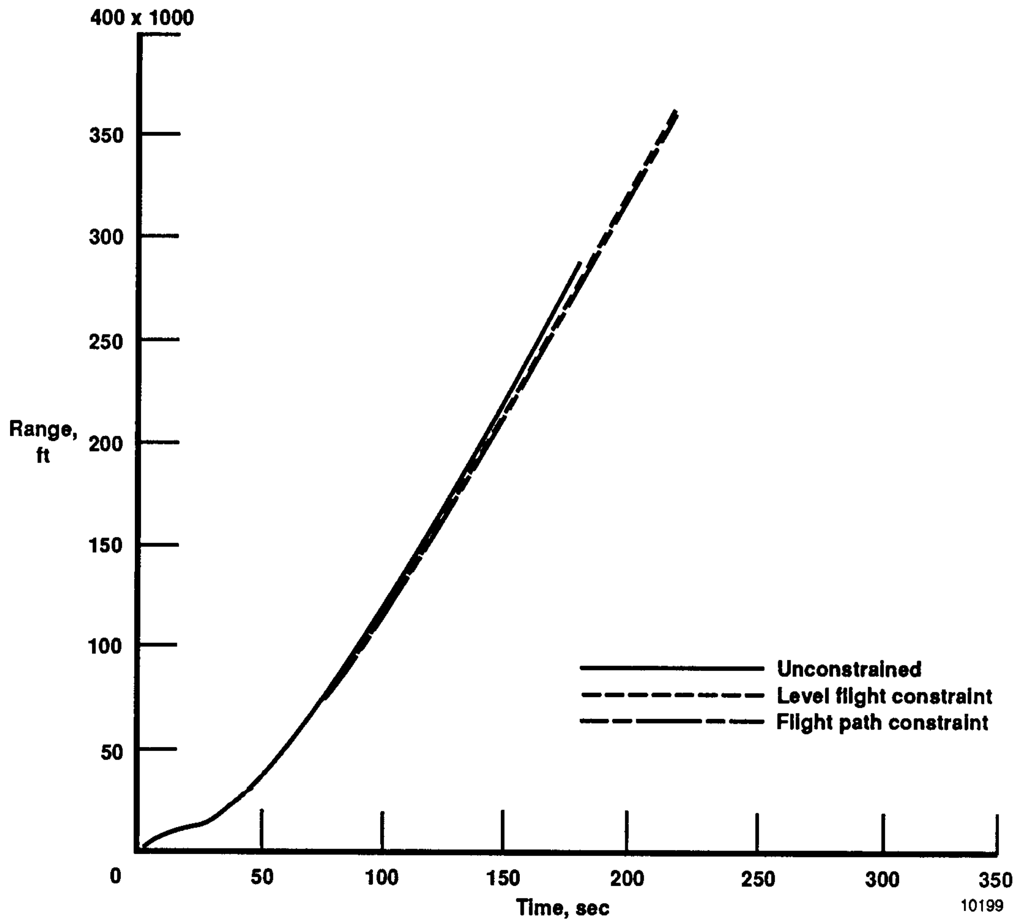
(b) Fuel used as a function of time.

Figure 7. Concluded.



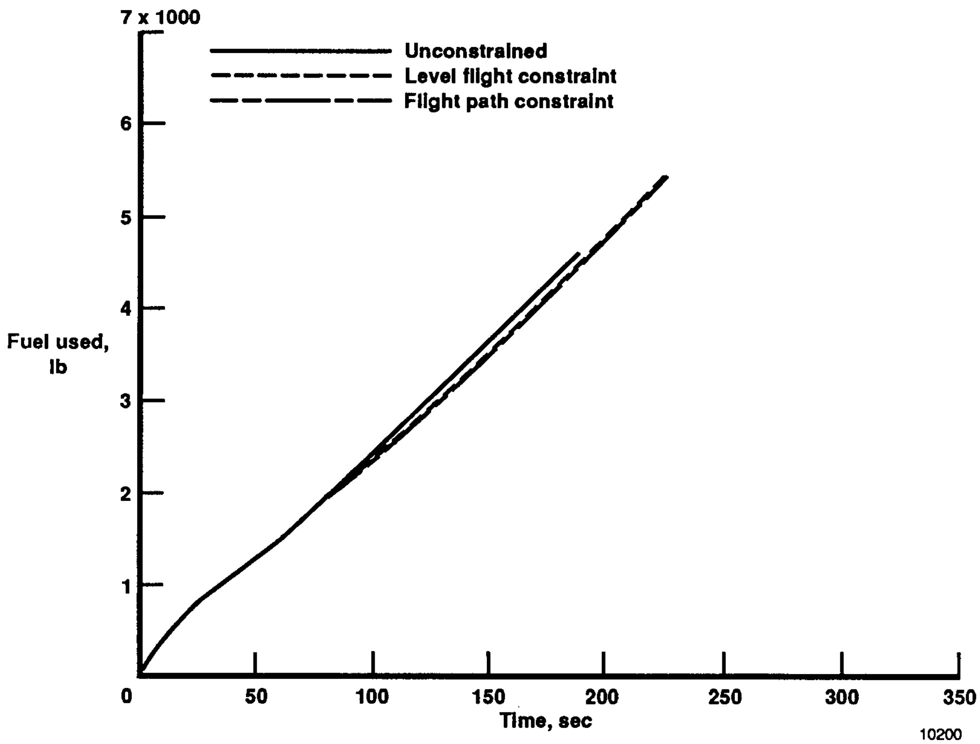
(a) Trajectory.

Figure 8. Effects of constraint conditions.



(b) Range as a function of time.

Figure 8. Continued.



(c) Fuel used as a function of time.

Figure 8. Concluded.

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16. Abstract

When analyzing the performance of an aircraft, certain simplifying assumptions, which decrease the complexity of the problem, can often be made. The degree of accuracy required in the solution may determine the extent to which these simplifying assumptions are incorporated. A complex model may yield more accurate results if it describes the real situation more thoroughly. However, a complex model usually involves more computation time, makes the analysis more difficult, and often requires more information to do the analysis. Therefore, to choose the simplifying assumptions intelligently, it is important to know what effects the assumptions may have on the calculated performance of a vehicle. This paper examines several simplifying assumptions, compares the effects of simplified models to those of the more complex ones, and draws conclusions about the impact of these assumptions on flight envelope generation and optimal trajectory calculation. Models which affect an aircraft are analyzed, but the implications of simplifying the model of the aircraft itself are not studied. The examples in this paper are atmospheric models, gravitational models, different models for equations of motion, and constraint conditions.

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