

Combinatorial Coverage Measurement

Rick Kuhn
NIST

Software Failure Analysis

- We studied software failures in a variety of fields including 15 years of FDA medical device recall data
- What **causes** software failures?
 - logic errors?
 - calculation errors?
 - inadequate input checking?
 - interaction faults? Etc.



Interaction faults: e.g., failure occurs if

`pressure < 10`

(1-way \leq all-values testing catches)

`pressure < 10 && volume > 300`

(2-way \leq all-pairs testing catches)

Example:

Failure when “altitude adjustment set on 0 meters and total flow volume set at delivery rate of less than 2.2 liters per minute.”

=> **2-way interaction**

Software Failure Internals

How does an interaction fault manifest itself in code?

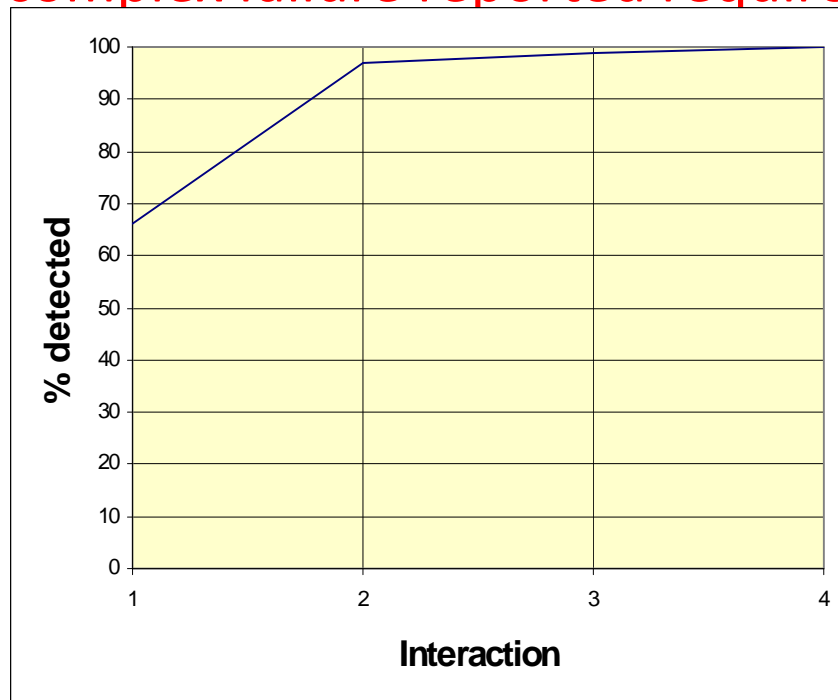
Example: `altitude_adj == 0 && volume < 2.2` (2-way interaction)

```
if (altitude_adj == 0 ) {  
    // do something  
    if (volume < 2.2) { faulty code!  BOOM! }  
    else { good code, no problem}  
} else {  
    // do something else  
}
```

A test that included `altitude_adj == 0` and `volume = 1` would trigger this failure

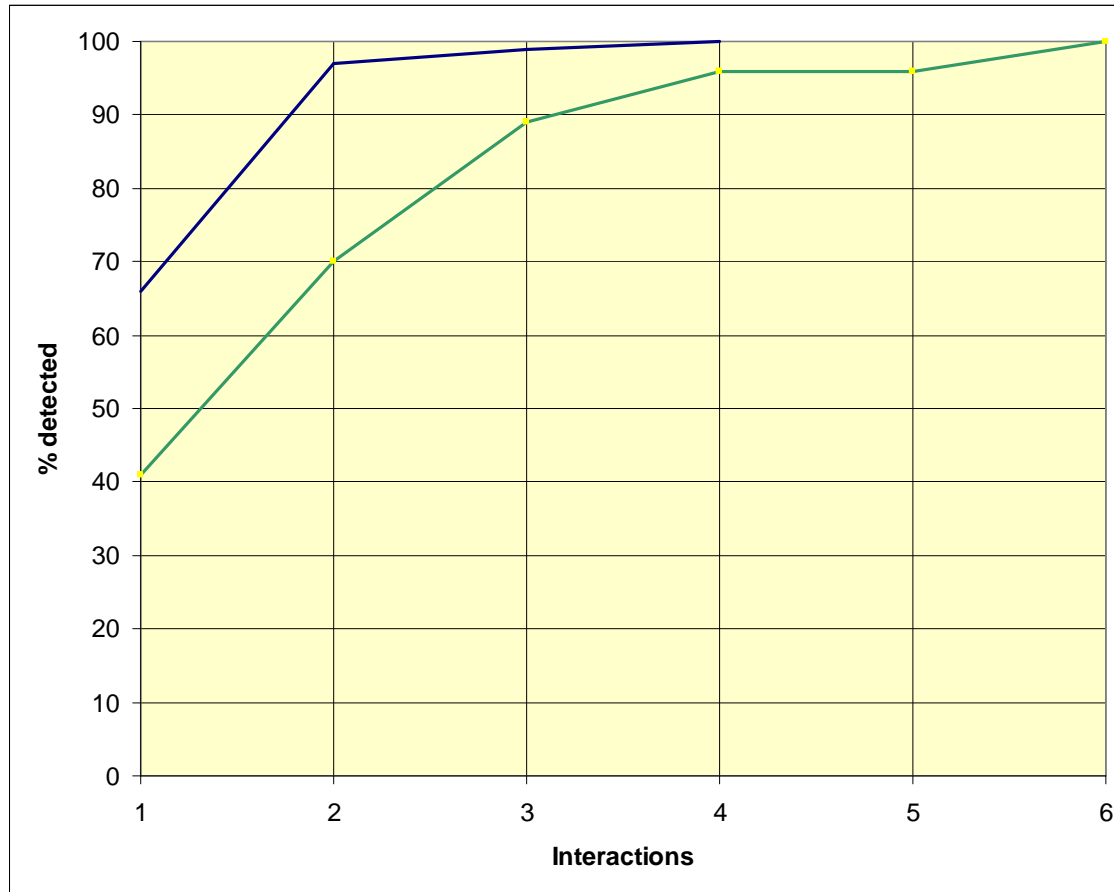
How about flaws that are harder to find ?

- Interactions e.g., failure occurs if
 - pressure < 10 (1-way interaction)
 - pressure < 10 & volume > 300 (2-way interaction)
 - pressure < 10 & volume > 300 & velocity = 5 (3-way interaction)
- The most complex failure reported required 4-way interaction to trigger



What about other applications?

Server (green)

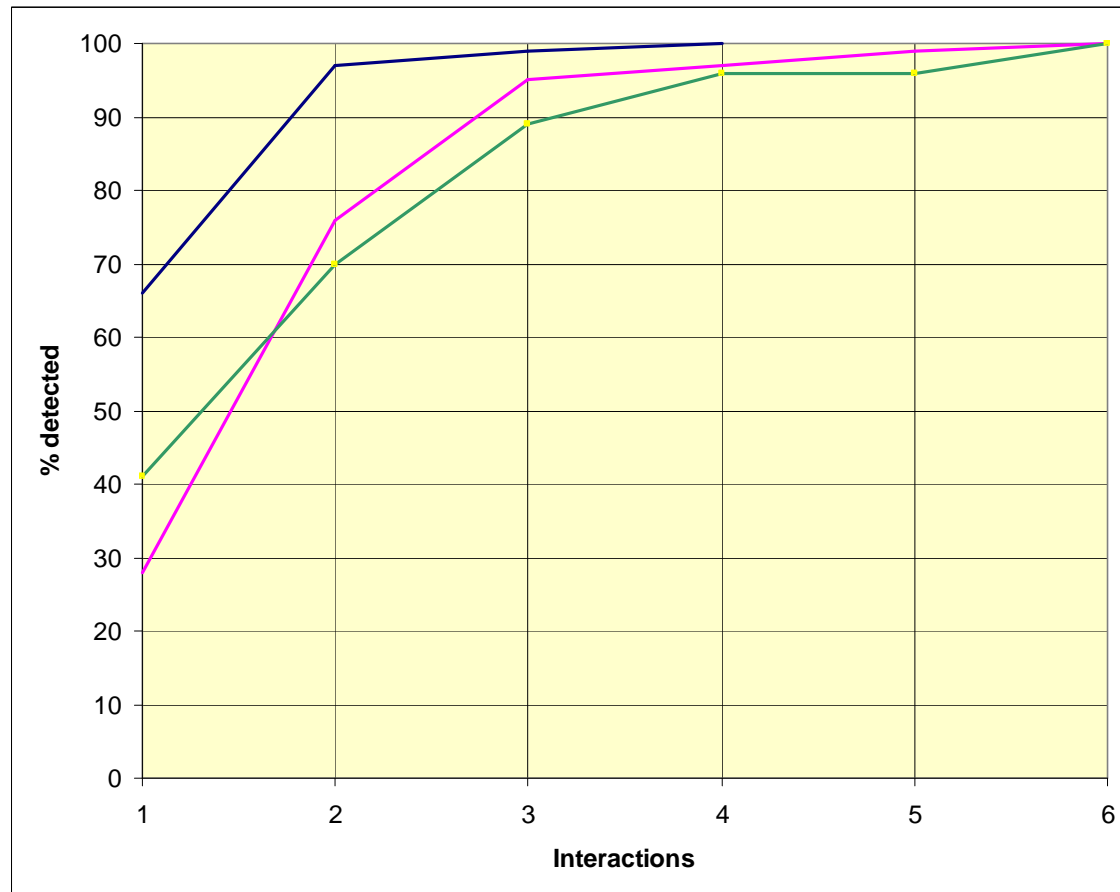


These faults more complex than medical device software!!

Why?

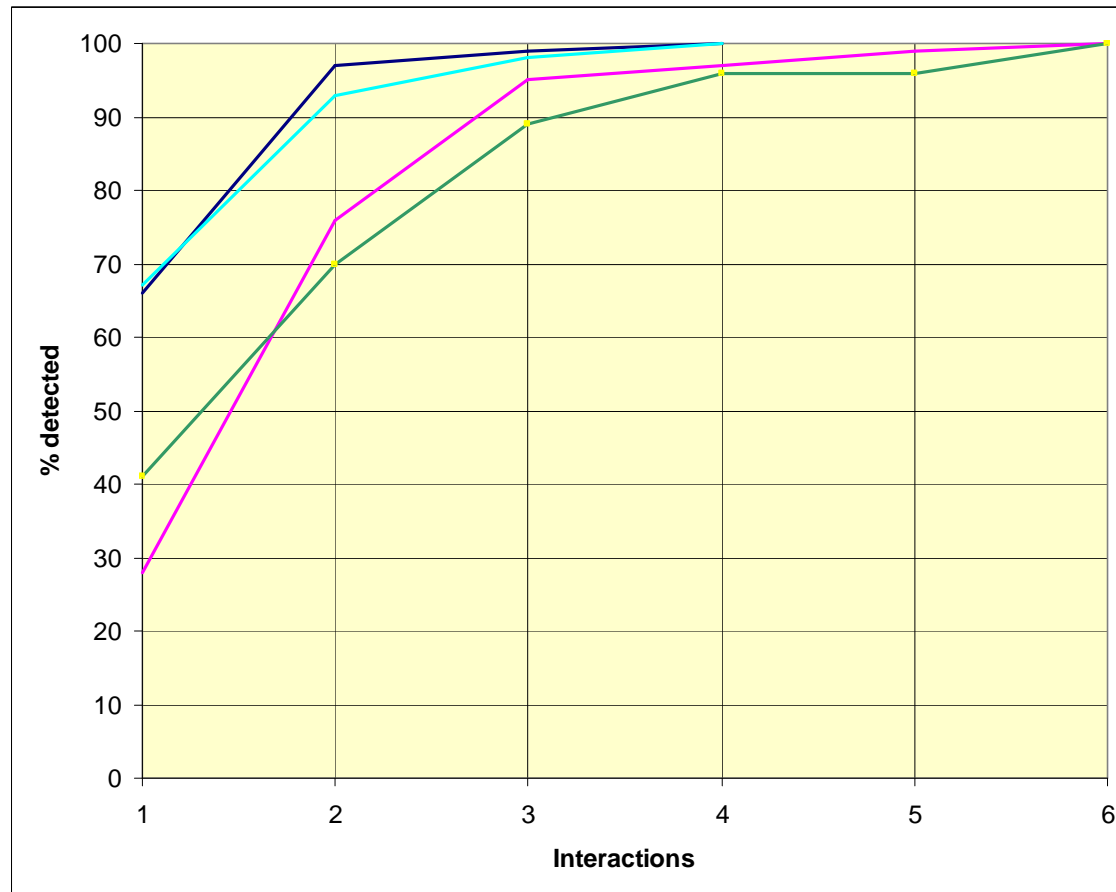
Others?

Browser (magenta)



Still more?

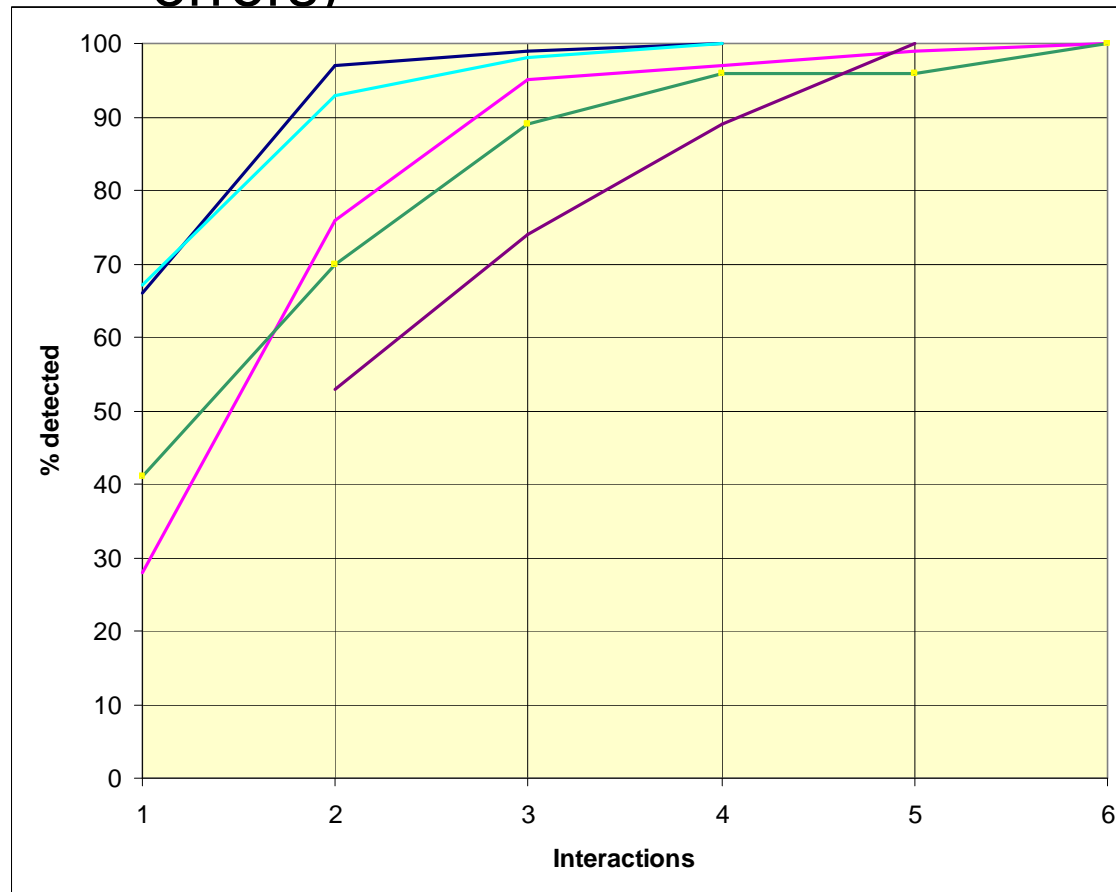
NASA Goddard distributed database (light blue)



Note:
development
data, others are
released
products

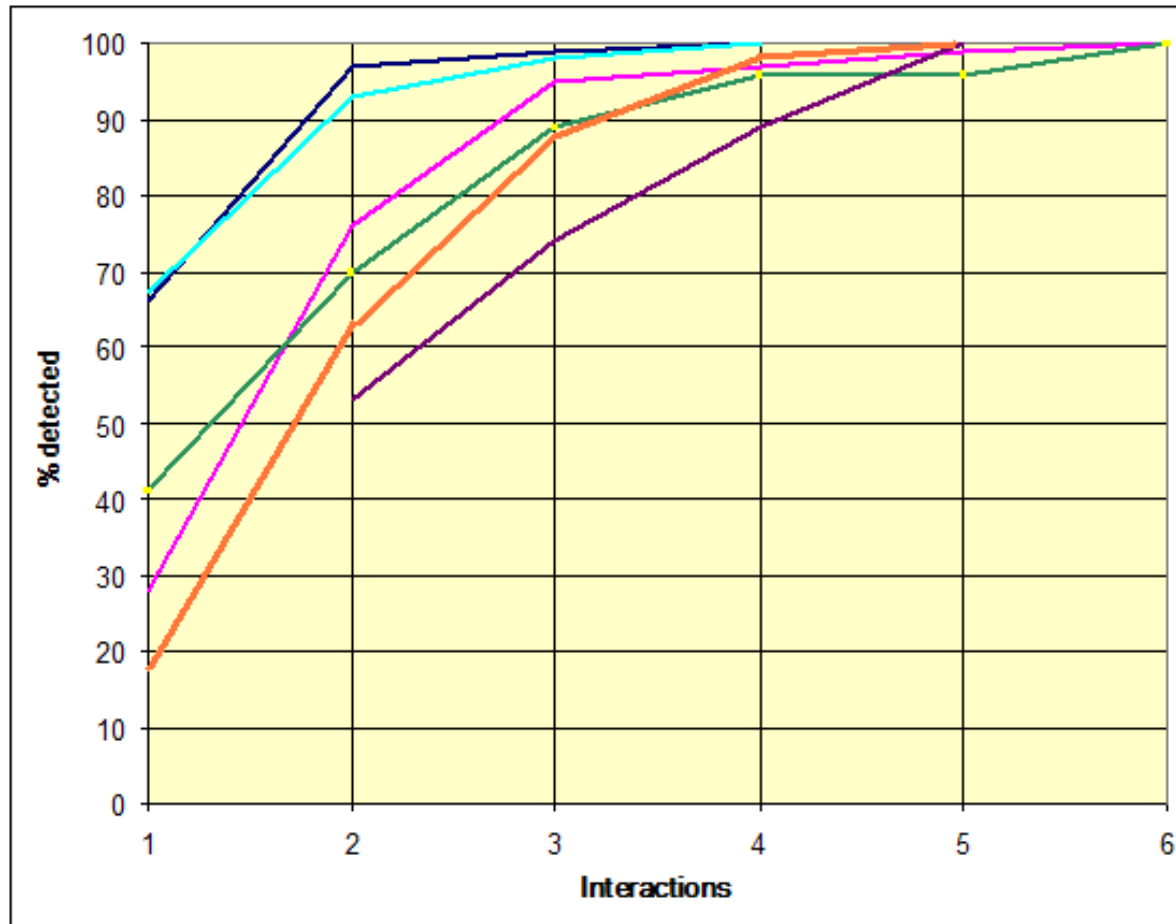
Even more?

FAA Traffic Collision Avoidance System module (seeded errors)
(purple)



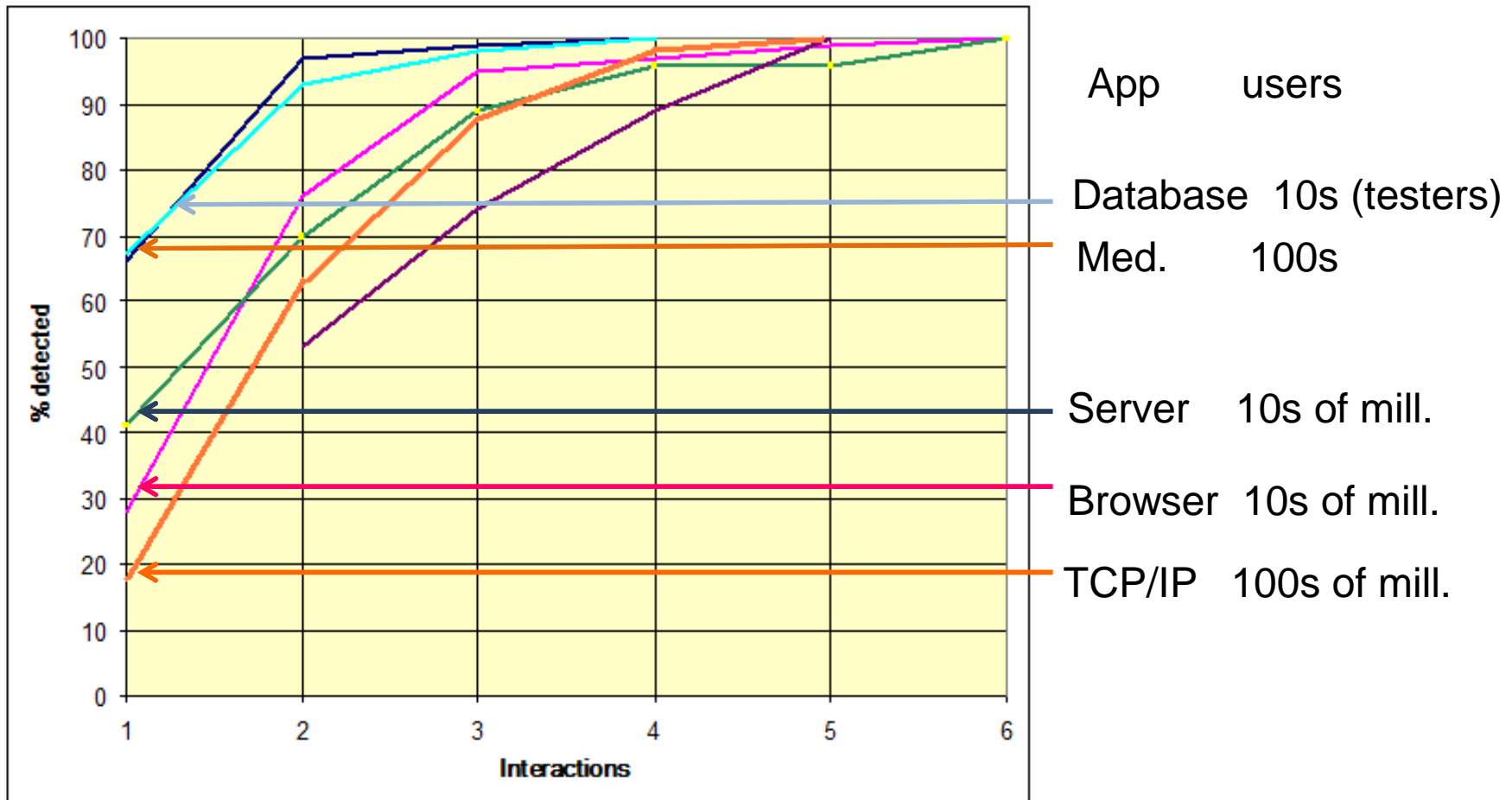
Finally

Network security (Bell, 2006) (orange)



Curves appear to be similar across a variety of application domains.

Fault curve pushed down and right as faults detected and removed?

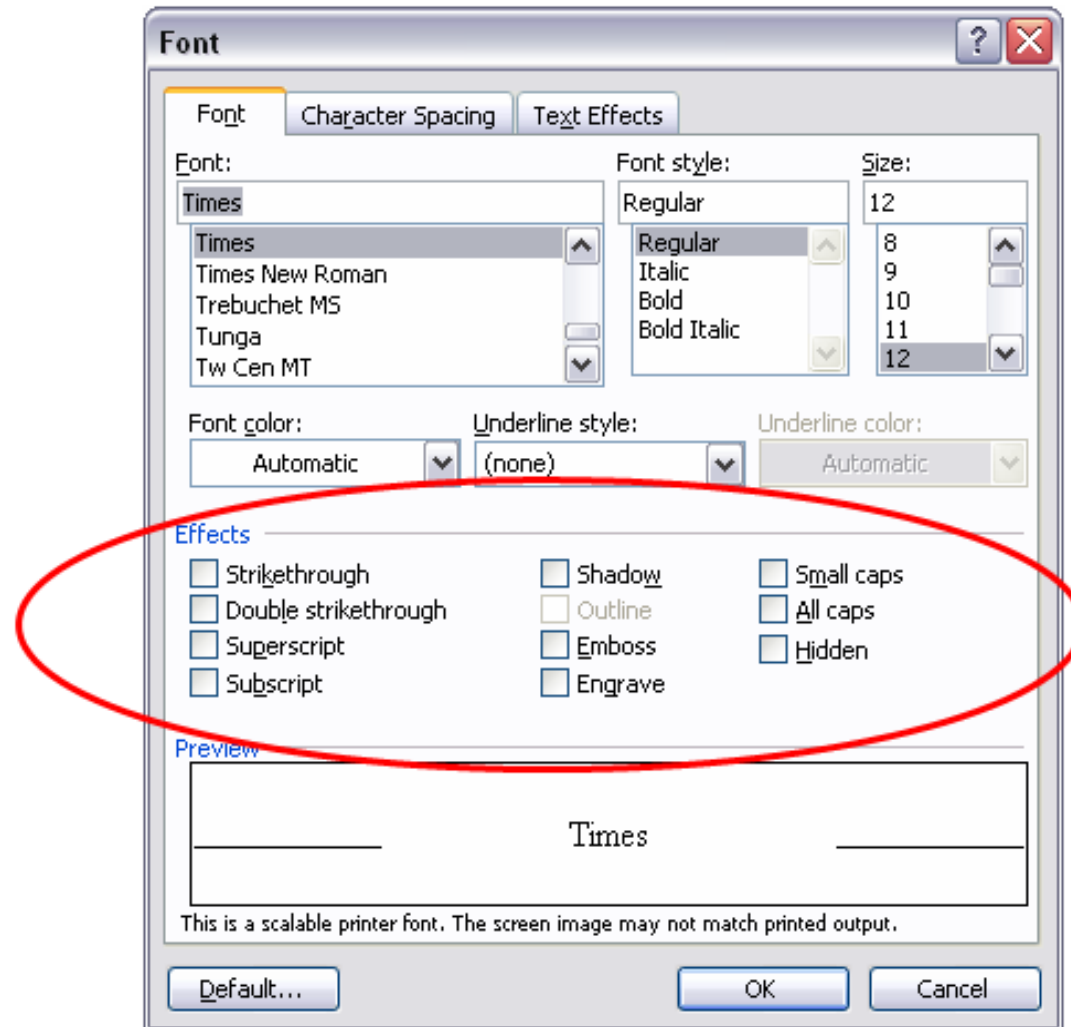


Interaction Rule

- How many parameters involved in faults? => *interaction rule*: most failures are triggered by one or two parameters, and progressively fewer by three, four, or more parameters, and the maximum interaction degree is small.
- *Maximum interactions* for fault triggering was 6
- Popular “pairwise testing” not enough
- More empirical work needed
- Reasonable evidence that maximum interaction strength for fault triggering is *relatively small*

How do we use this knowledge in testing?

A simple example

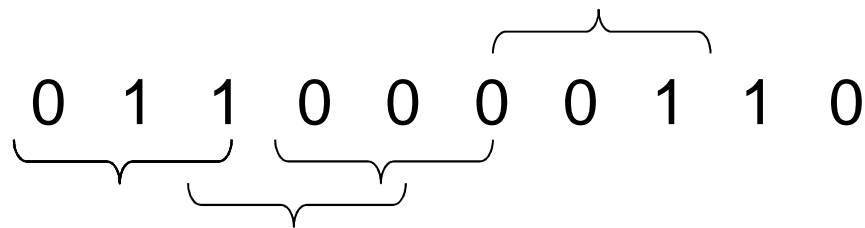


How Many Tests Would It Take?

- There are 10 effects, each can be on or off
- All combinations is $2^{10} = 1,024$ tests
- What if our budget is too limited for these tests?
- Instead, let's look at all 3-way interactions ...

Now How Many Would It Take?

- There are $\binom{10}{3} = 120$ 3-way interactions.
- Naively $120 \times 2^3 = 960$ tests.
- Since we can pack 3 triples into each test, we need no more than 320 tests.
- Each test exercises many triples:



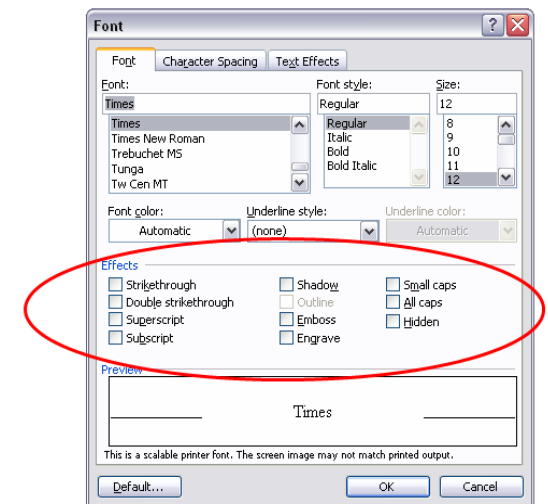
A covering array

All triples in only 13 tests, covering $\binom{10}{3} 2^3 = 960$ combinations

Each row is a test:

0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
1	1	1	0	1	0	0	0	0	1
1	0	1	1	0	1	1	0	1	0
1	0	0	0	1	1	1	0	0	0
0	1	1	0	0	1	0	1	1	0
0	0	1	0	1	0	1	1	1	0
1	1	0	1	0	0	0	0	1	0
0	0	0	1	1	1	1	0	0	1
0	0	1	1	0	0	0	1	0	0
0	1	0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	0	1	1
0	1	0	0	0	1	1	1	0	1

Each column is a parameter:



- Developed 1990s
- Extends Design of Experiments concept
- NP hard problem but good algorithms now

How does this knowledge help?

If all faults are triggered by the interaction of t or fewer variables, then testing all t -way combinations can provide strong assurance.

(taking into account: value propagation issues, equivalence partitioning, timing issues, more complex interactions, . . .)

Test coverage measurement

Path coverage

- Many varieties, studied for decades
- Path, branch, condition coverage, plus many variations

Combinatorial coverage

- The subject of this talk, new
- How should we measure it?

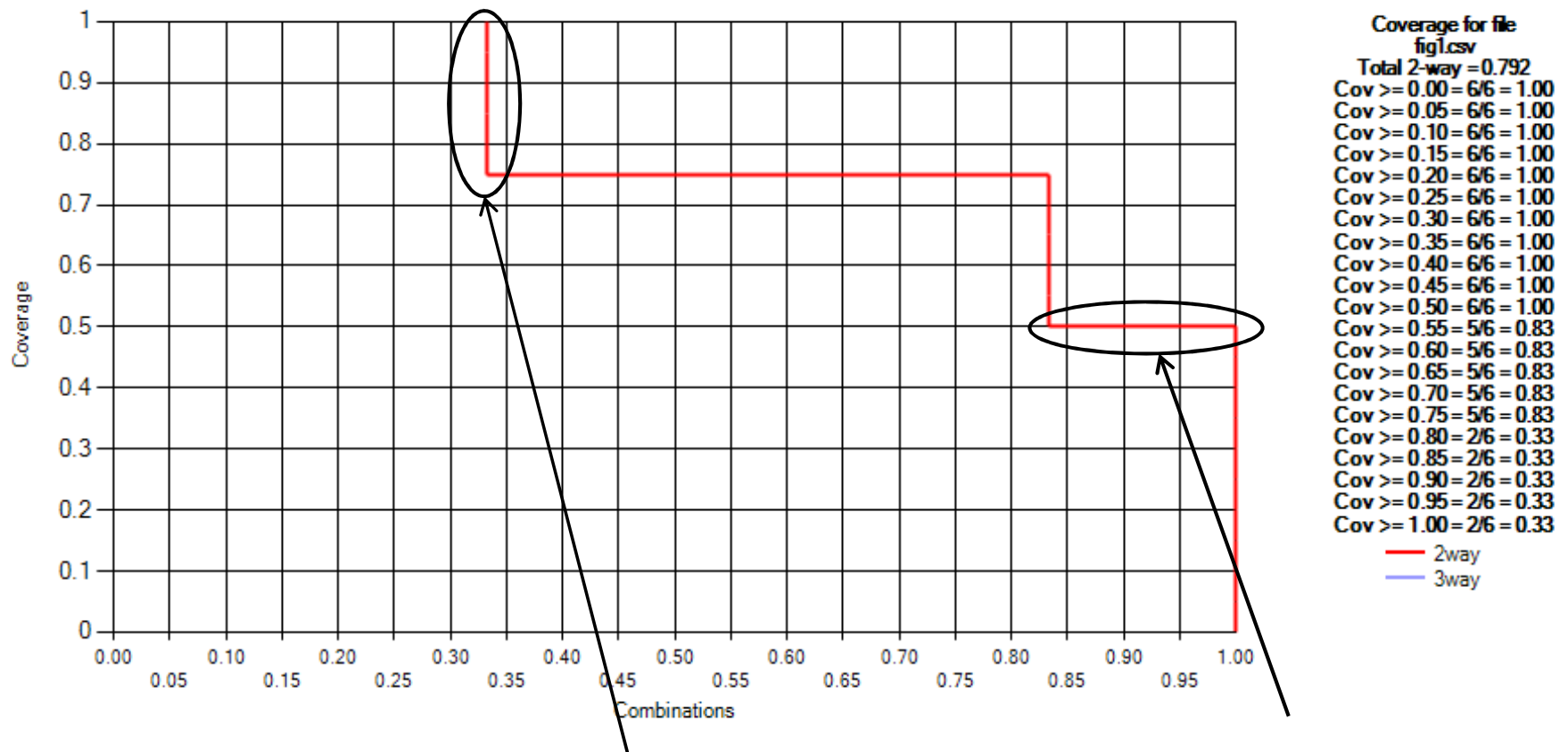
Combinatorial Coverage Measurement

Tests	Variables			
	a	b	c	d
1	0	0	0	0
2	0	1	1	0
3	1	0	0	1
4	0	1	1	1

Variable pairs	Variable-value combinations covered	Coverage
<i>ab</i>	00, 01, 10	.75
<i>ac</i>	00, 01, 10	.75
<i>ad</i>	00, 01, 11	.75
<i>bc</i>	00, 11	.50
<i>bd</i>	00, 01, 10, 11	1.0
<i>cd</i>	00, 01, 10, 11	1.0

100% coverage of 33% of combinations
 75% coverage of half of combinations
 50% coverage of 16% of combinations

Graphing Coverage - graphing

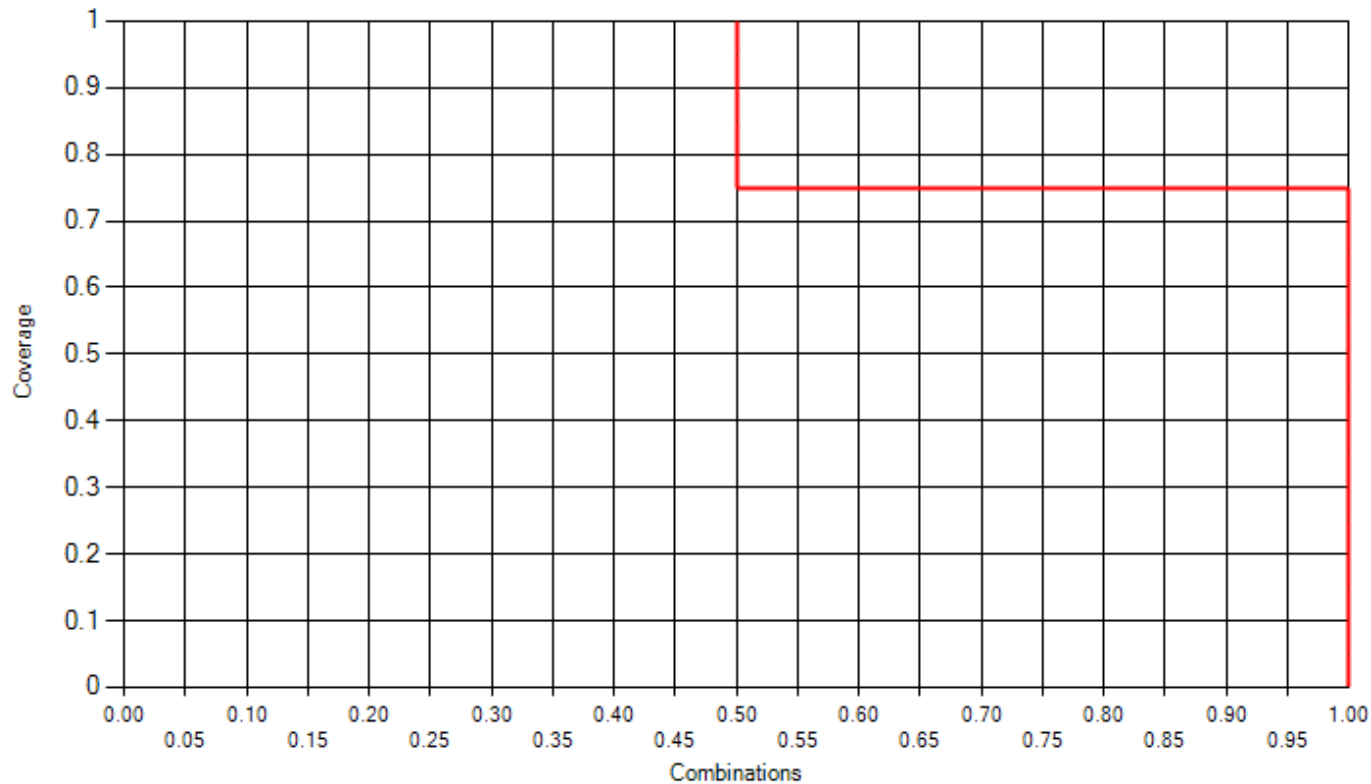


33% of combinations
covered 100%

Bottom line:
All combinations
covered to at least 50%

100% coverage of 33% of combinations
75% coverage of half of combinations
50% coverage of 16% of combinations

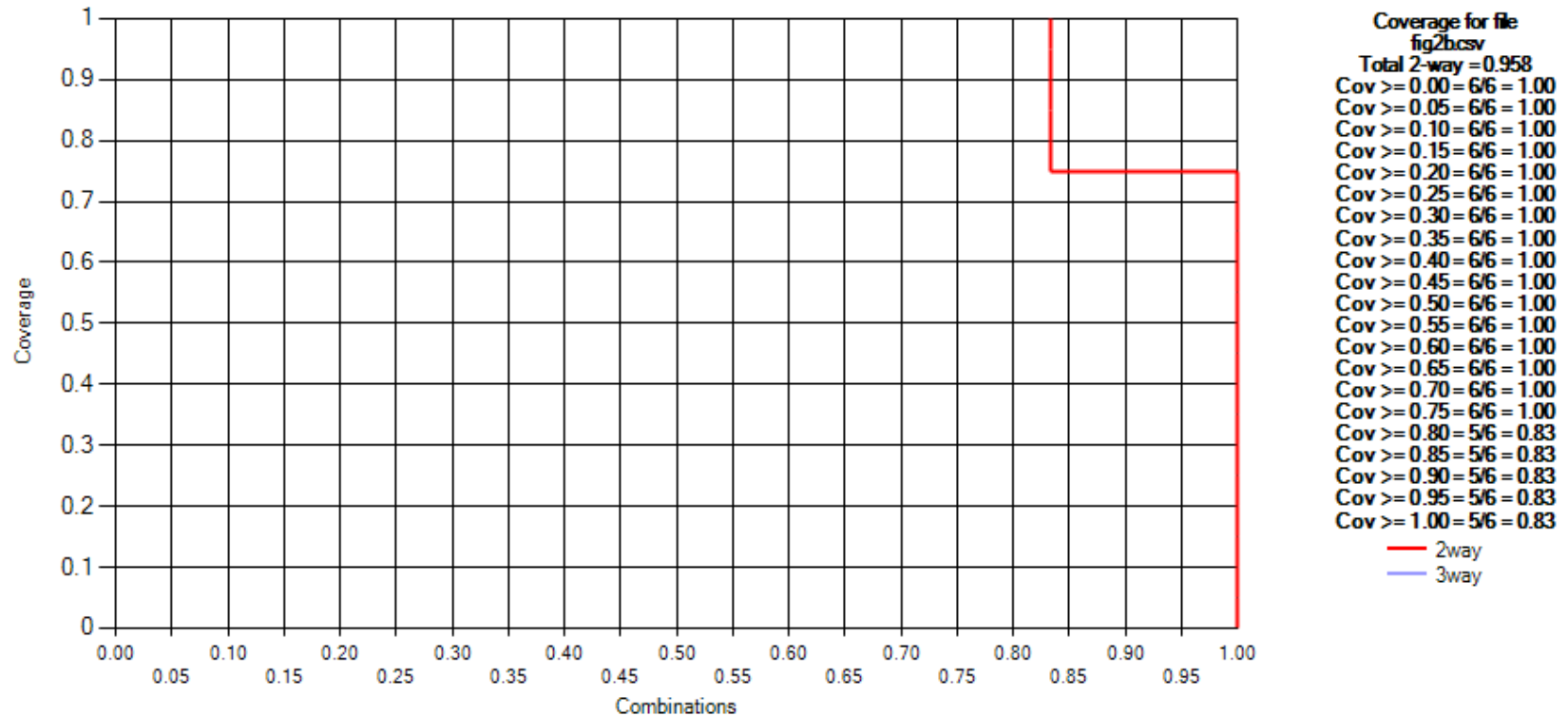
Adding a test



Coverage for file
fig2acsv
Total 2-way = 0.875
Cov >= 0.00 = 6/6 = 1.00
Cov >= 0.05 = 6/6 = 1.00
Cov >= 0.10 = 6/6 = 1.00
Cov >= 0.15 = 6/6 = 1.00
Cov >= 0.20 = 6/6 = 1.00
Cov >= 0.25 = 6/6 = 1.00
Cov >= 0.30 = 6/6 = 1.00
Cov >= 0.35 = 6/6 = 1.00
Cov >= 0.40 = 6/6 = 1.00
Cov >= 0.45 = 6/6 = 1.00
Cov >= 0.50 = 6/6 = 1.00
Cov >= 0.55 = 6/6 = 1.00
Cov >= 0.60 = 6/6 = 1.00
Cov >= 0.65 = 6/6 = 1.00
Cov >= 0.70 = 6/6 = 1.00
Cov >= 0.75 = 6/6 = 1.00
Cov >= 0.80 = 3/6 = 0.50
Cov >= 0.85 = 3/6 = 0.50
Cov >= 0.90 = 3/6 = 0.50
Cov >= 0.95 = 3/6 = 0.50
Cov >= 1.00 = 3/6 = 0.50
— 2way
— 3way

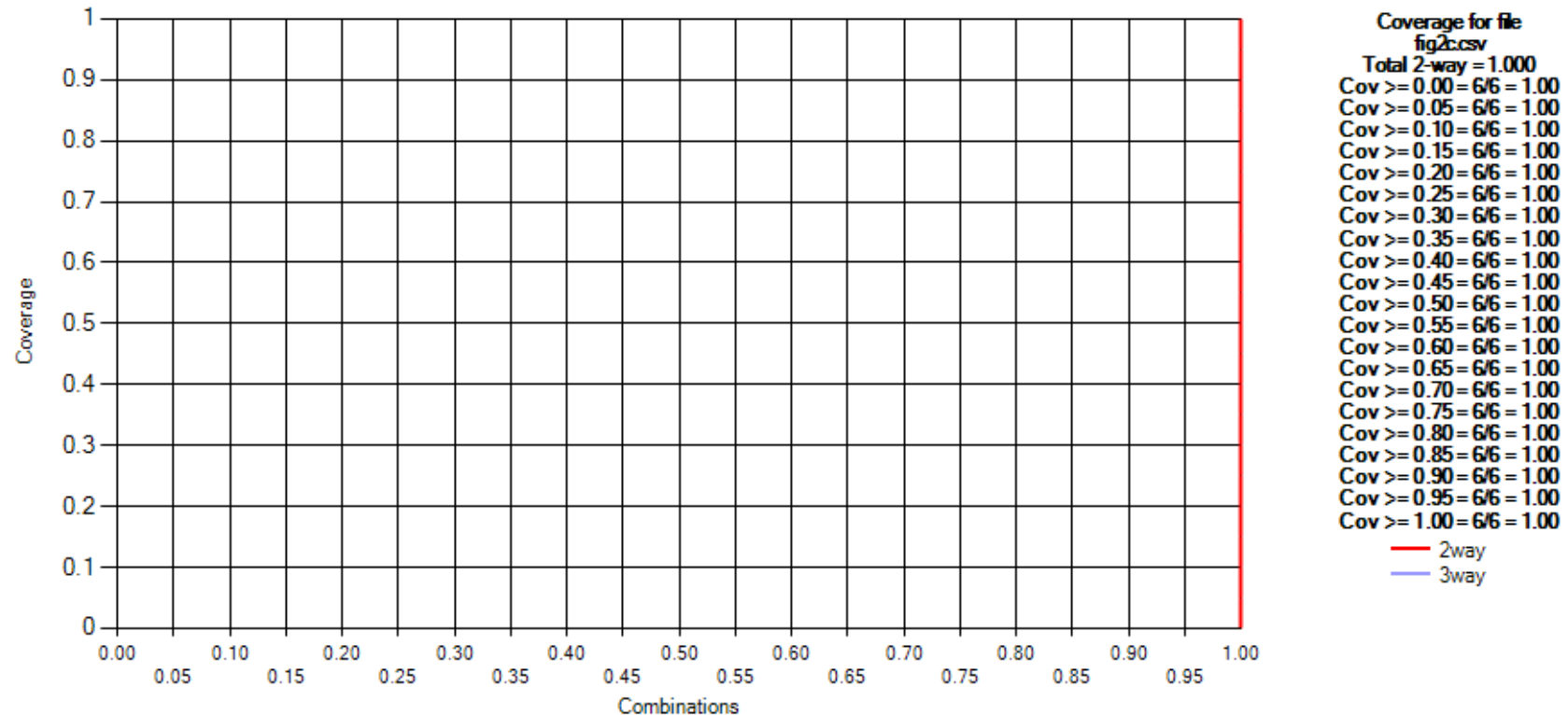
Coverage after adding test [1,1,0,1]

Adding another test



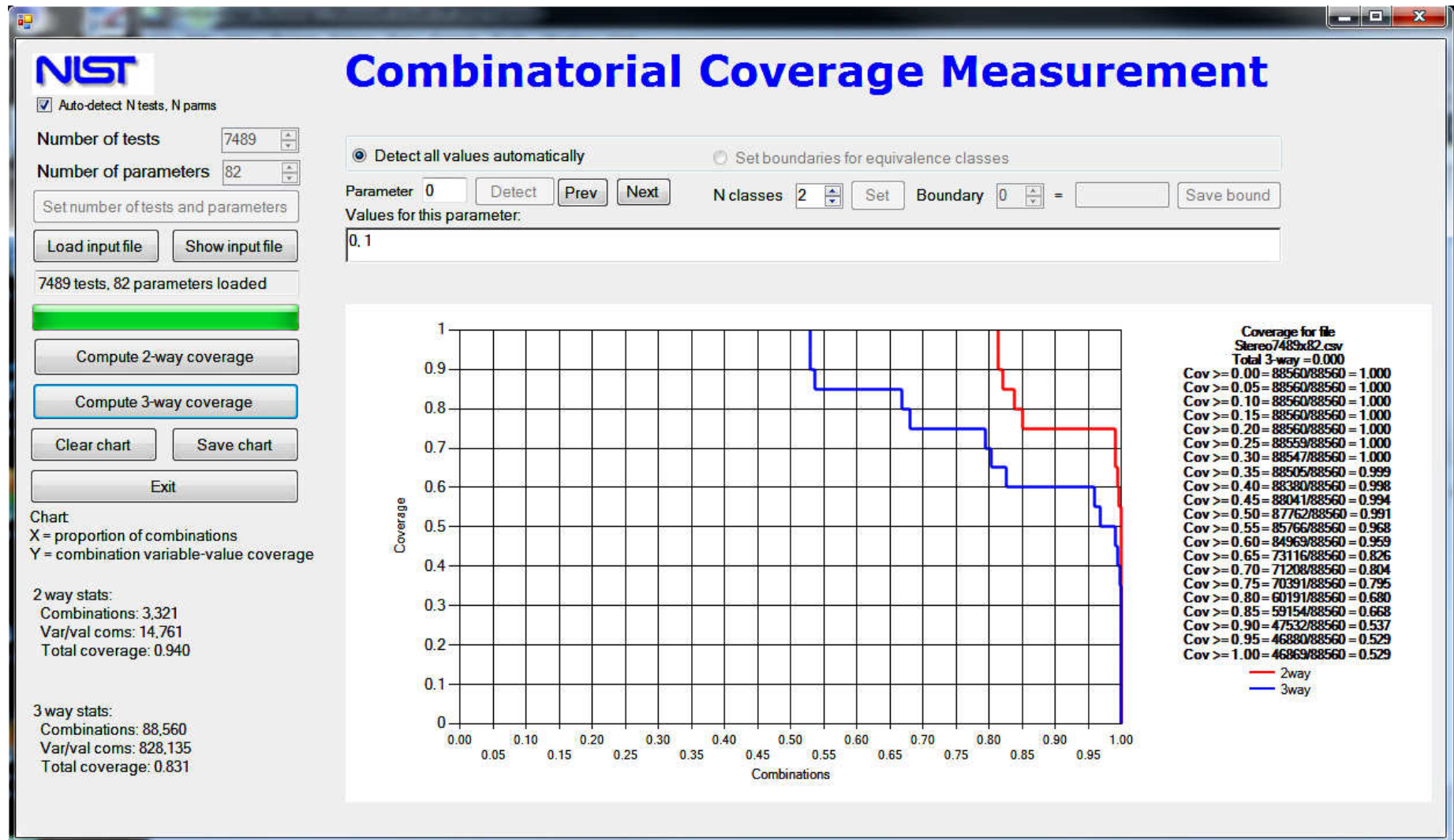
Coverage after adding test [1,0,1,1]

Additional test completes coverage



Coverage after adding test [1,0,1,0]
All combinations covered to 100% level,
so this is a covering array.

Coverage Measurement Tool



4 variables, mixed level

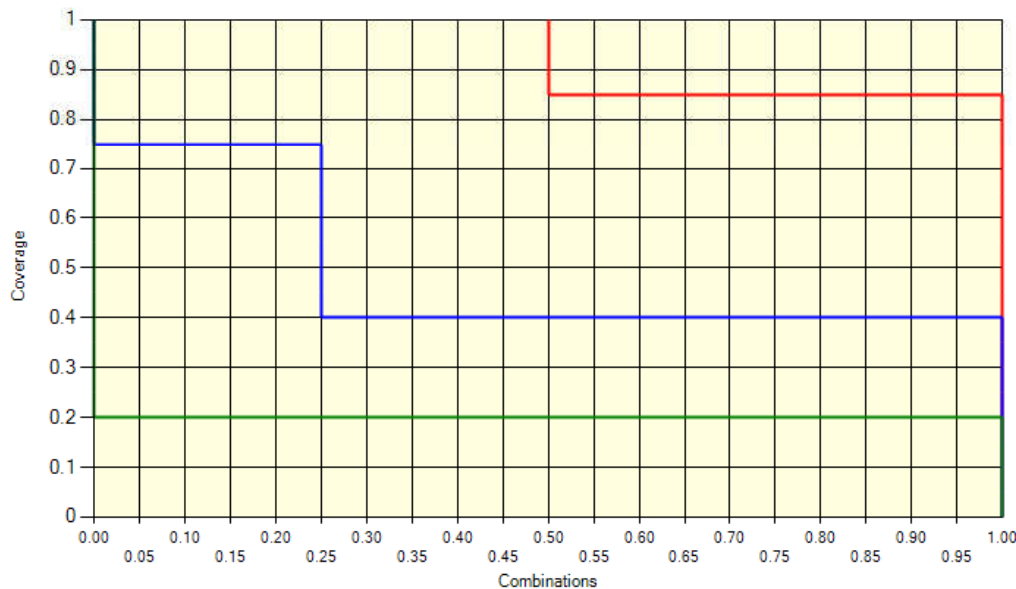
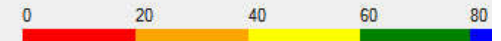
Combinatorial Coverage Measurement

☒ Detect all values automatically
 ☐ Set boundaries for equivalence classes

Parameter 0 [Detect] [Prev] [Next]
 N classes 2 [Set]
 Boundary 0 = [] [Save bound]

Values for this parameter:

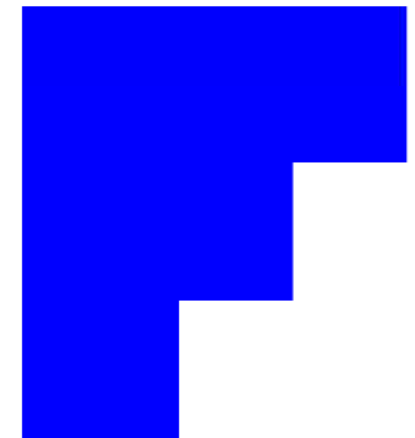
E-3, E-4



Coverage for file
tbl5.csv
Total 3-way = 0.457

Cov >= 0.00 = 4/4 = 1.000
Cov >= 0.05 = 4/4 = 1.000
Cov >= 0.10 = 4/4 = 1.000
Cov >= 0.15 = 4/4 = 1.000
Cov >= 0.20 = 4/4 = 1.000
Cov >= 0.25 = 4/4 = 1.000
Cov >= 0.30 = 4/4 = 1.000
Cov >= 0.35 = 4/4 = 1.000
Cov >= 0.40 = 4/4 = 1.000
Cov >= 0.45 = 1/4 = 0.250
Cov >= 0.50 = 1/4 = 0.250
Cov >= 0.55 = 1/4 = 0.250
Cov >= 0.60 = 1/4 = 0.250
Cov >= 0.65 = 1/4 = 0.250
Cov >= 0.70 = 1/4 = 0.250
Cov >= 0.75 = 1/4 = 0.250
Cov >= 0.80 = 0/4 = 0.000
Cov >= 0.85 = 0/4 = 0.000
Cov >= 0.90 = 0/4 = 0.000
Cov >= 0.95 = 0/4 = 0.000
Cov >= 1.00 = 0/4 = 0.000

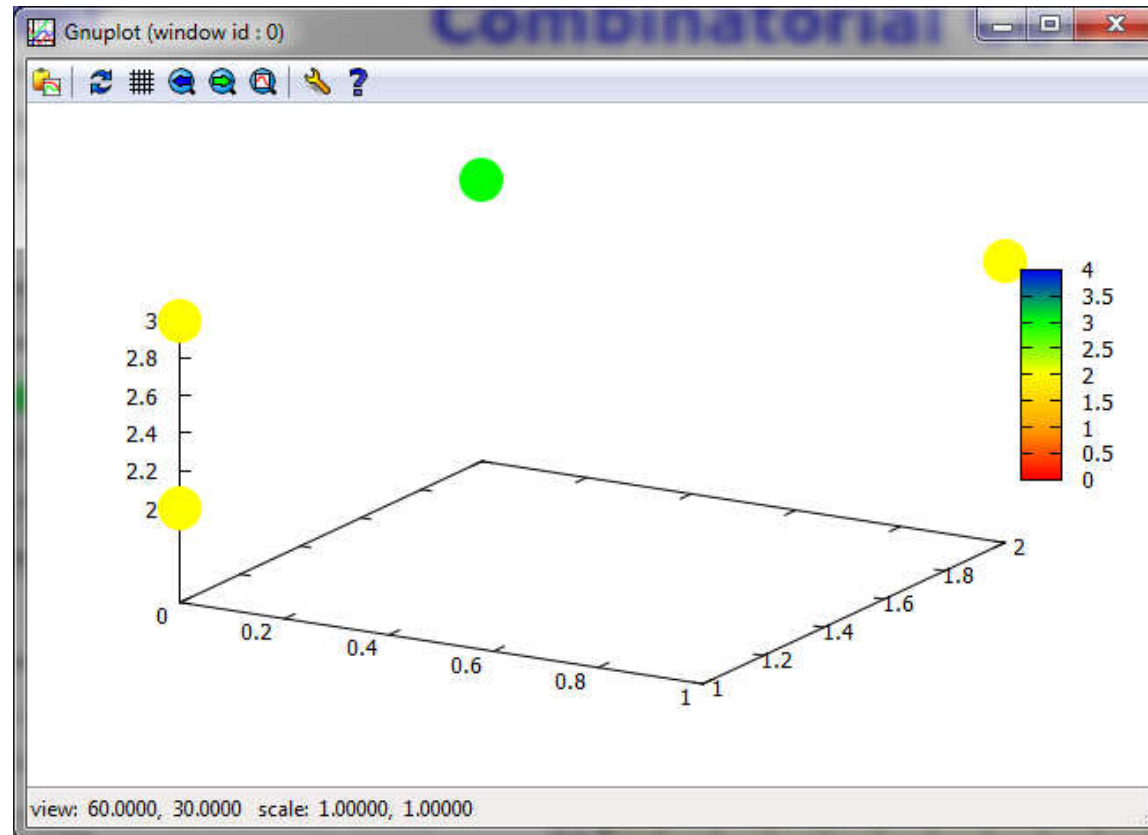
— 2way
 — 3way
 — 4way



-1 0 1 2

- Line graph for 2-way coverage shows 100% for half, 75% for half;
3-way coverage (blue line) at 75% for 25% of combinations, 40% coverage for 75% of combinations
- Number of 2-way combinations = $C(4,2) = 6$

Measurements of 3-way coverage



Comparing with line graph:

- Line graph shows 3-way coverage (blue line) at 75% for 25% of combinations, 40% coverage for 75% of combinations
- 3d graph shows one combination with 60%-80% coverage (green), and three with 40%-60% coverage (yellow)
- Number of 3-way combinations = $C(4,3) = 4$

7 variables, mixed level

Combinatorial Coverage Measurement

☒ Detect all values automatically

☐ Set boundaries for equivalence classes

Parameter 0

Detect

Prev

Next

N classes 3

Set

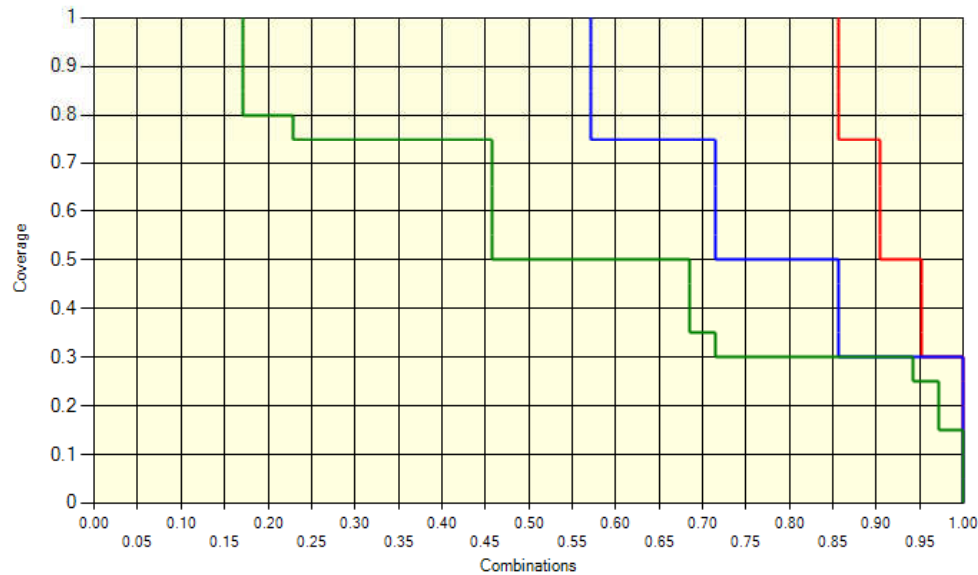
Boundary 0

=

Save bound

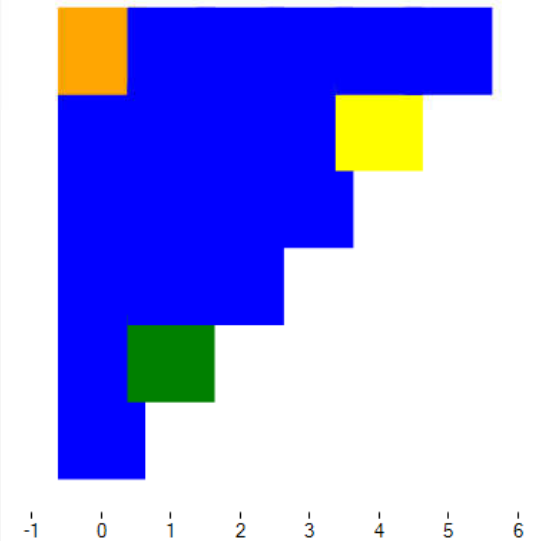
Values for this parameter:

Eglin_F-16, Nellis_F-16, Nellis_F-15



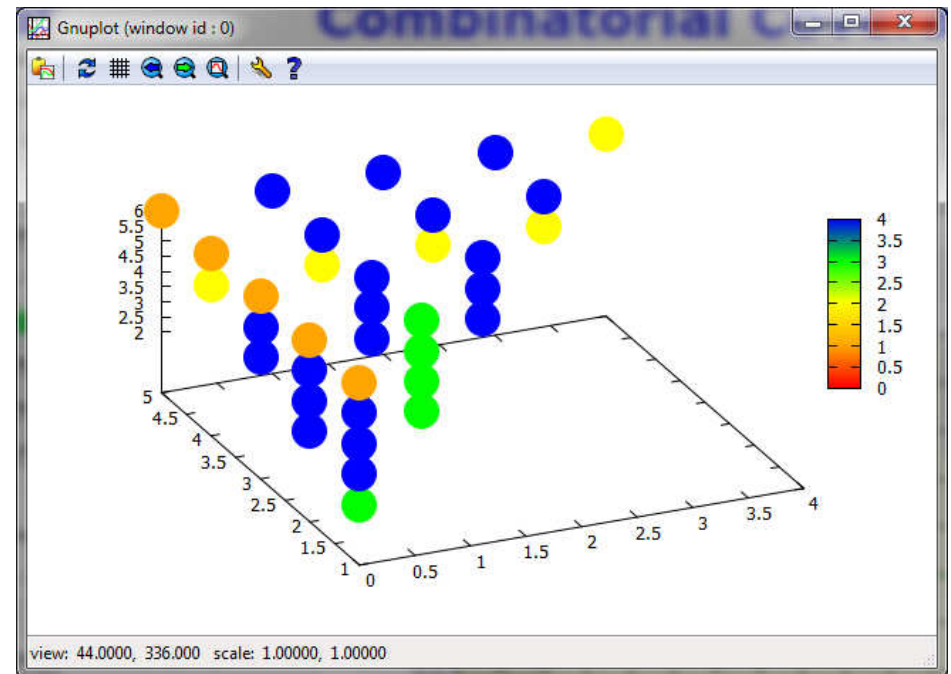
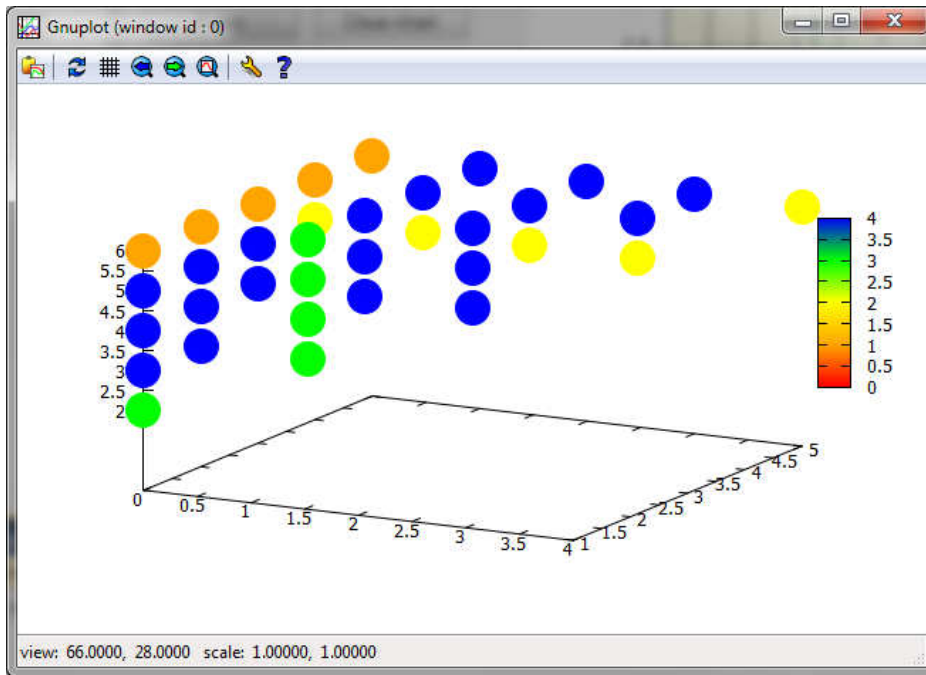
Coverage for file:
tbl-xi-1.csv
Total 3-way = 0.709
Cov >= 0.00 = 35/35 = 1.000
Cov >= 0.05 = 35/35 = 1.000
Cov >= 0.10 = 35/35 = 1.000
Cov >= 0.15 = 35/35 = 1.000
Cov >= 0.20 = 35/35 = 1.000
Cov >= 0.25 = 35/35 = 1.000
Cov >= 0.30 = 35/35 = 1.000
Cov >= 0.35 = 30/35 = 0.857
Cov >= 0.40 = 30/35 = 0.857
Cov >= 0.45 = 30/35 = 0.857
Cov >= 0.50 = 30/35 = 0.857
Cov >= 0.55 = 25/35 = 0.714
Cov >= 0.60 = 25/35 = 0.714
Cov >= 0.65 = 25/35 = 0.714
Cov >= 0.70 = 25/35 = 0.714
Cov >= 0.75 = 25/35 = 0.714
Cov >= 0.80 = 20/35 = 0.571
Cov >= 0.85 = 20/35 = 0.571
Cov >= 0.90 = 20/35 = 0.571
Cov >= 0.95 = 20/35 = 0.571
Cov >= 1.00 = 20/35 = 0.571

— 2way
— 3way
— 4way



Two views of the 3-way graph.

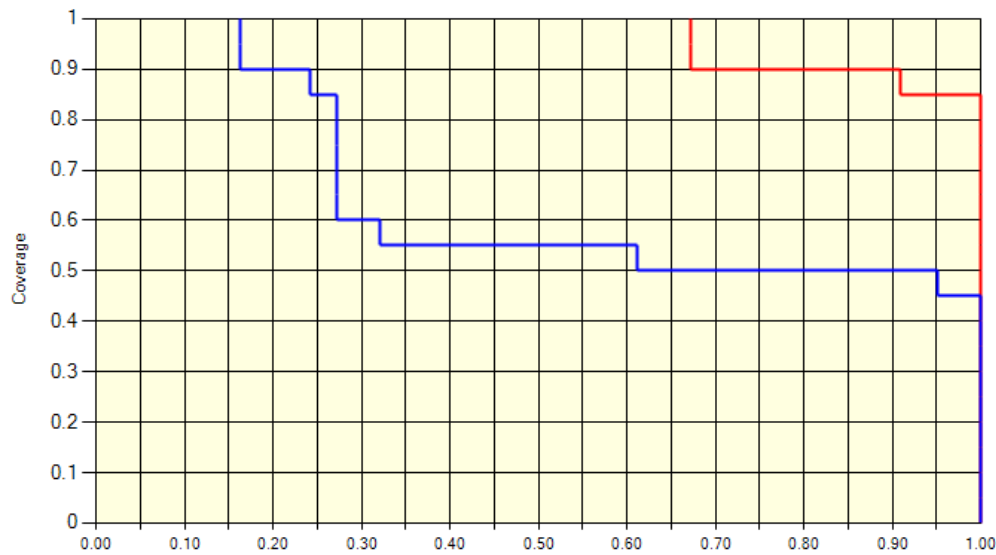
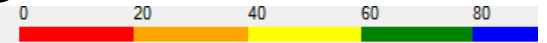
x, y, z are variable indices; color is coverage level.



What does this mean?

- Compared w/ 2-way, far fewer combinations with >80% coverage (blue), more with 60% .. 80% (green) than for 2-way
- Relatively few w/ <60% (red, orange, or yellow)
- One variable involved in low-coverage (orange) combinations, as seen by single line of markers
- Number of points = $C(7,3) = 35$

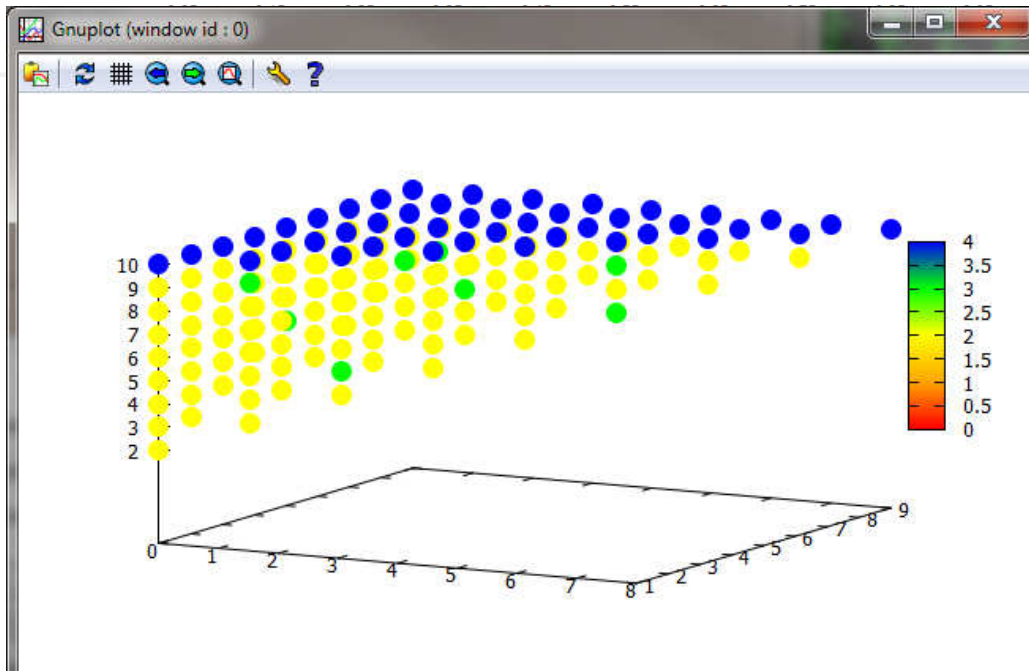
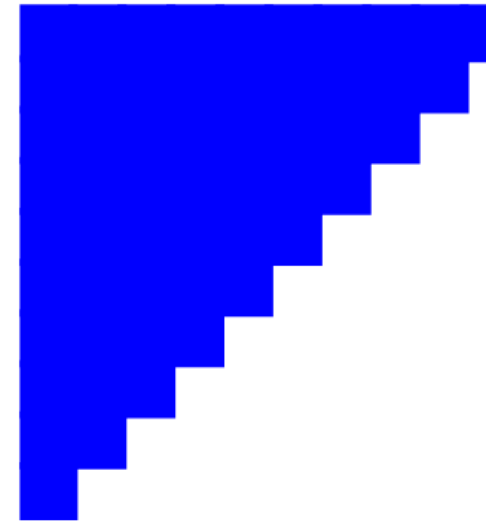
Random values, 0..3



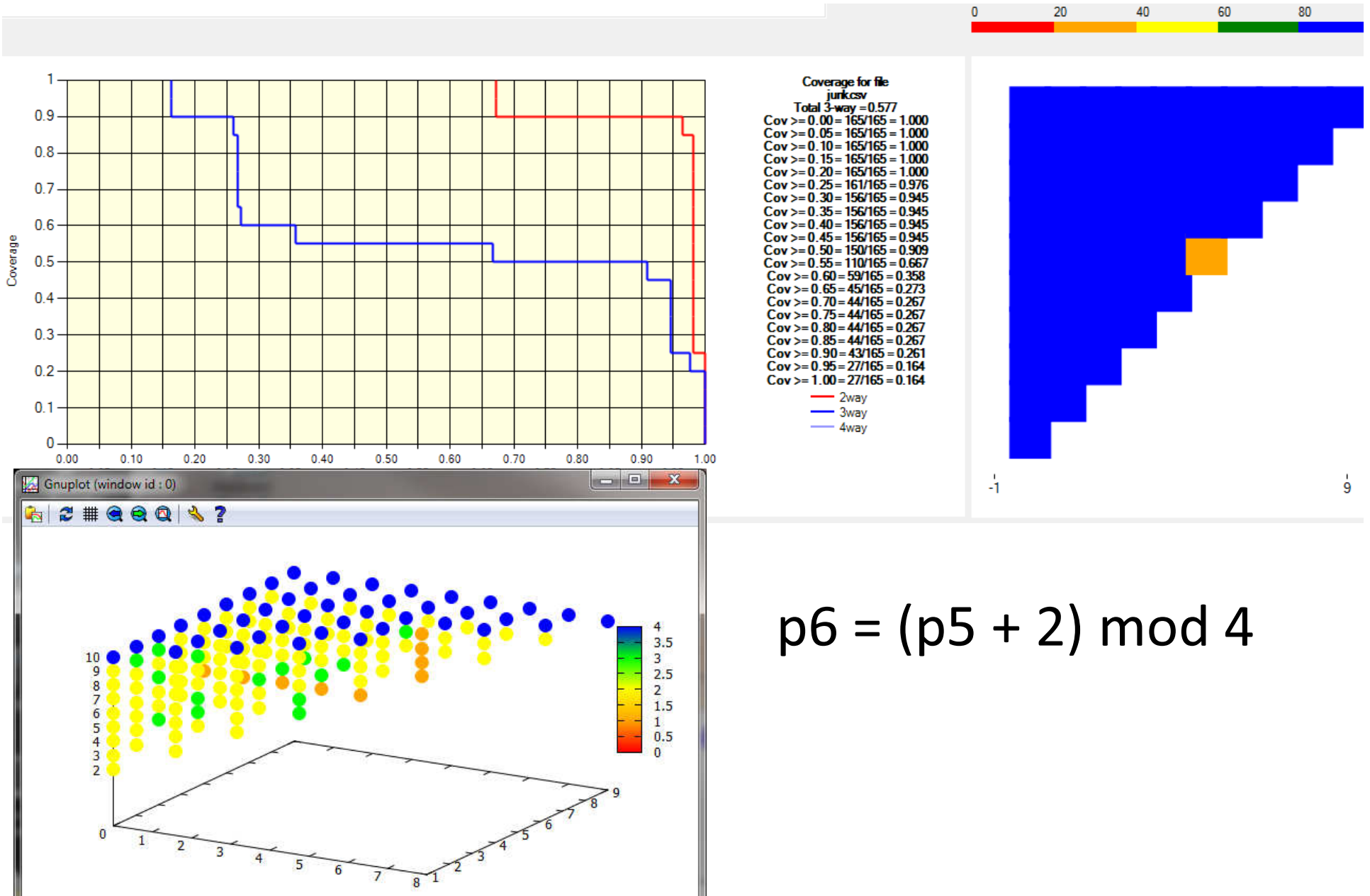
Coverage for file
rand.csv
Total 3-way = 0.586

Cov >= 0.00	= 165/165 = 1.000
Cov >= 0.05	= 165/165 = 1.000
Cov >= 0.10	= 165/165 = 1.000
Cov >= 0.15	= 165/165 = 1.000
Cov >= 0.20	= 165/165 = 1.000
Cov >= 0.25	= 165/165 = 1.000
Cov >= 0.30	= 165/165 = 1.000
Cov >= 0.35	= 165/165 = 1.000
Cov >= 0.40	= 165/165 = 1.000
Cov >= 0.45	= 165/165 = 1.000
Cov >= 0.50	= 157/165 = 0.952
Cov >= 0.55	= 101/165 = 0.612
Cov >= 0.60	= 53/165 = 0.321
Cov >= 0.65	= 45/165 = 0.273
Cov >= 0.70	= 45/165 = 0.273
Cov >= 0.75	= 45/165 = 0.273
Cov >= 0.80	= 45/165 = 0.273
Cov >= 0.85	= 45/165 = 0.273
Cov >= 0.90	= 40/165 = 0.242
Cov >= 0.95	= 27/165 = 0.164
Cov >= 1.00	= 27/165 = 0.164

— 2way
— 3way
— 4way



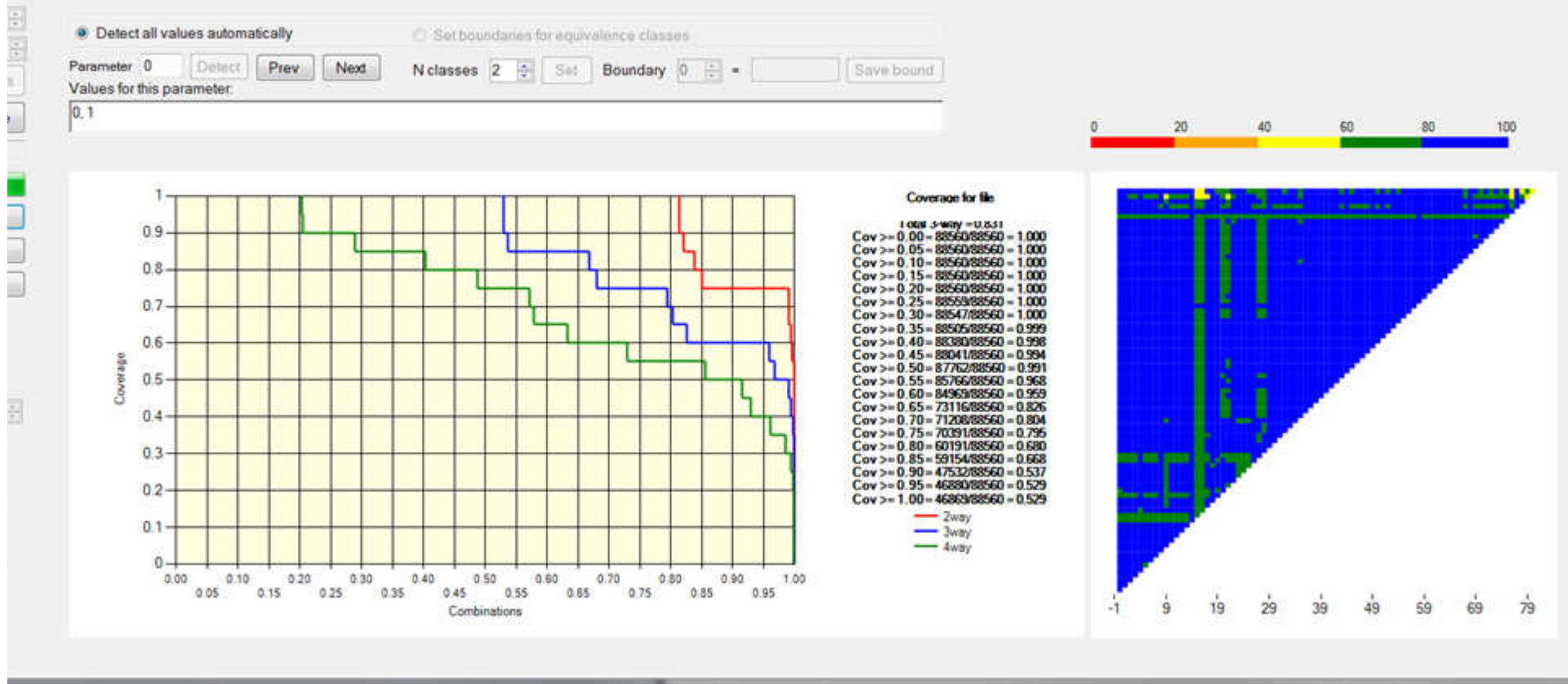
Same data, w/ one interaction



$$p6 = (p5 + 2) \bmod 4$$

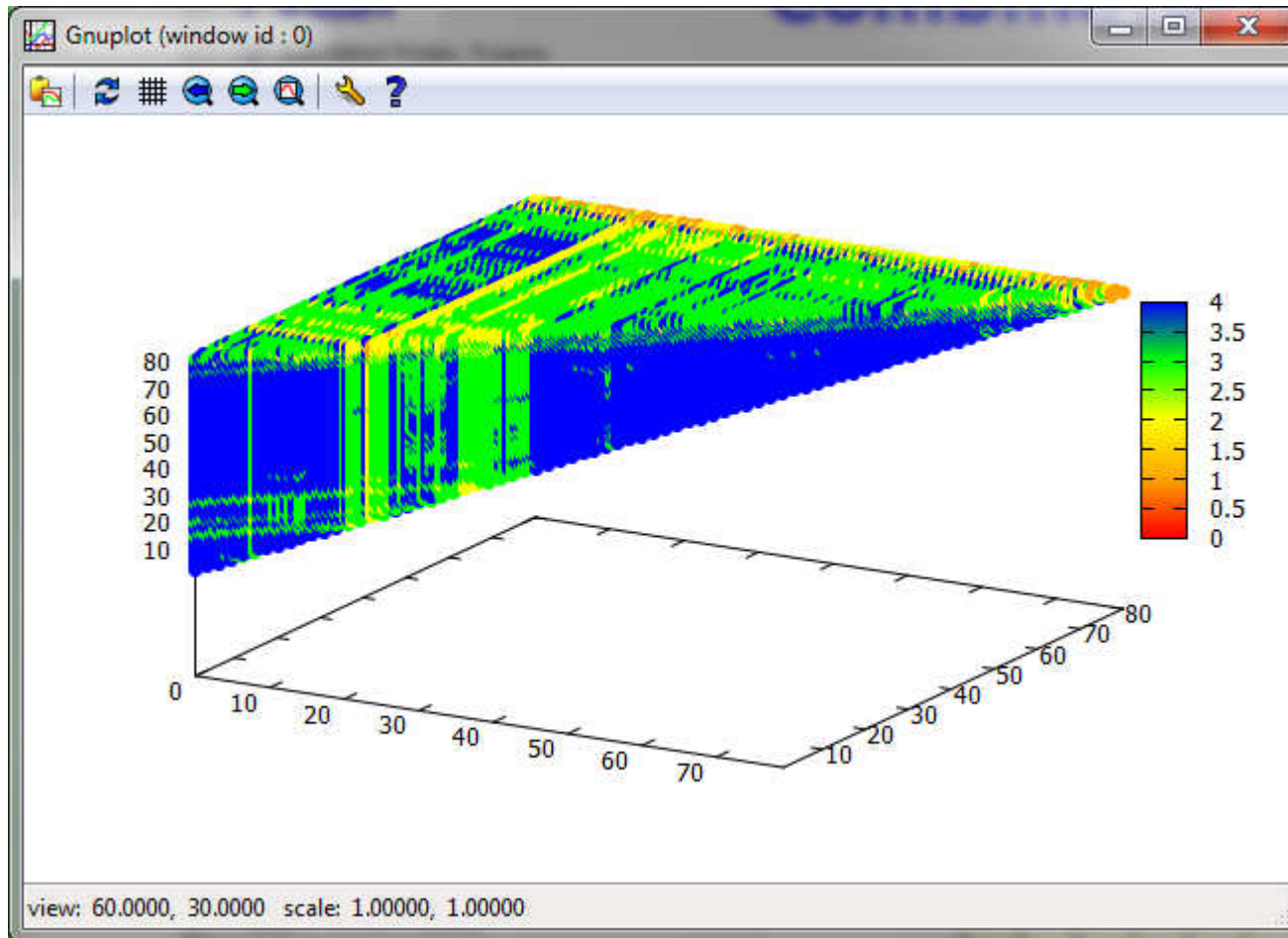
Spacecraft tests, 82 variables, mostly binary

Combinatorial Coverage Measurement



- Line graph shows 2-way (red), 3-way (blue), and 4-way (green) combination coverage.
- Heat map shows 2-way combination coverage; percentage coverage shown in color key above chart.

Heat map style graph of 3-way coverage



x, y, z are variable indices;
color is coverage level.

What does this mean?

- Compared w/ 2-way, far fewer combinations with >80% coverage (blue), more with 60% .. 80% (green)
- Relatively few w/ <60% (red, orange, or yellow)
- Small number of individual variables involved in low-coverage (orange) combinations
- Number of points = $C(82,3) = 82,560$

Summary

- Combinatorial coverage is an additional measurement that may be applied to system tests
 - applies to test data, rather than source code
 - may have utility for other data analysis?
- Has been applied to tests for NASA spacecraft
 - identify interactions that may not be tested sufficiently
 - can be used to automatically generate new tests to supplement coverage
- Part of overall combinatorial testing approach to software assurance
- Further information: Rick Kuhn - kuhn@nist.gov