



MATH AND SCIENCE @ WORK

AP* STATISTICS Educator Edition



THE INTEGRATED MEDICAL MODEL

Instructional Objectives

Students will

- apply binomial methods under proper conditions;
- use a random number generator to perform simulations;
- use simulations to construct empirical probability distributions; and
- use probability distributions to answer probability questions.

Degree of Difficulty

For the average student in AP Statistics, this problem is at a moderate difficulty level.

Class Time Required

This problem requires 55–70 minutes.

- Introduction: 5–10 minutes
 - Read and discuss the background section with the class before students work on the problem.
- Student Work Time: 40–50 minutes
- Post Discussion: 10 minutes

Background

This problem is part of a series of problems that apply Math and Science @ Work in NASA’s research facilities.

The National Aeronautics and Space Administration (NASA) has been placing humans in reduced gravity environments for over 50 years. Despite this extensive history, only now, are advances in technology and the acquisition of reliable, aggregate data enabling researchers and statisticians to fully evaluate the medical risks and dangers associated with short and long term exposure to reduced gravity.

As NASA is designing a new spacecraft capable of taking humans into deep space, and with the future advent of commercial spaceflight, a deeper and better understanding of medical risk has become even more vital for maintaining spaceflight safety and health for humans.

To address these critical issues, NASA’s Human Research Program (HRP) is addressing the medical hazards using a formalized risk management

Grade Level

11–12

Key Topic

Monte Carlo simulations

Degree of Difficulty

Moderate

Teacher Prep Time

10–15 minutes

Class Time Required

55–70 minutes

Technology

- TI-Nspire™ Learning Handhelds
- TI-Nspire document: *Medical_Model.tns*

AP Course Topics

Exploring Data:

- Constructing and interpreting graphical displays of distributions of univariate data

Anticipating Patterns:

- Probability

NCTM Standards

Data Analysis and Probability

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approach—to effectively identify and reduce acute and chronic medical risks to human spaceflight.



Figure 1: Artist concept of astronauts on the Mars surface

One of HRP's tools to assess the relative impact of medical events to astronauts and the mission is the Integrated Medical Model (IMM). The IMM is a simulation-based tool designed to quantify the probability of the medical risks and potential consequences that astronauts could experience during a mission.

The IMM incorporates “best evidence” with data from past missions, computer models, and comparable populations on Earth, to provide a quantifiable assessment of medical risk for a given mission scenario. The IMM also identifies medical resources, such as equipment and supplies, which are necessary for treating the medical conditions most likely to occur during the mission.

Using the Monte Carlo simulation technique (a random sampling of the data inputs as described by their statistical distribution), the IMM can forecast medical outcomes, helping to provide more appropriate medical support for flight crews. The medical risks addressed by the IMM range from minor conditions (such as headaches and nasal congestion) to more serious conditions (such as sudden cardiac arrest and kidney failure).

AP Course Topics

Exploring Data: Describing patterns and departures from patterns

- Constructing and interpreting graphical displays of distributions of univariate data (dotplot, stemplot, histogram, cumulative frequency plot)

Anticipating Patterns: Exploring random phenomena using probability and simulation

- Probability
 - Interpreting probability, including long-run relative frequency interpretation
 - Simulation of random behavior and probability distributions

NCTM Standards

Data Analysis and Probability

- Formulate questions that can be addressed with data, and collect, organize, and display relevant data to answer them
- Understand and apply basic concepts of probability



Problem and Solution Key (One Approach)

Students are given the following problem information within the TI-Nspire document, *Medical_Model.tns*, which should be distributed to their TI-Nspire Handhelds. They are provided with instructions on how to seed the random number generator in their handhelds prior to continuing with the activity below.

Due to the potential for ventilation issues and exposure to high concentrations of carbon dioxide on the International Space Station (ISS), a carbon dioxide (CO₂) headache is a common condition experienced by ISS crewmembers. In this activity, you will use the Monte Carlo technique to simulate and predict the probabilities of CO₂ headaches.

Assume the probability that a single crewmember experiences a CO₂ headache on a mission is 0.732.

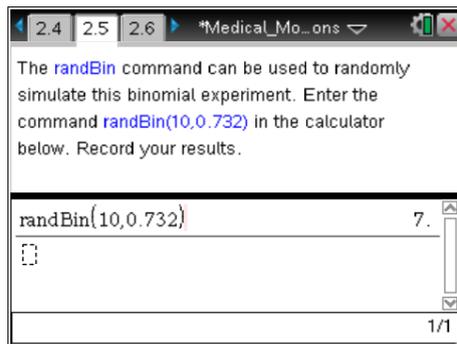
- A. Follow the instructions and answer the questions on pages 2.4–2.6 to estimate the number of crewmembers (from a 10–person crew) that will experience a headache on that mission.

2.4. What assumptions should be made in order to use a binomial approximation?

- *There is a fixed number of observations (crewmembers).*
- *Each trial has exactly two possible outcomes (headache or no headache).*
- *Outcomes of crewmembers experiencing headaches are independent.*
- *Each outcome of experiencing a headache has the same probability.*

- 2.5. The `randBin` command can be used to randomly simulate this binomial experiment. Enter the command `randBin(10,0.732)` in the calculator below. Record your results.

Answers will vary.

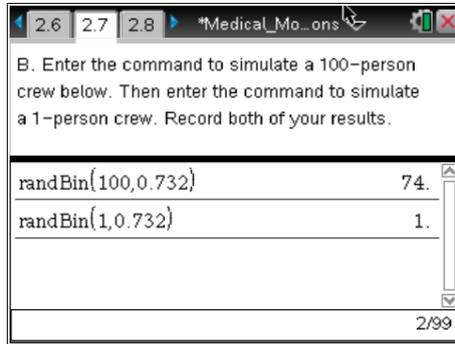


- 2.6. The result of your `randBin` command is the number of crewmembers in your simulation that experienced a headache on the mission. Compare with other students' results. Are the results the same? Why or why not?

No. Each student has a different random seed, and therefore, different results.

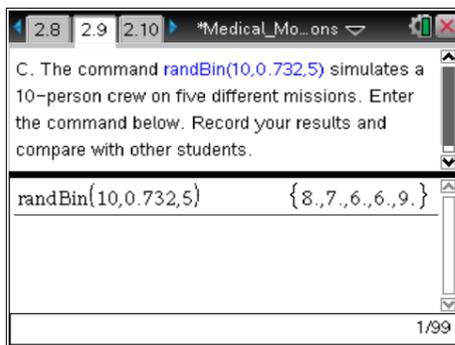
- B. Enter the command to simulate a 100–person crew below. Then enter the command to simulate a 1–person crew. Record both of your results.

Results will vary.



- C. The command **randBin(10,0.732,5)** simulates a 10-person crew on five different missions. Enter the command. Record your results and compare with other students.

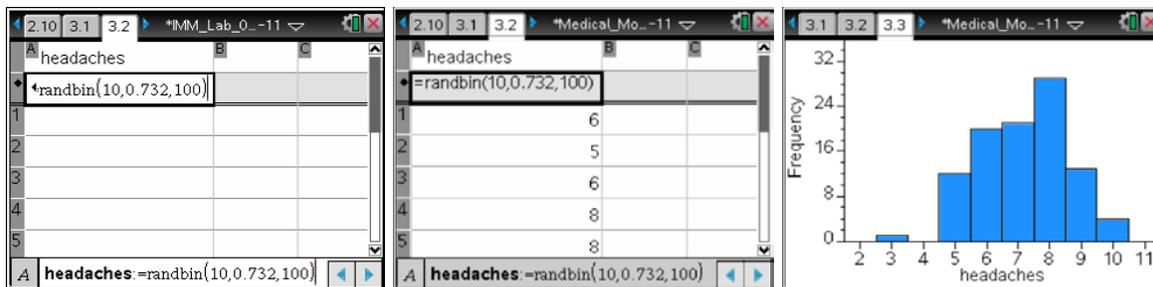
Answers will vary.



- 2.9. Discuss the meaning of the results.

The individual values in the list are the number of crewmembers with headaches on each of the five missions.

- D. Enter the command **randBin(10,0.732,100)** in the shaded formula entry cell of the column labeled “headaches” on page 3.2, then view the histogram of headaches created on page 3.3.



- 3.4. Use your histogram to estimate the probability that at least half the crew will experience headaches on a mission.

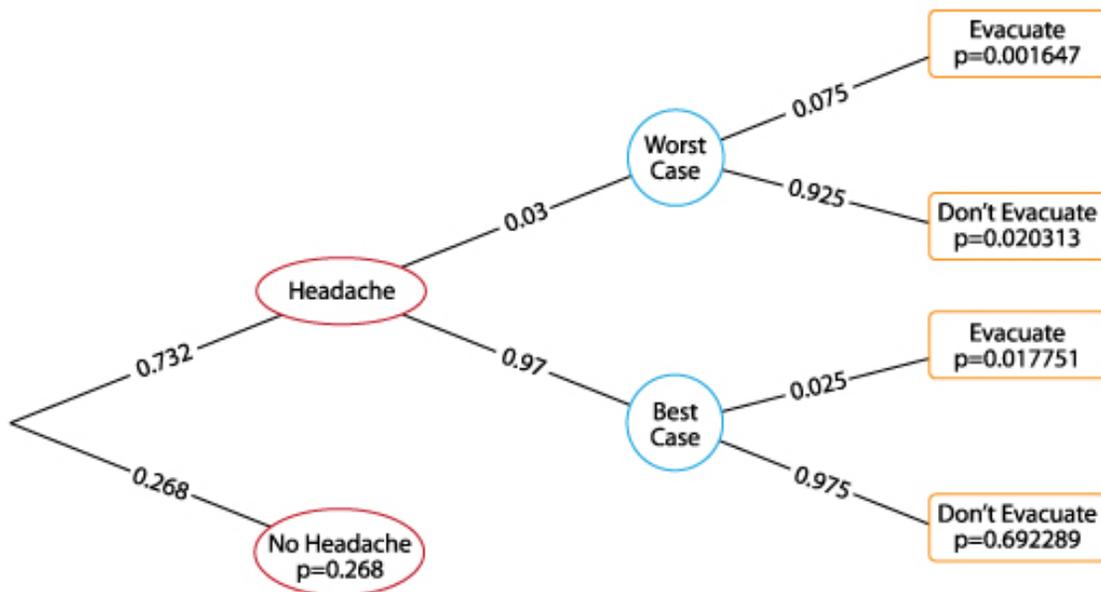
Results will vary. The histogram shown gives an estimate of 99% probability.



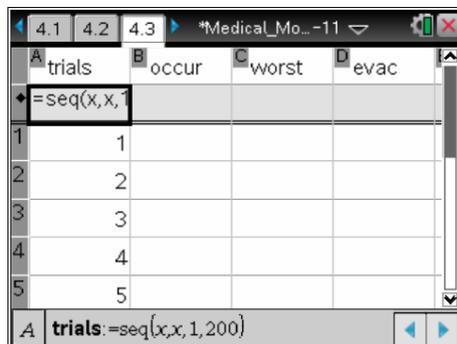
- E. It is possible for a headache to become severe enough that a crewmember would need to be evacuated. Assume (in the best case) that the probability a headache requires an evacuation is 0.025; and in the worst case, the probability a headache requires an evacuation is 0.075. Assume 3% of all headaches are worst case. What is the probability that an evacuation of a crewmember is needed?

A simulation can be used to estimate this probability. Enter the following commands into the spreadsheet on page 4.3 in order to complete the simulation.

Note to Educator: You may want to use the tree diagram shown here to help explain the flow of the equations as the students follow the steps below or ask the students to construct it.



- Enter the command **seq(x,x,1,200)** in the shaded formula cell of column A. This command simulates 200 crewmembers.



- Enter the command **randBin(1,0.732,200)** in the shaded formula cell of column B. This simulates whether or not each crewmember gets a headache. (1=headache, 0=no headache)



trials	occur	worst	evac
1	1	0	
2	2	1	
3	3	1	
4	4	1	
5	5	1	

- In cell C1, enter:
$$= \begin{cases} \text{randBin}(1,0.03), & b1 = 1 \\ 0, & b1 = 0 \end{cases}$$

This simulates whether each headache is worst case or not. (1=worst case, 0=best case)

Don't forget the "=" sign. The 2-piecewise function template can be found by pressing the **math template key**, located above the multiplication key.

- Use the fill feature in the data menu to copy cell C1 to the rest of column C. Do this by pressing **Menu** while the cursor is in cell C1. Select **Data > Fill**. Press the **down cursor arrow** until cell C200 is reached, and then press **Enter**.

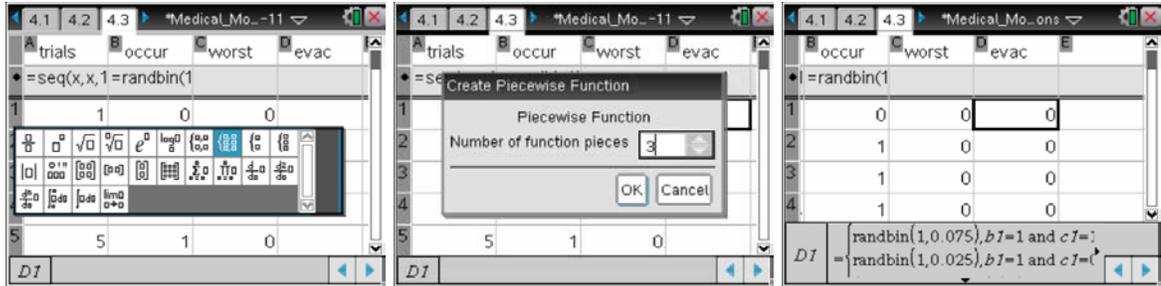
trials	occur	worst	evac
1	1	0	0
2	2	1	0
3	3	1	0
4	4	1	0

- In cell D1, enter:
$$= \begin{cases} \text{randBin}(1,0.075), & b1 = 1 \text{ and } c1 = 1 \\ \text{randBin}(1,0.025), & b1 = 1 \text{ and } c1 = 0 \\ 0, & b1 = 0 \end{cases}$$

This simulates whether each headache case requires evacuation (1=evacuation, 0=no evacuation).

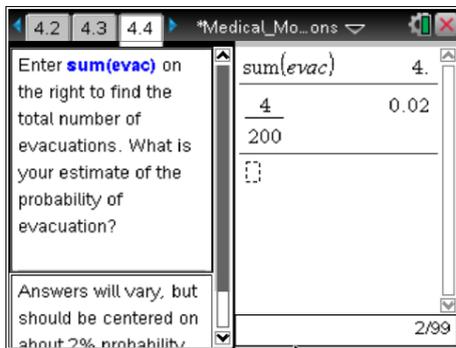
Don't forget the "=" sign before the template. The *n*-piecewise function can be found by pressing the **math template key**. Once you select the template, choose "3" as the number of function pieces.

- Use the fill feature in the data menu to copy cell D1 to the rest of column D.



4.4. Enter **sum(evac)** to find the total number of evacuations. What is your estimate of the probability of evacuation?

Answers will vary, but should be centered on about 2% probability.



F. What is the probability that out of 200 crewmembers, 5 or more will need to be evacuated?

To answer this question, combine the total number of evacuations found on page 4.4 with the results of the other students in the class.

Answers will vary. Students could combine their results in a histogram, and then find the relative percent of outcomes greater than or equal to 5.



Scoring Guide

Suggested 9 points total to be given.

Question	Distribution of points
A <i>2 points</i>	1 point for correct assumptions for binomial methods 1 point for correct random binomial command and correct comparison and explanation of student results
B <i>1 point</i>	1 point for correct random binomial commands
C <i>1 point</i>	1 point for correct execution of command and explanation of results
D <i>2 points</i>	1 point for correct histogram 1 point for correct use of histogram to approximate probability
E <i>2 points</i>	1 point for correct creation of spreadsheet 1 point for correct use of spreadsheet to estimate probability
F <i>1 point</i>	1 point for correct use of class results to estimate probability

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school AP Statistics instructors.

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References

Fitts, M. A.; Kerstman, E.; Butler, D. J.; Walton, M. E.; Minard, C. G.; Saile, L. G.; Toy, S.; Myers, J. "The Integrated Medical Model: Statistical Forecasting of Risks to Crew Health and Mission Success." Internet Archive. NASA Technical Documents. 4 Feb 2008. Retrieved 19 Sept 2011.

Myers, J; Lewandowski, B; Brooker, J; Weaver, S. "Assessing the Likelihood of Rare Medical Events in Astronauts." IEEE Xplore Digital Library. Reliability and Maintainability Symposium (RAMS), Lake Buena Vista, FL. 24-27 Jan 2011. Retrieved 19 Sept 2011.