



# MATH AND SCIENCE @ WORK

AP\* PHYSICS Educator Edition



## LUNAR LANDING

### Instructional Objectives

Students will

- choose a coordinate system best suited to the problem;
- apply equations of motion and force to solve for unknowns;
- determine magnitude and direction of vectors; and
- calculate a spring constant.

### Degree of Difficulty

This problem is a straightforward application of the equations of motion and force.

- For the average AP Physics student, the problem may be moderately difficult.

### Class Time Required

This problem requires 25-35 minutes.

- Introduction: 5 minutes
- Student Work Time: 15-20 minutes
- Post Discussion: 5-10 minutes

### Background

*This problem is part of a series of problems that apply math and science principles to human space exploration at NASA.*

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries, and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

A new era of NASA space exploration began on July 20, 1969 with the Apollo 11 mission. That year, the lunar module Eagle landed on the surface of the Moon (see Figure 1). Landing on the Moon was an important step towards future space exploration and it was one that was viewed by the world. It is estimated that one billion people watched Neil

**Grade Level**  
11-12

**Key Topic**  
Equations of Motion and Force

**Degree of Difficulty**  
Basic to Moderate

**Teacher Prep Time**  
5 minutes

**Class Time Required**  
25-35 minutes

**Technology**  
- TI-Nspire™ Learning Handhelds  
- TI-Nspire document: *Lunar\_Landing.tns*

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**AP Course Topics**  
Newtonian Mechanics:  
-Kinematics  
-Newton's Laws of Motion  
-Work, energy, power

**NSES  
Science Standards**  
- Physical Science  
- History and Nature of Science

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Armstrong's first step onto the surface of the Moon and heard him say the words, "That's one small step for man, one giant leap for mankind!"

Since that time, tremendous strides have been made in science and technology, with developments continuing to advance. Space exploration is on the forefront of this progress. NASA hopes to send humans on extended missions to the Moon and other planetary bodies. Technological advances will allow astronauts to build outposts, conduct new research, and learn to live on a surface different from Earth's. This will pave the way for eventual journeys to Mars and beyond.

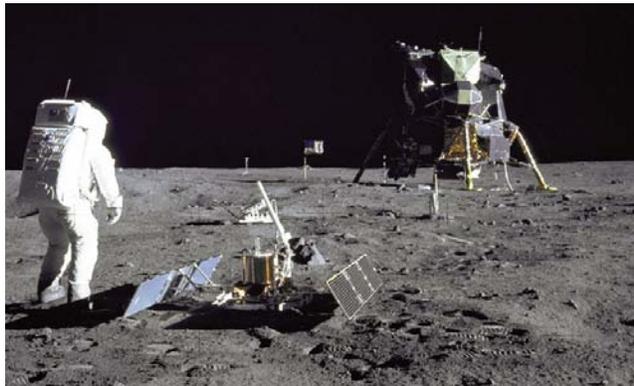


Figure 1: Apollo 11 lunar module Eagle



Figure 2: New lunar lander (NASA concept)

The human exploration missions to the Moon that took place during the Apollo program, and the robotic exploration missions of the Ranger, Surveyor, and Lunar Orbiter programs that preceded the Apollo astronauts, returned a wealth of information that helped develop a new scientific understanding of the Moon. After Apollo, additional scientific knowledge about the Moon was gained from the Clementine, Lunar Prospector, and SMART-1 missions - all robotic spacecraft that explored the Moon from lunar orbit. Based on the information gathered from all of these past missions, scientists, and engineers have proposed future landing sites that could provide clues to still unanswered questions about the Moon.

The proposed landing sites would bring us to locations where potential resources may be located. Hydrogen might be found in the form of water ice in permanently shadowed craters near the lunar polar region. Another chemical element that may be found in some of the potential landing sites is oxygen, probably bound in the crystalline minerals of the lunar soil.

Although it's been done before, landing spacecraft on the lunar surface is not easy. The Moon's gravitational force is 0.165 times the surface gravity on Earth. This difference in surface gravity affects the amount of thrust, or opposing gravitational force, needed to land. Another critical decision for landing on the lunar surface is the landing location. Landing safely means avoiding rocks, holes, or slopes large enough to damage the spacecraft. This is called hazard avoidance. Selecting landing sites that will be advantageous and interesting for science and engineering is also important.

On Apollo missions, the crew looked out the window and visually picked a safe landing location. NASA's latest development projects, with hazard avoidance technology, will enable astronauts to safely land and explore the surface of the Moon. On unpiloted missions, the spacecraft will have systems on board to automatically find the safe areas and land the spacecraft (see Figure 2).

Landing safely and learning to live on the Moon or other planetary bodies will give NASA a head start in exploring Mars and other destinations in the solar system.



## AP Course Topics

### Newtonian Mechanics

- Kinematics:
  - Motion in one dimension
- Newton's Laws of Motion:
  - Dynamics of a single particle (second law)
- Work, energy, power:
  - Forces and potential energy

## NSES Science Standards

### Physical Science

- Motion and Forces

### History and Nature of Science

- Science as a Human Endeavour
- Historical Perspectives

## Problem and Solution Key (One Approach)

Students are given the following problem information within the TI-Nspire document, *Lunar\_Landing.tns*. The questions are embedded within the TI-Nspire document.

During a lunar landing, the spacecraft is 100 meters (m) above the lunar surface and comes straight down to a safe landing location with a constant thrust. The velocity of the spacecraft is 10 meters per second (m/s) at this point. The lander should touch down on the surface at 0.5 m/s due to the constant thrust (see Figure 3, also found on TI-Nspire page 1.3).

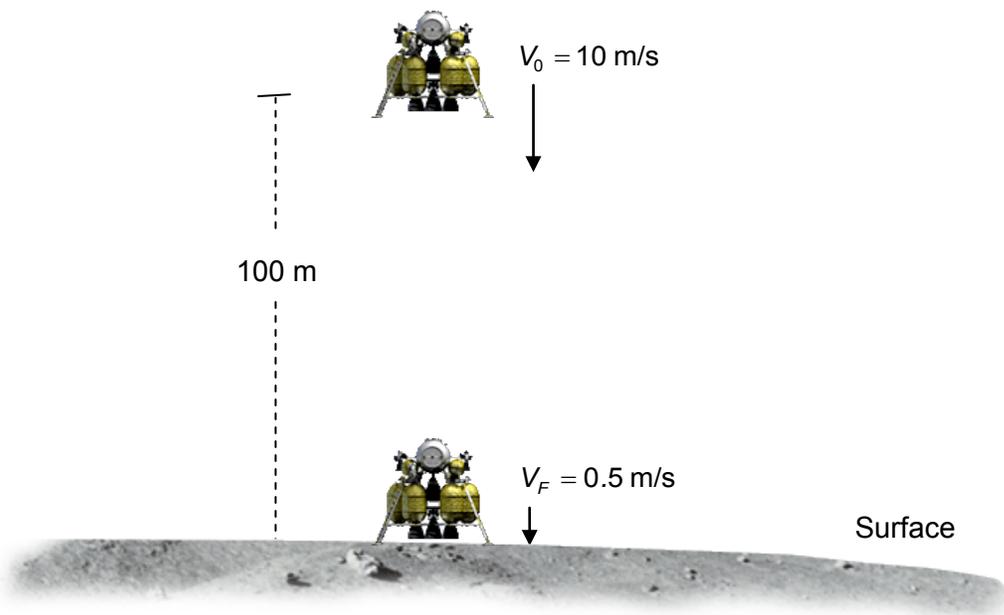


Figure 3: Problem diagram



- A. What is the magnitude and direction of the acceleration of the spacecraft? Express your answer in  $\text{m/s}^2$ .

Since the force (thrust) is constant, the acceleration is constant; so we can use the equations for constant acceleration. Place the origin of our measurement system at the point of impact with up being positive and down being negative.

$$v_f^2 - v_i^2 = 2 \cdot a \cdot s$$

$$v_f^2 - v_i^2 = 2 \cdot a \cdot s$$

$$\frac{v_f^2 - v_i^2}{2 \cdot s} = a$$

$$\frac{v_f^2 - v_i^2}{2 \cdot s} = a$$

A. What is the magnitude and direction of the acceleration of the spacecraft? Express your answer in  $\text{m/s}^2$ .

$$a = \frac{\left(\frac{-0.5 \cdot \text{m}}{\text{s}}\right)^2 - \left(\frac{-10 \cdot \text{m}}{\text{s}}\right)^2}{2 \cdot -100 \cdot \text{m}}$$

$$a = \frac{0.499 \cdot \text{m}}{\text{s}^2}$$

© Acceleration is positive and thus up.

A. What is the magnitude and direction of the acceleration of the spacecraft? Express your answer in  $\text{m/s}^2$ .

Note that the displacement,  $s$ , is really the final position minus the original position.  $0 \text{ m} - 100 \text{ m} = -100 \text{ m}$ , symbolizing the vehicle traveling 100 m down.

- B. How long does it take to reach the surface from 100 m?

$$v_f = v_i + a \cdot t$$

$$v_f = a \cdot t + v_i$$

$$(v_f = a \cdot t + v_i) - v_i$$

$$v_f - v_i = a \cdot t$$

$$\frac{v_f - v_i}{a} = t$$

$$\frac{-0.5 \cdot \text{m} \quad -10 \cdot \text{m}}{0.499 \cdot \text{m/s}^2} = t = 19.038 \cdot \text{s}$$

B. How long does it take to reach the surface from 100 m?

$$t = \frac{\frac{-0.5 \cdot \text{m}}{\text{s}} - \frac{-10 \cdot \text{m}}{\text{s}}}{\frac{0.499 \cdot \text{m}}{\text{s}^2}}$$

$$t = 19.038 \cdot \text{s}$$

B. How long does it take to reach the surface from 100 m?

- C. If the spacecraft has a mass of 20,000 kilograms (kg), what thrust in newtons (N) must the engines be exerting? Ignore the change in mass of the spacecraft due to propellant use. Include a force diagram with your solution.

The spacecraft has two forces working on it. The thrust ( $T$ ), which we are trying to find and the force caused by the Moon's gravitational pull ( $m g_m$ ). The total force ( $F$ ) on the spacecraft is then  $T - m g_m$ . This takes into account the direction of  $g_m$ .



©  $F = m \cdot a$ , &  $F = T - m \cdot g$ , so  
 $T - m \cdot g = m \cdot a$ , or  
 $T = m \cdot a + m \cdot g$ , or  
 $T = m(a + g)$   
 $t = 20000 \cdot (0.499 + 0.165 \cdot 9.8)$   
 $t = 42320.000$

D. In the NASA concept shown on page 1.1, the lander has four legs, each with a plate on the end that will set firmly on the surface. Each leg has a spring to act as a shock absorber and each shock will compress 30 centimeters (cm) when the leg plates touch the surface at a velocity of 0.5 m/s. Assume the engines' thrust goes instantly to zero at the time of contact with the surface and the lunar surface does not compress.

I. What is the average acceleration of the vehicle after it touches the surface?

$$a = \frac{v_f^2 - v_i^2}{2 \cdot s}$$

$$a = \frac{(0 \cdot \frac{m}{s})^2 - (-0.5 \cdot \frac{m}{s})^2}{2 \cdot 0.3 \cdot m}$$

$$a = \frac{0.417 \cdot m}{s^2}$$

i) What is the average acceleration of the vehicle after it touches the surface?

II. What is the spring constant for each spring?

© First find the force on each leg

$$f_4 = 20000 \cdot 0.417$$

$$f_4 = 8340.000$$

$$f_1 = \frac{8340}{4}$$

$$f_1 = 2085$$

© Find the spring constant for each spring

ii) What is the spring constant for each spring?

© Find the spring constant for each spring

$$f = -kx$$

$$\frac{f}{x} = -k$$

$$k = \frac{-2085}{-0.3}$$

$$k = 6950.000$$

ii) What is the spring constant for each spring?

**Scoring Guide**

Suggested 12 points total to be given.

| Question                 | Distribution of points  |
|--------------------------|---|
| <b>A</b> <i>3 points</i> | 1 point for use of constant acceleration equation<br>1 point for correct substitution and process used to find the magnitude of acceleration<br>1 point for identifying correct direction of acceleration   |
| <b>B</b> <i>2 points</i> | 1 point for use of correct equation<br>1 point for correct substitution and process used to find the time   |
| <b>C</b> <i>3 points</i> | 1 point for correct force diagram that identifies both forces (thrust and the force from the Moon's gravitational pull)<br>1 point for using correct equation to find total force<br>1 point for correct substitution and process used to find total force                            |
| <b>D</b> <i>4 points</i> | 1 point for correct equation to find average acceleration<br>1 point for correct substitution and process used to find average acceleration<br>1 point for correct process used to find force on each leg<br>1 point for correct process used to find spring constant for each spring |



## Contributors

Thanks to the subject matter experts for their contributions in developing this problem:

### **NASA Experts**

*NASA Johnson Space Center*

Dr. Chiold D. Epp  
Assistant for Exploration  
Aeroscience and Flight Mechanics Division  
Engineering Directorate

John Gruener  
Flight Systems Engineer  
Lunar Surface System Project Office  
Constellation Program

### **Problem Development**

*NASA Langley Research Center*

Chris Giersch  
Communications and Education Lead  
Exploration and Flight Projects Directorate

*NASA Johnson Space Center*

Natalee Lloyd  
Educator, Secondary Mathematics  
Human Research Program Education and Outreach

Monica Trevathan  
Education Specialist  
Human Research Program Education and Outreach

Traci Knight  
Graphics Specialist  
Human Research Program Education and Outreach

*National Institute of Aerospace*

Norman “Storm” Robinson, III  
Education Specialist

*Texas Instruments*

Todd Morstein  
Texas Instruments T<sup>3</sup> (Teachers Teaching with Technology™) National Instructor