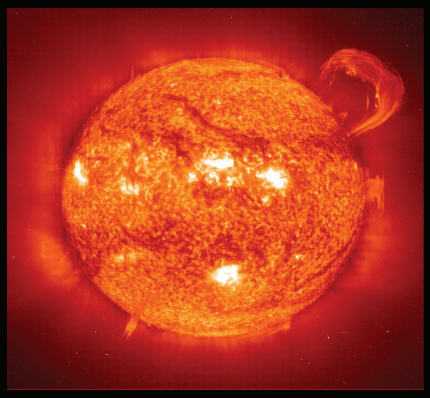
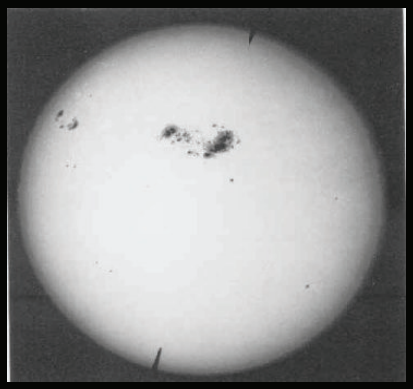
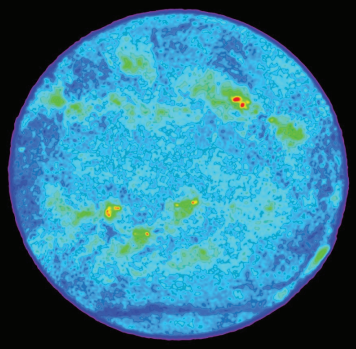
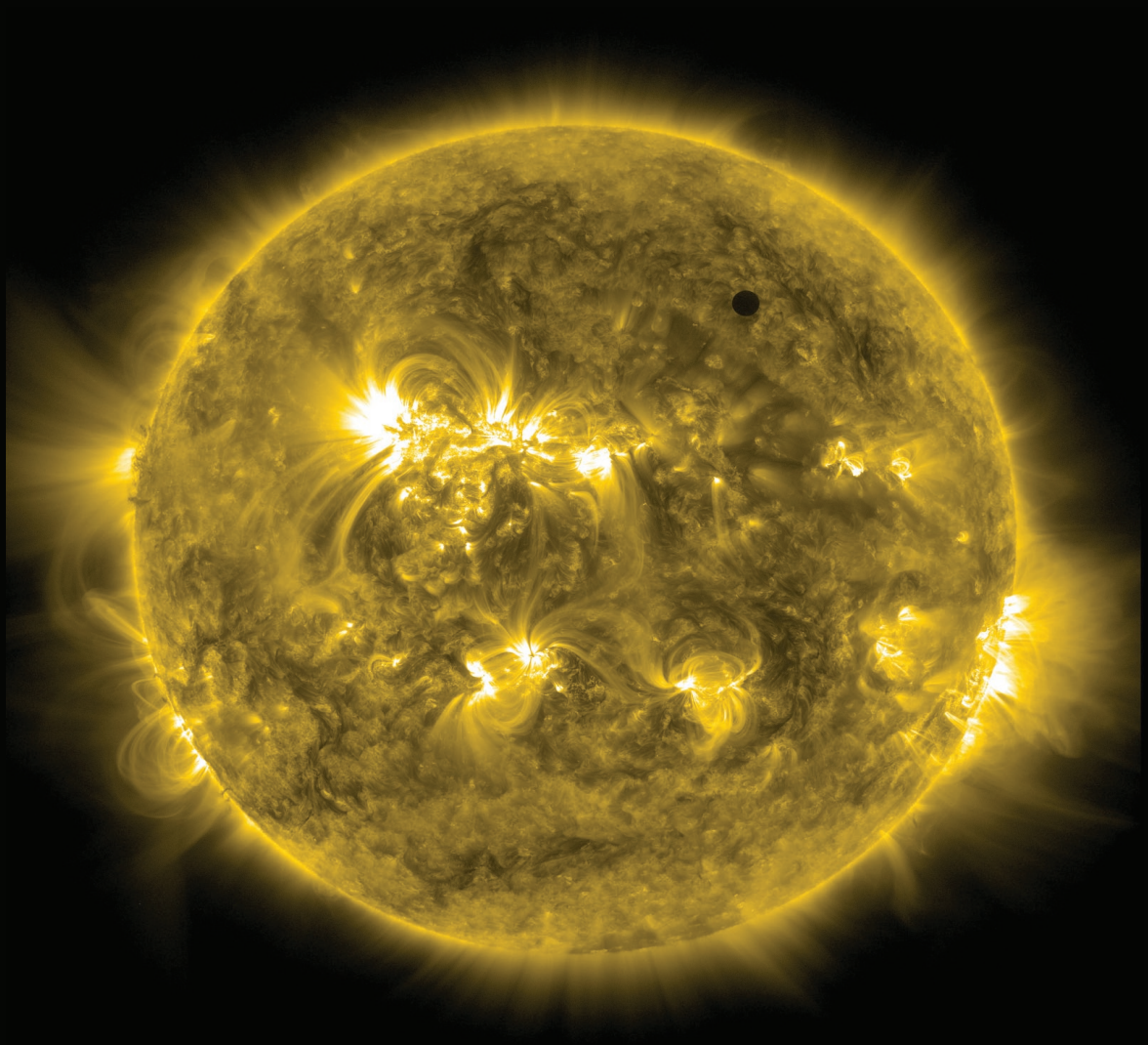
National Aeronautics and Space Administration









Solar Math

This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2005-2012 school years. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 5 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

This book is an expanded edition of the previous Hinode math solar problem book, with many new problems added to the original collection that were created by *SpaceMath@NASA* since 2009

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

Front and back cover credits: Front) Main image: SDO Solar Image of Transit of Venus, June 6, 2012; Small images, top left to right - SOHO/EIT Helium 304A image; Optical image KPNO; 20cm Radio Image, NRAO/VLA Back) Hinode soft x-ray imagery (Hinode/JAXA/NRL)

This booklet was created through an education grant NNH06ZDA001N-EPO from NASA's Science Mission Directorate.

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Mathematics Topic Matrix

Topic	Problem Numbers																														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry											X	X																			
Technology, rulers		X	X	X	X	X	X	X	X	X																				X	X
Numbers, patterns, percentages											X	X	X	X	X	X	X	X					X	X	X		X				
Averages															X	X	X				X	X				X	X			X	
Time, distance, speed																															
Areas and volumes														X					X	X											
Scale drawings		X	X	X	X	X	X	X	X	X									X											X	X
Geometry									X																						
Scientific Notation																															
Unit Conversions																			X												
Fractions	X																														
Graph or Table Analysis												X	X	X	X	X	X			X	X	X	X	X	X	X	X	X	X		
Solving for X																															
Evaluating Fns																															
Modeling																															
Probability																							X	X							
Rates/Slopes																															
Logarithmic Fns																															
Polynomials																															
Power Fns																															
Conics																															
Piecewise Fns																															
Trigonometry																															
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																														
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
Inquiry									X																						
Technology, rulers	X	X						X		X	X	X	X	X															X		X
Numbers, patterns, percentages			X	X											X			X		X		X	X	X		X			X	X	
Averages																X		X		X							X	X			
Time, distance, speed			X		X			X	X	X	X	X	X	X		X	X	X													X
Areas and volumes		X																													
Scale drawings	X	X		X						X	X	X	X	X															X		X
Geometry		X			X														X												
Scientific Notation													X																	X	
Unit Conversions																															
Fractions																															
Graph or Table Analysis						X	X								X		X	X		X		X	X	X	X	X	X	X	X		
Solving for X																															
Evaluating Fns													X														X	X			
Modeling																															
Probability																					X				X						
Rates/Slopes			X				X																							X	
Logarithmic Fns																															
Polynomials																															X
Power Fns																															
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Trigonometry																															
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																														
	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
Inquiry							X													X											
Technology, rulers																															
Numbers, patterns, percentages	X																														
Averages																															
Time, distance ,speed																															X
Areas and volumes					X				X	X														X							
Scale drawings		X	X	X																				X							
Geometry		X	X	X		X															X			X		X	X				X
Scientific Notation					X				X	X		X	X	X	X								X	X							
Unit Conversions														X																	
Fractions																															
Graph or Table Analysis	X										X				X					X			X		X	X	X	X	X		X
Solving for X																															
Evaluating Fns						X		X				X	X	X	X							X	X							X	
Modeling																				X											X
Probability																X	X	X	X												
Rates/Slopes														X																	
Logarithmic Fns											X																				
Polynomials																														X	
Power Fns																															
Exponential Fns																															
Conics																															
Piecewise Fns																															
Trigonometry																					X				X	X	X				
Matrices																													X		
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																														
	94	95	96	97	98	99																									
Inquiry		X																													
Technology, rulers		X				X																									
Numbers, patterns, percentages																															
Averages																															
Time, distance ,speed	X		X			X																									
Areas and volumes																															
Scale drawings		X				X																									
Geometry																															
Scientific Notation	X				X																										
Unit Conversions																															
Fractions																															
Graph or Table Analysis	X			X	X																										
Solving for X																															
Evaluating Fns	X		X																												
Modeling																															
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Integration																															
Differentiation																															
Limits																															

How to use this book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as *"access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information."* 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for all of the Space Math applications.

Solar Math is one in a series of booklets developed by Space Math @ NASA, designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math, science and, as in this scenario, technology classroom, through a more intricate method for using **Solar Math**. Read the scenario that follows:

Ms. Black teaches a class about the use of technology. She integrates math and science in her class; she was excited when she saw the possibilities in the **Solar Math** book. She wanted her students to learn about the technology that is used on a single spacecraft to research space, in this case the Sun. She challenged each student team with math problems from the **Solar Math** book. The students were to use the facts available in the **Solar Math** book to develop an observing the Sun bulletin board display. What we can learn through the use of Technology!

NASA's YouTube gives some additional information that students can use, http://www.youtube.com/watch?v=DSfAI6_7bjU

Solar Math can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

Alignment with Standards

AAAS: Project:2061 Benchmarks

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1 **(6-8)** Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1 **(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

NCTM: Principles and Standards for School Mathematics

Grades 6–8 :

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; .
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- use graphs to analyze the nature of changes in quantities in linear relationships.
- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system.

Grades 9–12 :

- judge the reasonableness of numerical computations and their results.
- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- draw reasonable conclusions about a situation being modeled.

Teacher Comments

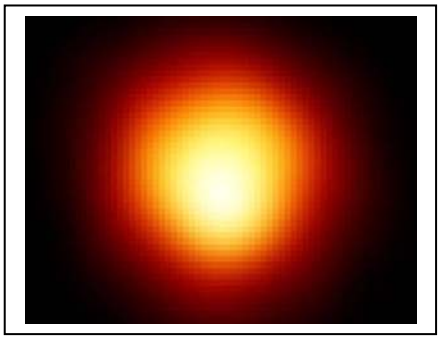
"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than "Consider a particle of mass 'm' and speed 'v' that…" (Associate Professor of Physics)

The relative sizes of the sun and stars



Stars come in many sizes, but their true appearances are impossible to see without special telescopes. The image to the left was taken by the Hubble Space telescope and resolves the red supergiant star Betelgeuse so that its surface can be just barely seen. Follow the number clues below to compare the sizes of some other familiar stars!

Problem 1 - The sun's diameter if 10 times the diameter of Jupiter. If Jupiter is 11 times larger than Earth, how much larger than Earth is the Sun?

Problem 2 - Capella is three times larger than Regulus, and Regulus is twice as large as Sirius. How much larger is Capella than Sirius?

Problem 3 - Vega is 3/2 the size of Sirius, and Sirius is 1/12 the size of Polaris. How much larger is Polaris than Vega?

Problem 4 - Nunki is 1/10 the size of Rigel, and Rigel is 1/5 the size of Deneb. How large is Nunki compared to Deneb?

Problem 5 - Deneb is 1/8 the size of VY Canis Majoris, and VY Canis Majoris is 504 times the size of Regulus. How large is Deneb compared to Regulus?

Problem 6 - Aldebaran is 3 times the size of Capella, and Capella is twice the size of Polaris. How large is Aldebaran compared to Polaris?

Problem 7 - Antares is half the size of Mu Cephi. If Mu Cephi is 28 times as large as Rigel, and Rigel is 50 times as large as Alpha Centauri, how large is Antares compared to Alpha Centauri?

Problem 8 - The Sun is 1/4 the diameter of Regulus. How large is VY Canis Majoris compared to the Sun?

Inquiry: - Can you use the information and answers above to create a scale model drawing of the relative sizes of these stars compared to our Sun.

Answer Key

The relative sizes of some popular stars is given below, with the diameter of the sun = 1 and this corresponds to an actual physical diameter of 1.4 million kilometers.

Betelgeuse	440	Nunki	5	VY CMa	2016	Delta Bootis	11
Regulus	4	Alpha Cen	1	Rigel	50	Schedar	24
Sirius	2	Antares	700	Aldebaran	36	Capella	12
Vega	3	Mu Cephi	1400	Polaris	24	Deneb	252

Problem 1 - Sun/Jupiter = 10, Jupiter/Earth = 11 so Sun/Earth = 10 x 11 = **110 times.**

Problem 2 - Capella/ Regulus = 3.0, Regulus/Sirius = 2.0 so Capella/Sirius = 3 x 2 = **6 times.**

Problem 3 - Vega/Sirius = 3/2 Sirius/Polaris=1/12 so Vega/Polaris = 3/2 x 1/12 = **1/8 times**

Problem 4 - Nunki/Rigel = 1/10 Rigel/Deneb = 1/5 so Nunki/Deneb = 1/10 x 1/5 = **1/50.**

Problem 5 - Deneb/VY = 1/8 and VY/Regulus = 504 so Deneb/Regulus = 1/8 x 504 = **63 times**

Problem 6 - Aldebaran/Capella = 3 Capella/Polaris = 2 so Aldebaran/Polaris = 3 x 2 **= 6 times**.

Problem 7 - Antares/Mu Cep = 1/2 Mu Cep/Rigel = 28 Rigel/Alpha Can = 50, then Antares/Alpha Can = 1/2 x 28 x 50 = **700 times**.

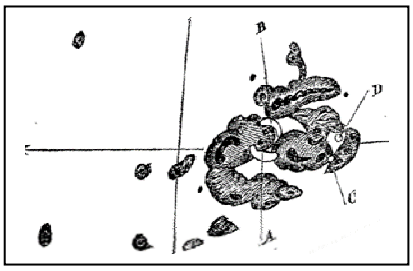
Problem 8 - Regulus/Sun = 4 but VY CMA/Regulus = 504 so VY Canis Majoris/Sun = 504 x 4 = **2016 times the sun's size!**

Inquiry: Students will use a compass and millimeter scale. If the diameter of the Sun is 1 millimeter, the diameter of the largest star VY Canis Majoris will be 2016 millimeters or about 2 meters!

Sunspots have been observed for thousands of years because, from time to time, the sun produces spots that are so large they can be seen from Earth with the naked eye…with the proper protection. Ancient observers would look at the sun near sunrise or sunset when Earth's atmosphere provided enough shielding to very briefly look at the sun for a few minutes. Astronomers keep track of these large 'super spots' because they often produce violent solar storms as their magnetic fields become tangled up into complex shapes.

Below are sketches and photographs of some large sunspots that have been observed during the last 150 years. They have been reproduced at scales that make it easy to study their details, but do not show how big they are compared to each other.

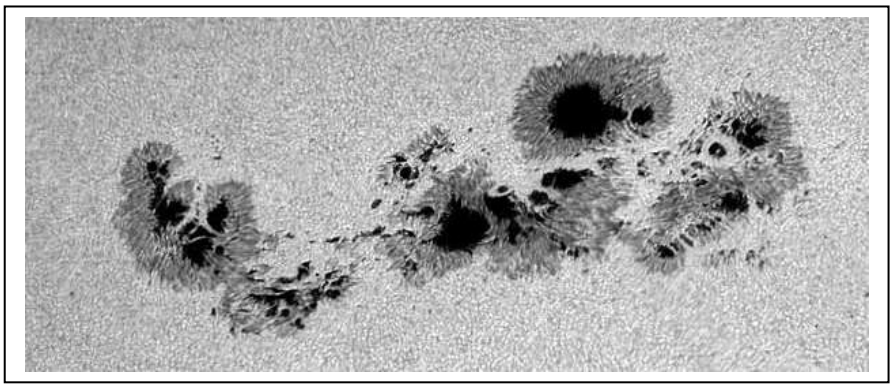
Problem: By using a millimeter ruler, use the indicated scales for each image to compute the physical sizes of the three sunspots in kilometers. Can you sort them by their true physical size?

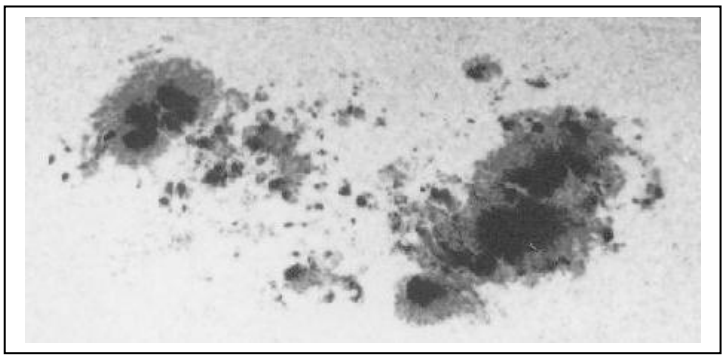


Top is the sunspot drawn by Richard Carrington on August 28, 1859 at a scale of 5,700 kilometers/mm.

Middle is a photograph of a sunspot seen on March 29, 2000 at a scale of 23,500 kilometers/cm.

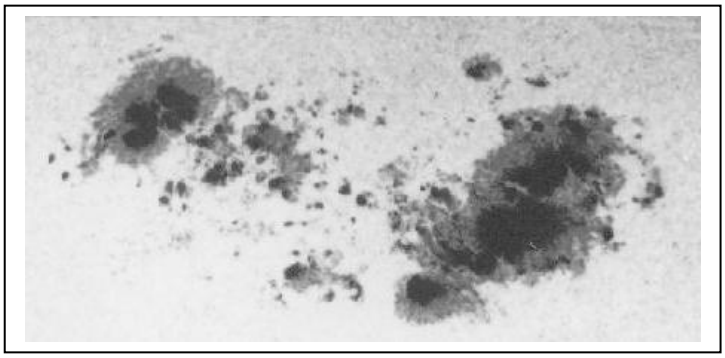
Bottom is a sunspot seen on April 8, 1947 at a scale of 100,000 kilometers/inch



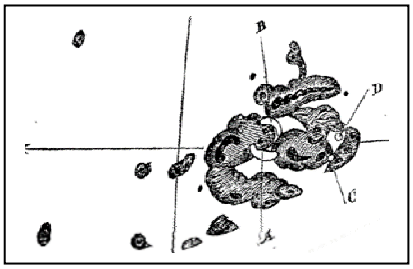


Answer Key:

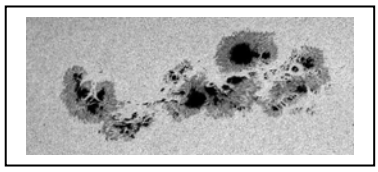
Images ordered from largest to smallest and to scale:



Sunspot seen on April 8, 1947 reproduced at a scale of 100,000 kilometers/inch. The linear extent on the page is 7 centimeters, so the length in inches is 7 / 2.5 = 2.8 inches. The true length is then 2.8 x 100,000 = <u>280,000 kilometers.</u>

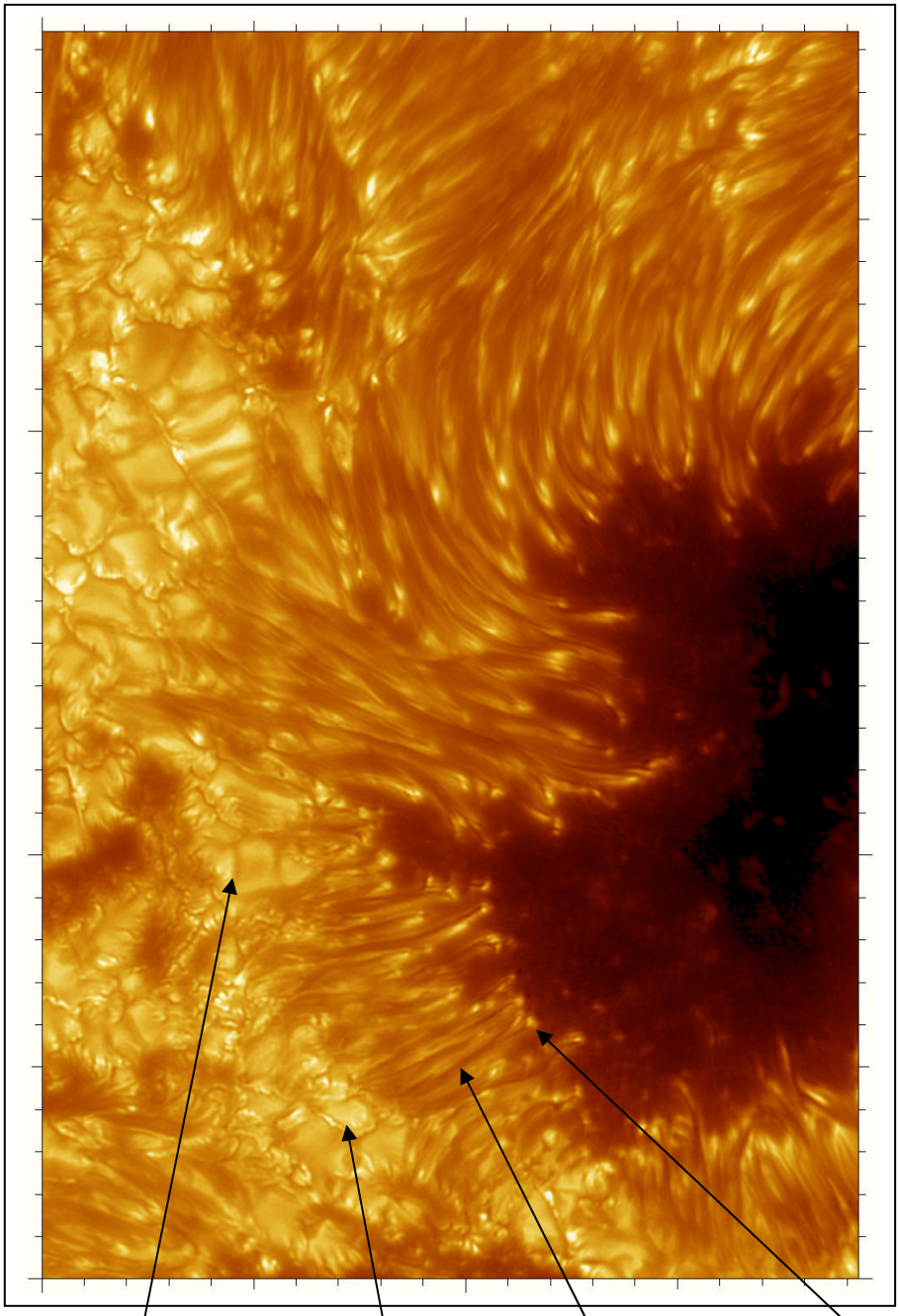


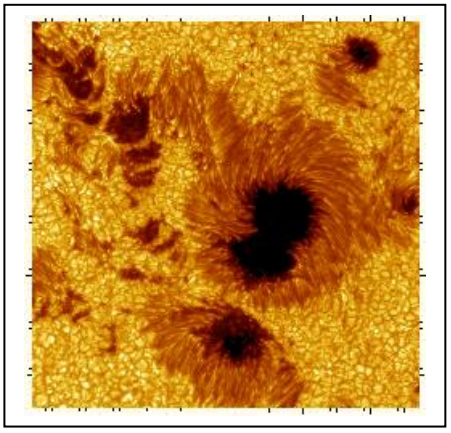
The sunspot drawn by Richard Carrington on August 28, 1859 at a scale of 5,700 kilometers/mm. With a ruler, the distance from the left to the right of the group is about 40 millimeters, so the true length is about 40 x 5,700 = <u>228,000 kilometers.</u>



A photograph of a sunspot seen on March 29, 2000 at a scale of 23,500 kilometers/cm. The length of the spot is 90 millimeters or 9 centimeters. The true length is then 211,500 kilometers.

The sun is our nearest star. From Earth we can see its surface in great detail. The images below were taken with the 1-meter Swedish Vacuum Telescope on the island of La Palma, by astronomers at the Royal Swedish Academy of Sciences. The image to the right is a view of sunspots on July 15, 2002. The enlarged view to the left shows never-before-seen details near the edge of the largest spot. Use a millimeter ruler, and the fact that the dimensions of the left image are 19,300 km x 29,500 km, to determine the scale of the photograph, and then answer the questions. See the arrows below to identify the various solar features mentioned in the questions.





Problem 1 - What is the scale of the image in km/mm?

Problem 2 – What is the smallest feature you can see in the image?

Problem 3 – What is the average size of a solar granulation region?

Problem 4 – How long and wide are the Dark Filaments?

Problem 5 – How large are the Bright Spots?

Problem 6 – Draw a circle centered on this picture that is the size of Earth (radius = 6,378 km). How big are the features you measured compared to familiar Earth features?

Granulation Boundary

Solar Granulation

Dark Filament

Bright Spot

Problem 1 - What is the scale of the image in km/mm? **Answer:** the image is about 108mm x 164mm so the scale is 19300/108 = 179 km/mm.

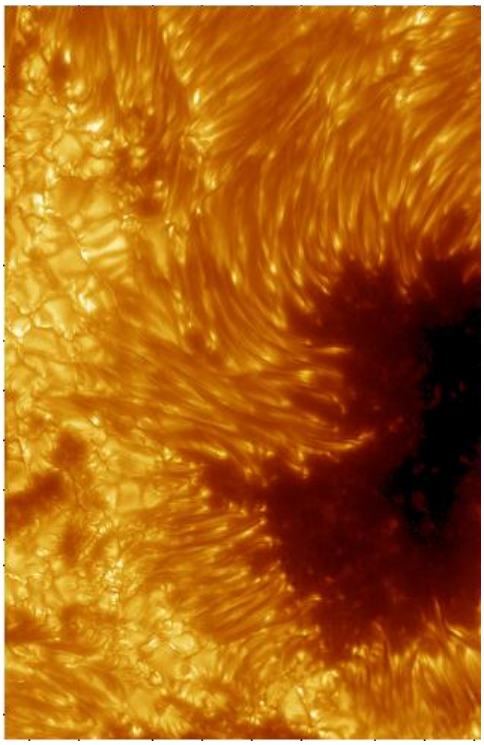
Problem 2 – What is the smallest feature you can see in the image? **Answer:** Students should be able to find features, such as the Granulation Boundaries, that are only 0.5 mm across, or 0.5 x 179 = 90 km across.

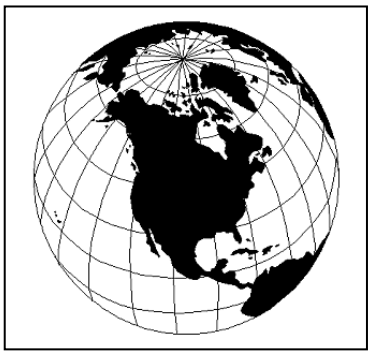
Problem 3 – What is the average size of a solar granulation region? **Answer:** Students should measure several of the granulation regions. They are easier to see if you hold the image at arms length. Typical sizes are about 5 mm so that 5 x 179 is about 900 km across.

Problem 4 – How long and wide are the Dark Filaments? **Answer:** Students should average together several measurements. Typical dimensions will be about 20mm x 2mm or 3,600 km long and about 360 km wide.

Problem 5– How large are the Bright Spots? **Answer:** Students should average several measurements and obtain values near 1 mm, for a size of about 180 km across.

Problem 6 – Draw a circle centered on this picture that is the size of Earth (radius = 6,378 km). How big are the features you measured compared to familiar Earth features? **Answer:** See below.



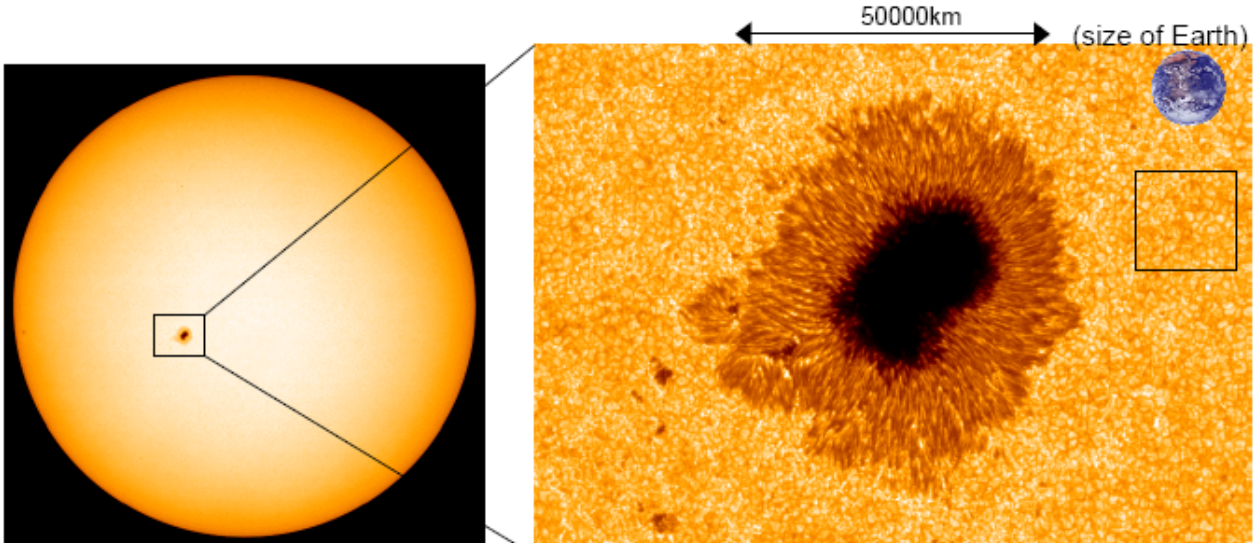


Granulation Region – Size of a large US state.

Bright Spot – Size of a small US state or Hawaii

Filament – As long as the USA, and as narrow as Baja California or Florida.

Close-up of a Sunspot by Hinode



After a successful launch on September 22, 2006 the Hinode solar observatory caught a glimpse of a large sunspot on November 4, 2006. An instrument called the Solar Optical Telescope (SOT) captured this image, showing sunspot details on the solar surface.

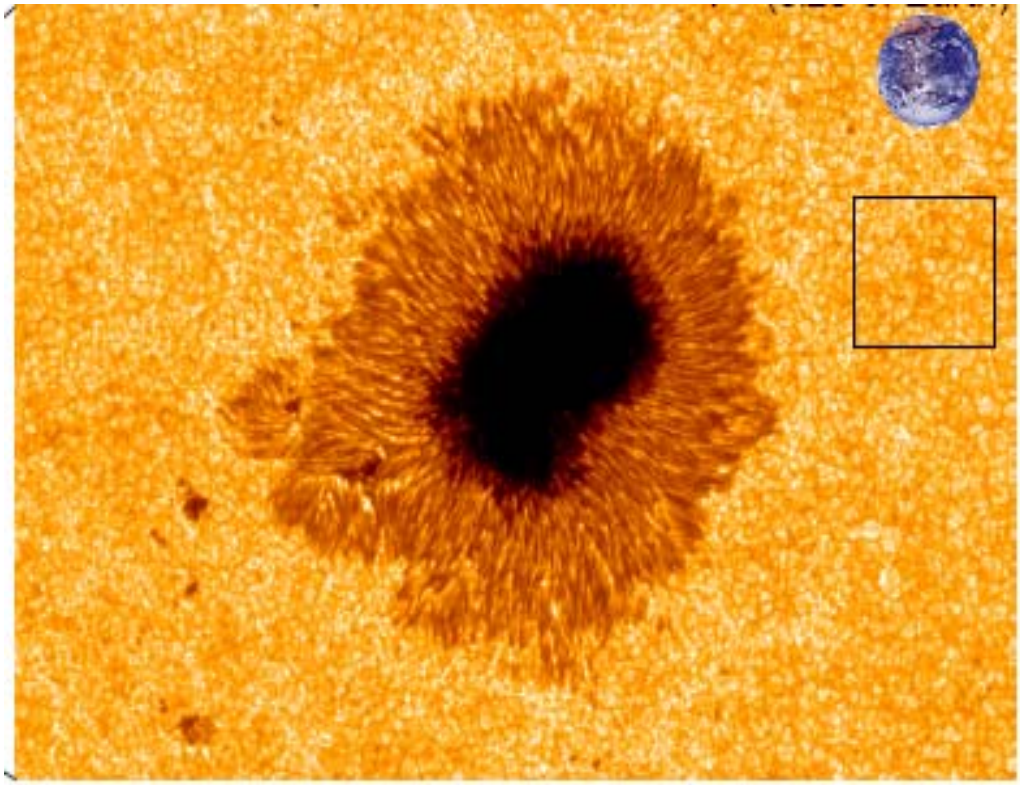
Problem 1 - Based on the distance between the arrow points, what is the scale of the image on the right in units of kilometers per millimeter?

Problem 2 - What is the size of the smallest detail you can see in the image?

Problem 3 - Compared to familiar things on the surface of Earth, how big would the smallest feature in the solar image be?

Problem 4 - The gold-colored, textured surface is the photosphere of the sun. The texturing is produced by heated gas that is flowing up to the surface from the hot interior of the sun. The convecting gases form cells, called granulations, at the surface, with upwelling gas flowing from the center of each cell, outwards to the cell boundary, where it cools and flows back down to deeper layers. What is the average size of a granulation cell within the square?

Problem 5 - Measure several granulation cells at different distances from the sunspot, and plot the average size you get versus distance from the spot center. Do granulation cells have about the same size near the sunspot, or do they tend to become larger or smaller as you approach the sunspot?



Problem 1 - From the 40 millimeter length of the 50,000 km arrow marker, the scale of the image is 50,000 km/40 mm = 1250 kilometers per millimeter

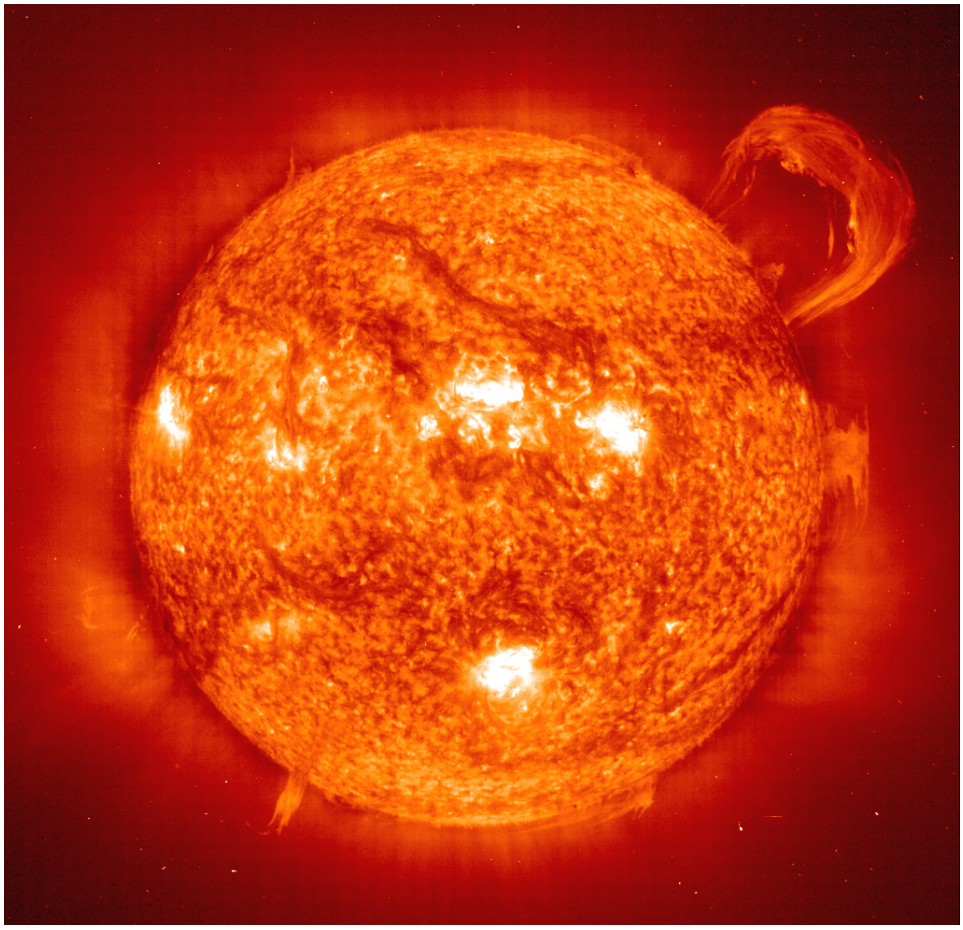
Problem 2 - Depending on the copy quality, the smallest detail is about 0.5 millimeters or 0.5 x 1250 = 625 kilometers across but details that are 1 or 2 mm across are also acceptable.

Problem 3 - Similar features on Earth would be continents like Greenland (1,800 km) or England (700 km).

Problem 4 - Measure about 5 cells to get: 1.5 mm, 1.0 mm, 0.8mm, 1.2mm and 1.4 mm. The average is about 1.2 mm, so the average size is (1.2) x 1250km = 1,500 km.

Problem 5 - Students should measure about 5 granulation cells in three groups; Group 1 should be far from the center of the spot. Group 3 should be as close to the outer, tan-colored, 'penumbra' of the spot as possible, and Group 2 should be about half-way in between Group 1 and 3. The average granulation sizes do not change significantly.

An Extreme Ultraviolet Image of the Sun



False-color image taken by the Solar and Heliospheric Observatory (SOH)) satellite's Extreme Ultraviolet Imaging Telescope (EIT) using ultraviolet light emitted by helium ions at temperatures between 60,000 and 80,000 K.

The image, obtained at a wavelength of 30 nm and a frequency of 10 petaHertz (1.0×10^{16} Hz) shows large active regions associated with sunspot groups (yellow-white); prominences and other filamentary features caused by matter being ejected from the sun into the corona; dark swaths of cooler hydrogen plasma that are prominences seen from above projected onto the solar disk; and the speckled surface caused by solar granulation cells.

Problem – Use a millimeter ruler to determine the scale of the image. What was the height and diameter of the huge prominence seen at the upper right edge of the sun? (Sun diameter = 1,300,000 km)

Problem - What was the height and diameter of the huge prominence seen at the upper right edge of the sun? (Sun diameter = 1,300,000 km)

Answer: The sun disk has a diameter of about 95 millimeters, so the scale is 1,300,000 km/95 mm = 14,000 km/mm. The maximum height of the center of the prominence is about 25 mm above the disk edge, for a height of 25 mm x 14,000 km/mm = **350,000 kilometers**. The diameter of one of its 'legs' is about 4 mm near the photosphere or about **56,000 kilometers**. At the top of the loop, the diameter is about 10 mm or **140,000 kilometers**.

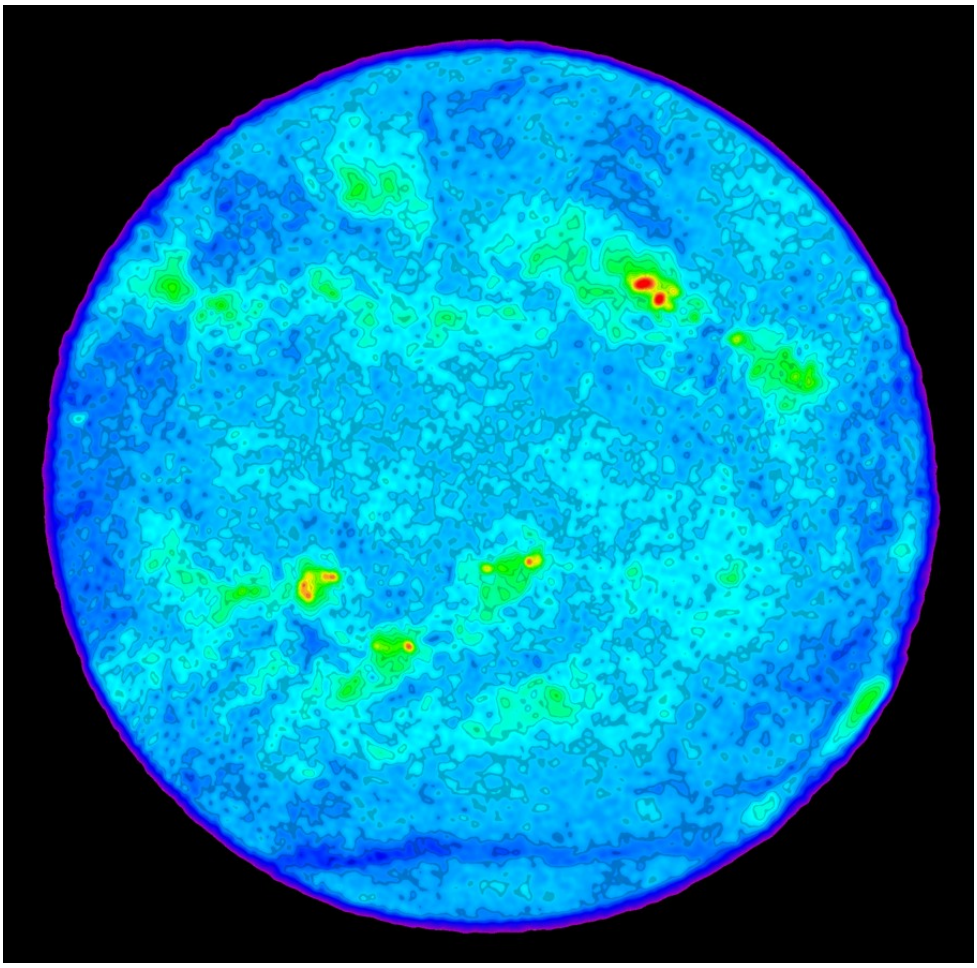
The brightness of the sun at this wavelength varies depending on how many active regions are present. Near sunspot minimum the brightness is about

$$F = 3 \times 10^{-29} \text{ watts/meter}^2$$

But near sunspot maximum it can be higher than

$$F = 3 \times 10^{-27} \text{ watts/meter}^2$$

Radio Image of the Sun



High resolution false-color image obtained at a frequency of 4.7 GHz (0.06 meters) or 5.0 gigaHertz (5.0×10^9 Hz) by the Very Large Array radio telescope (VLA) of the 'quiet sun' at a resolution of 12 arcseconds, from plasma emitting at 30,000 K. The brightest features (red) in this false-color image have temperatures of about 100,000 degrees K and coincide with sunspots. The green features are cooler and show where the Sun's atmosphere is very dense. At this frequency the radio-emitting surface of the Sun has an average temperature of 30,000 K, and the dark blue features are cooler yet. (Courtesy: Stephen White, University of Maryland, and NRAO/AUI).

Problem - From the scale of this image, what is the size of the smallest feature compared to the diameter of Earth (12,500 km)? (Sun diameter = 1,300,000 km)

Problem - From the scale of this image, what is the size of the smallest feature compared to the diameter of Earth (12,500 km)?

Answer: The sun disk is about 115 millimeters in diameter so the scale is 1,300,000 km/115 mm = 11,300 km/mm. The smallest features are the dark blue 'freckles' which hare about 1-2 millimeters across, corresponding to a physical size of 11,000 to 23,000 kilometers. This is about **1 to 2 times the size of Earth.**

The brightness of the sun at this wavelength varies from

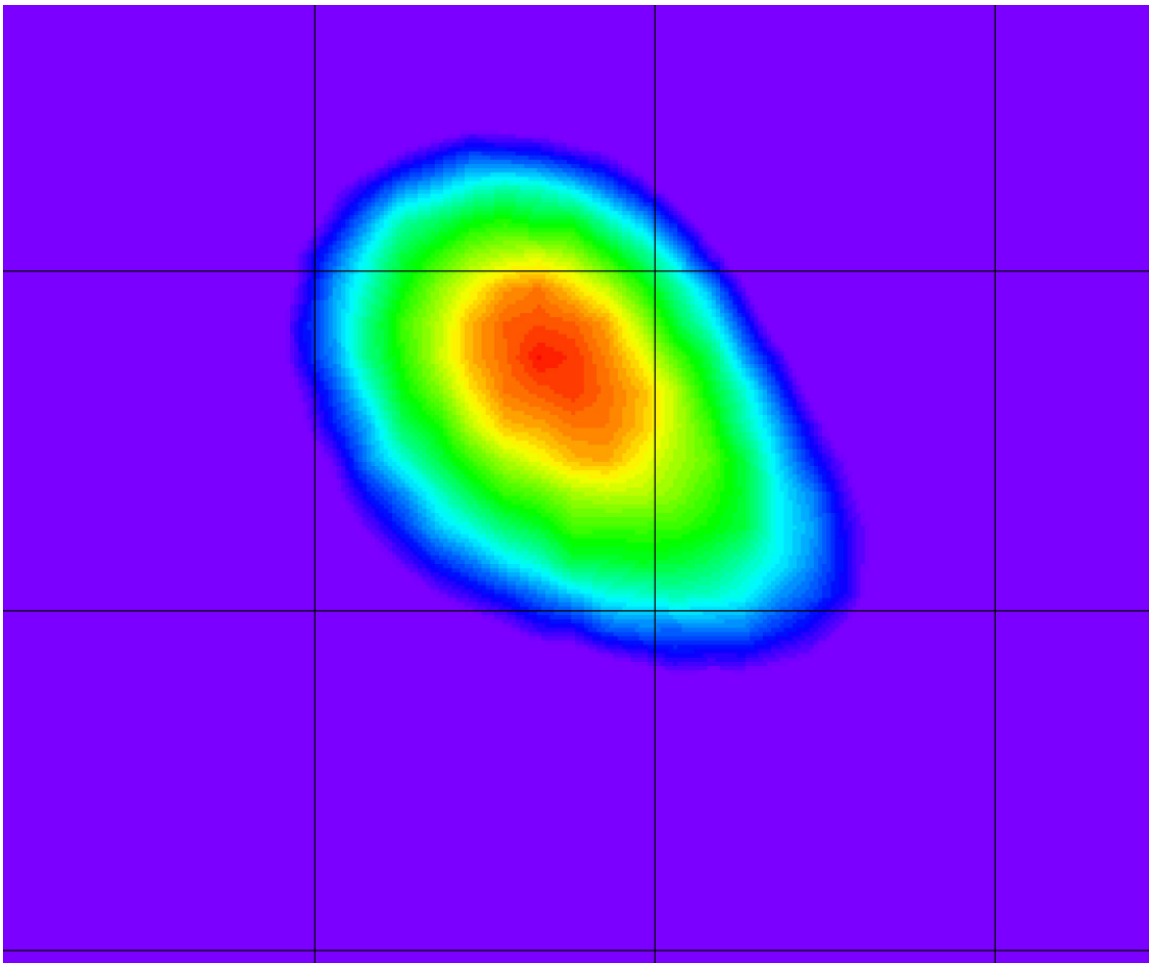
$$F = 2 \times 10^{-21} \text{ watts/meter}^2$$

to

$$F = 1 \times 10^{-19} \text{ watts/meter}^2$$

depending on the level of solar 'sunspot' activity.

A Gamma-Ray Image of the Sun



This is a colorized plot of the 40 MeV gamma-ray light from the sun during a major solar flare event on June 15, 1991. The equivalent wavelength of the light is 0.00003 nm and a frequency of 1.0×10^{22} Hertz. The image was created by NASA's Compton Gamma Ray Observatory (CGRO). The squares in the grid measure 10 degrees on a side, so the sun (0.5 degrees) cannot be resolved.

This image provided the first evidence that the sun can accelerate particles for several hours. This phenomenon was not observed before CGRO and represents a new understanding of solar flares. (Courtesy: COMPTEL team, University of New Hampshire).

Problem - How big would the disk of the sun be at the scale of this gamma-ray image?

Problem - How big would the disk of the sun be at the scale of this gamma-ray image?

Answer: The grid interval measures 45 millimeters, so the scale is 0.22 degrees/mm. The sun disk would be **2 millimeters** in diameter at this scale.

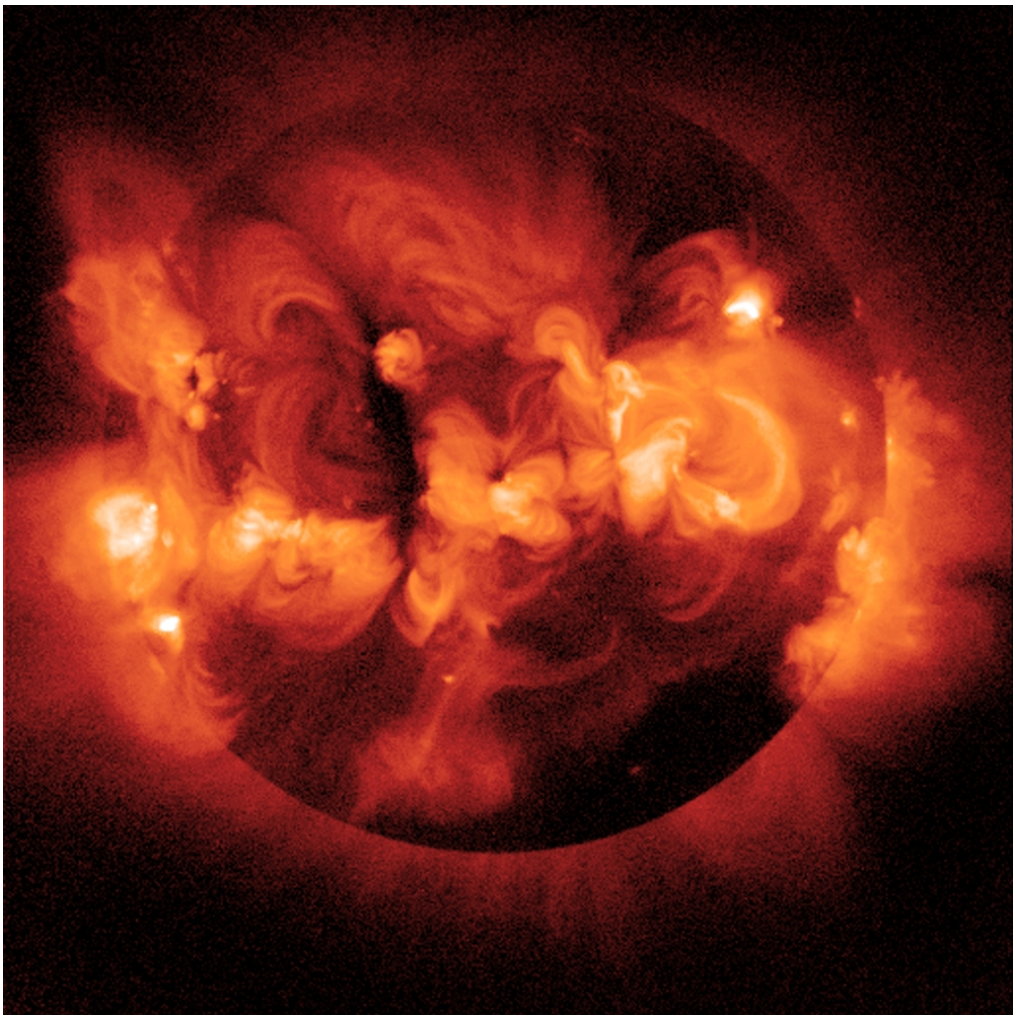
The brightness of the sun at gamma-ray energies is generally

$$F = 0.0 \text{ watts/meter}^2$$

But during intense flares it can brighten to

$$F = 10^{-35} \text{ watts/meter}^2$$

X-Ray Image of the Sun



This image was taken by the Yohkoh X-ray Satellite using X-rays emitted by the Sun at energies between 0.25 - 4 keV. The equivalent wavelength is 5.0 nm corresponding to a frequency of 60 petaHertz (6.0×10^{16} Hz). The Yohkoh satellite had a resolution of 2.5 arcseconds, which is similar to the newer, more sensitive, x-ray imager on the Hinode satellite.

The image shows a very active sun during the maximum activity period of sunspot cycle 23 in 2000. The billowy clouds are plasma heated to several million degrees above intense sunspot groups on the surface. The magnetically-confined plasma can be detected through the light produced by heavily ionized iron atoms at these temperatures. Most of this structure is located in the Sun's inner corona.

Problem 1 - What features of these clouds in this image suggest that magnetic fields may be confining them?

Problem 2 - How far above the solar photosphere does the coronal structure extend? (Sun diameter = 1,300,000 km).

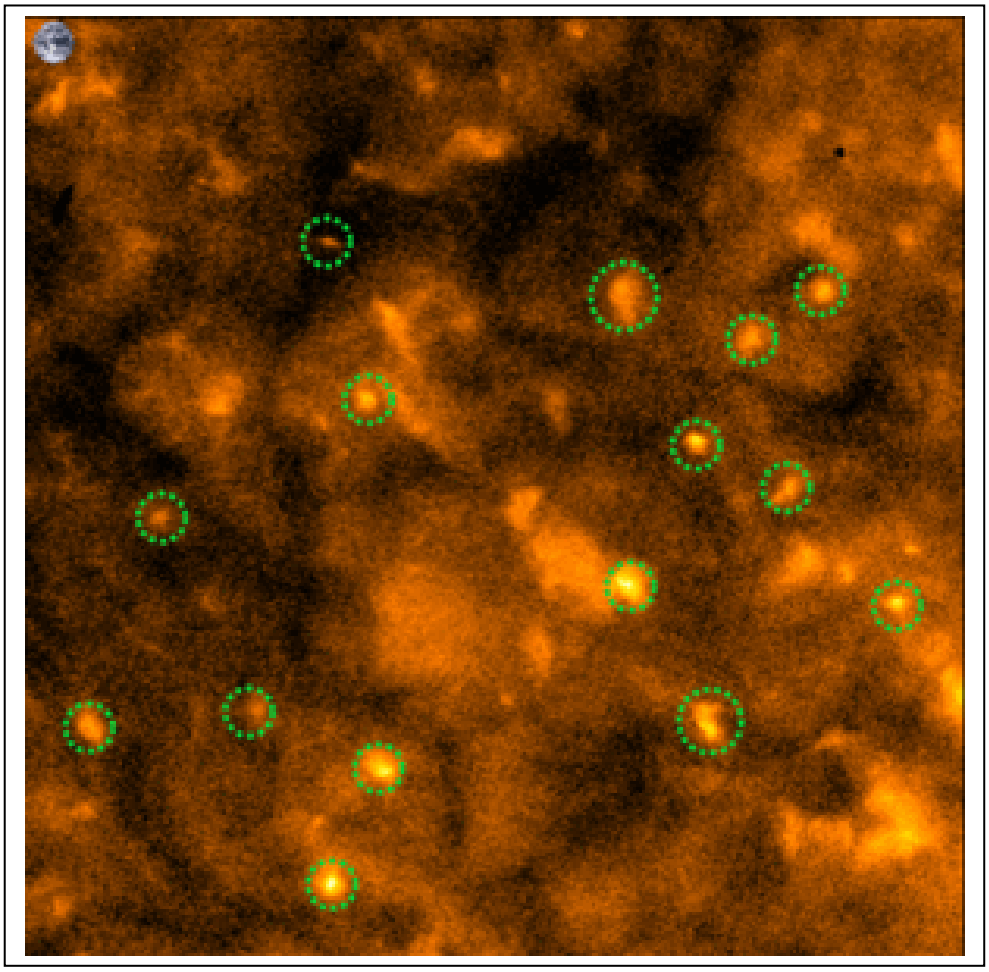
Problem 1 - What features of these clouds in this image suggest that magnetic fields may be confining them?

Answer: There appear to be faint filaments within the cloud shapes that resemble the magnetic fields you see when you sprinkle iron filings on a bar magnet.

Problem 2 - How far above the solar photosphere does the coronal structure extend? (Sun diameter = 1,300,000 km).

Answer: The diameter of the sun disk is about 115 mm, so the scale of the image is 1,300,000 km/115mm = 11,000 km/mm. Measuring the height of the clouds on the edge of the Sun, they seem to be about 20 mm above the disk edge ,for a height of 20 mm x 11,000 km/mm = **220,000 kilometers**.

Hinode Sees Mysterious Solar Micro-flares



The Sun's surface is not only speckled with sunspots, it is also dotted with intense spots of X-ray light called 'X-ray Bright Points'. Although sunspots can be over 100,000 kilometers across and easily seen with a telescope, X-ray Bright Points are so small even the largest solar telescope only sees a few of them with enough detail to reveal their true shapes. X-ray Bright Points release their energy by converting tangled magnetic fields into smoother ones. This liberates large quantities of stored magnetic energy. For that reason, these Bright Points can be thought of as micro-flares.

Hinode's X-ray Telescope (XRT) can now see the details in some of the Bright Points and allow scientists to see small magnetic loops. In the image above, individual bright points are circled in green. A few of them can be resolved into tiny magnetic loops. These data were taken on March 16, 2007. The image is 300 x 300 pixels in size. Each pixel views an area on the sun that is 1 arcsecond x 1 arcsecond on a side.

Problem 1: If the diameter of the Sun is 1800 arcseconds, and has a radius of 696,000 km, what is the scale of the above image in A) kilometers per arcsecond? B) kilometers/millimeter?

Problem 2: What are the dimensions, in kilometers, of the smallest circled Bright Point in the image?

Problem 3: How many Bright Points cover the solar surface if the above picture is typical?

Answer Key:

Problem 1: If the diameter of the sun measures 1800 arcseconds and has a radius of 696,000 km, what is the scale of the above image in kilometers per arcsecond?

Answer: A) The solar radius is 1800 arcseconds/2 = 900 arcseconds which physically equals 696,000 km, so the scale is 696,000/900 = **773 kilometers/arcsecond**.

B) The image is 300 pixels across, which measures 115 millimeters with a ruler. Each pixel is 1 arcsecond in size, so this represents 773 km/arcsec x 300 = 232,000 km. The ruler says that this equals 115 mm, so the image scale is 232,000 km/115 mm = **2,020 km/mm**.

Problem 2: What are the dimensions of the smallest circled Bright Point in the image?

Answer: With a ruler, the circled Bright Point at the top of the picture seems to be the smallest. It measures about 2 millimeters across and 1 millimeter wide. This corresponds to about **4000 x 2000 km.**

Problem 3: How many Bright Points cover the solar surface if the above picture is typical?

Answer:

The sun is a sphere with a radius of 696,000 kilometers. The area of a sphere is given by $4 \pi R^2$, so the surface area of the sun is $4 \times 3.141 \times (696{,}000 \text{ km})^2 = 6.1 \times 10^{12}$ kilometers^2.

The size of the Hinode image is 300 pixels x 773 km/pixel = 232,000 km on a side. The area covered is about (232,000 km x 232,000 km) = 5.4×10^{10} km^2. Note, this is an approximation because of the distortion of a flat image attempting to represent a curved spherical surface. The actual solar surface area covered is actually a bit larger.

The solar surface is about 6.1×10^{12} km^2 / 5.4×10^{10} km^2 = 113 times larger than the Hinode image.

There are 16 Bright Points in the Hinode image, so there would be 15 x 113 = **1,695** Bright Points covering the full solar surface if the Hinode image is typical.

X-Ray Image of the Sun

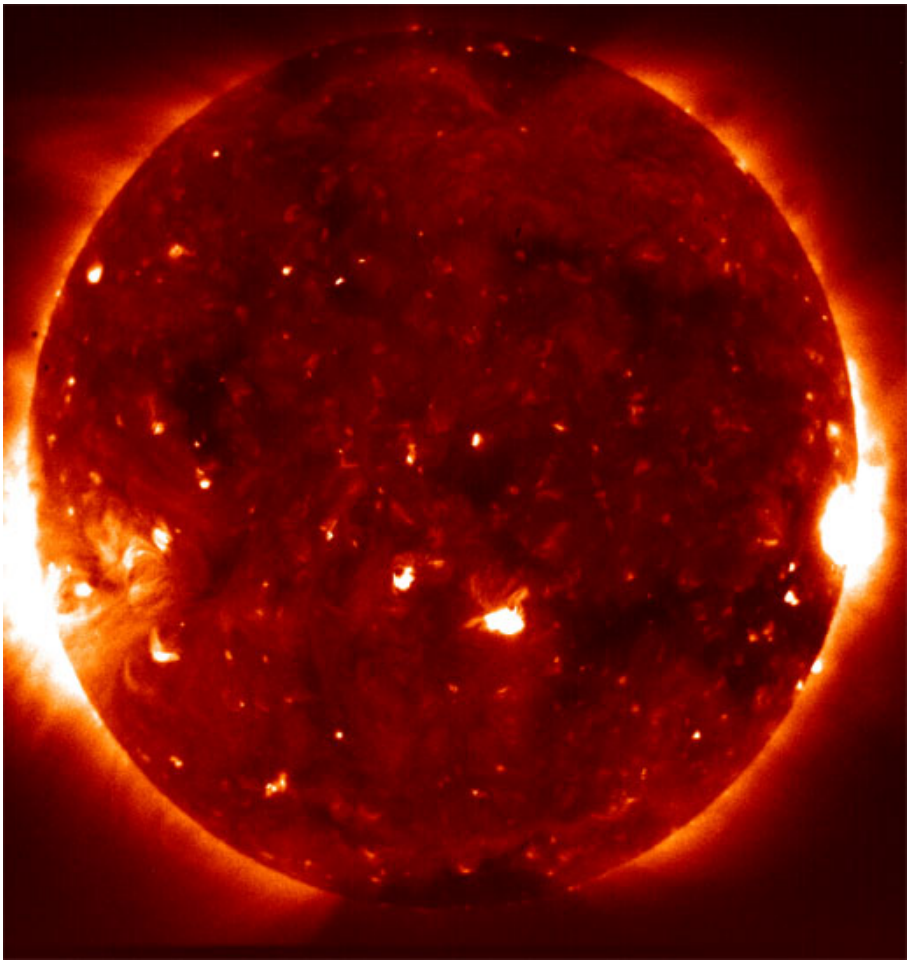


Image taken by the Hinode satellite's X-ray Telescope (XRT) using x-rays emitted by the sun at energies between 1,000 to 10,000 electron volts (1 to 10 keV). The equivalent wavelength is 0.2 nm or a frequency of 1.5 petaHertz (1.5×10^{18} Hz). The resolution is 2 arcseconds. At these energies, only plasma heated to over 100,000 degrees K produce enough electromagnetic energy to be visible. The solar surface, called the photosphere, at a temperature of 6,000 K is too cold to produce x-ray light, and so in X-ray pictures it appears black.

The Hinode image shows for the first time that the typically dark areas of the sun can contain numerous bright 'micro-flares' that speckle the surface, releasing energy into the corona.

Problem 1 - How big are the micro-flares compared to Earth? (Sun diameter = 1,300,000 km; Earth diameter = 12,500 km).

Problem 2 - About how many micro-flares can you see on this hemisphere of the Sun, and how many would you estimate exist at this time for the entire solar surface?

Problem 1 - How big are the micro-flares compared to Earth? (Sun diameter = 1,300,000 km; Earth diameter = 12,500 km).
Answer: The disk of the Sun measures about 110 millimeters in diameter, so the scale is 1,300,000 km/110 mm = 12,000 km/mm. The micro-flares are just over 1 millimeter in diameter or 12,000 kilometers, which is **similar to the diameter of Earth.**

Problem 2 - About how many micro-flares can you see on this hemisphere of the Sun, and how many would you estimate exist at this time for the entire solar surface?

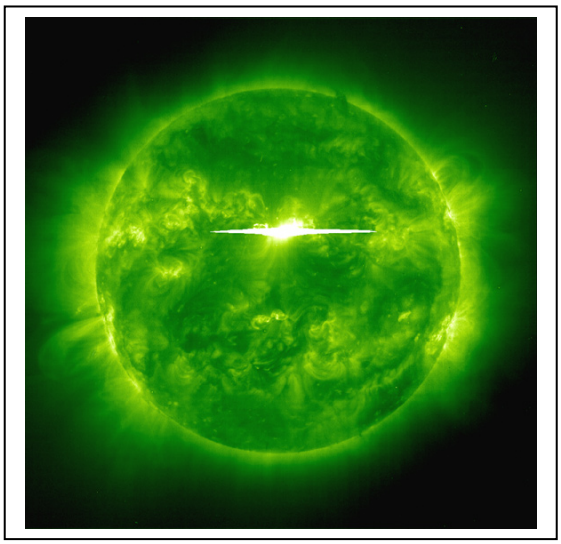
Answer: A careful count should find **about 200** of these 'spots' some bright and some faint. Over the entire solar surface there would be **about 400.**

The brightness of the sun at x-ray wavelengths varies depending on the level of solar activity from

$$F = 3 \times 10^{-29} \text{ watts/meter}^2$$

to

$$F = 3 \times 10^{-27} \text{ watts/meter}^2$$



On July 15, 2001 a solar storm was tracked from the Sun to Earth by a number of research satellites and observatories. This activity lets you perform time and day arithmetic to figure out how long various events lasted. This is a very basic process that scientists go through to study an astronomical phenomenon. The image to the left was taken by the TRACE satellite and shows the x-ray flare on the Sun.

Photo courtesy SOHO/NASA

The Story: On July 14, 2000, NASA's TRACE satellite spotted a major X5.7-class solar flare erupting at 09:41 from Active Region 9077. The flare continued to release energy until 12:31. At 10:18:27, radio astronomers using the Nancay radio telescope detected the start of a radio-frequency Type-I noise storm. This storm strengthened, and at 10:27:27, four moving radio sources appeared. Meanwhile, the satellite, GOES-10 detected the maximum of the x-ray light from this flare at 10:23. The SOHO satellite, located 92 million miles from the Sun, and 1 million miles from Earth, recorded a radiation storm from fast-moving particles, that caused data corruption at 10:41. The SOHO satellite's LASCO imager also detected the launch of a coronal mass ejection (CME) at 10:54. The CME arrived at the satellite at 14:17 on July 15. Then at 14:37 on July 15, the CME shock wave arrived at Earth and compressed Earth's magnetic field. The IMAGE satellite recorded the brightening of the auroral oval starting at 14:25. Aurora were at their brightest at 14:58. The aurora expanded to the lowest latitude at 17:35. By 20:00, Earth's magnetic field has slightly decreased in strength in the equatorial regions. By 16:47 on July 16, the IMAGE satellite recorded the recovery of Earth's magnetosphere to normal conditions. On January 12, 2001, the CME was detected by the Voyager I satellite located 63 AU from the Sun.

Problem 1 - From this information, create a time line of the events mentioned.

Problem 2 – How long did it take for the CME to reach Earth?

Inquiry: What other questions can you explore using this timing information?

Answer Key

Problem 1

July 14,
- 09:41 - X5.7-class solar flare
- 10:19 - Radio astronomers detect Type-I radio storm.
- 10:23 - GOES-10 detected the maximum of the x-ray light from this flare
- 10:27 - Four moving radio sources appeared on sun.
- 10:41 - SOHO satellite radiation storm and data corruption.
- 10:54 - SOHO sees launch of CME

July 15,
- 14:17 - CME shock wave arrived at Earth
- 14:25 – IMAGE satellite sees brightening of the auroral oval
- 14:58 - Aurora at brightest
- 17:35 – Aurora expand to lowest latitudes
- 20:00 - Earth's magnetic field has slightly decreased in strength

July 16
- 16:47 - IMAGE satellite recorded the recovery of Earth's magnetosphere

January 12, 2001, CME detected by Voyager I satellite 63 AU from the Sun.

Problem 2 - The CME was launched on July 14 at 10:54 and arrived at Earth on July 15 at 14:17. The elapsed time is 1 full day (24 hours) and the difference between 10:54 and 14:17 which is 14:17 – 10:54 = 13:77 – 10:54 = 3hours and 77-54 = 23 minutes. The total elapsed time is then 24h + 3h 23m = 27hours 23minutes.

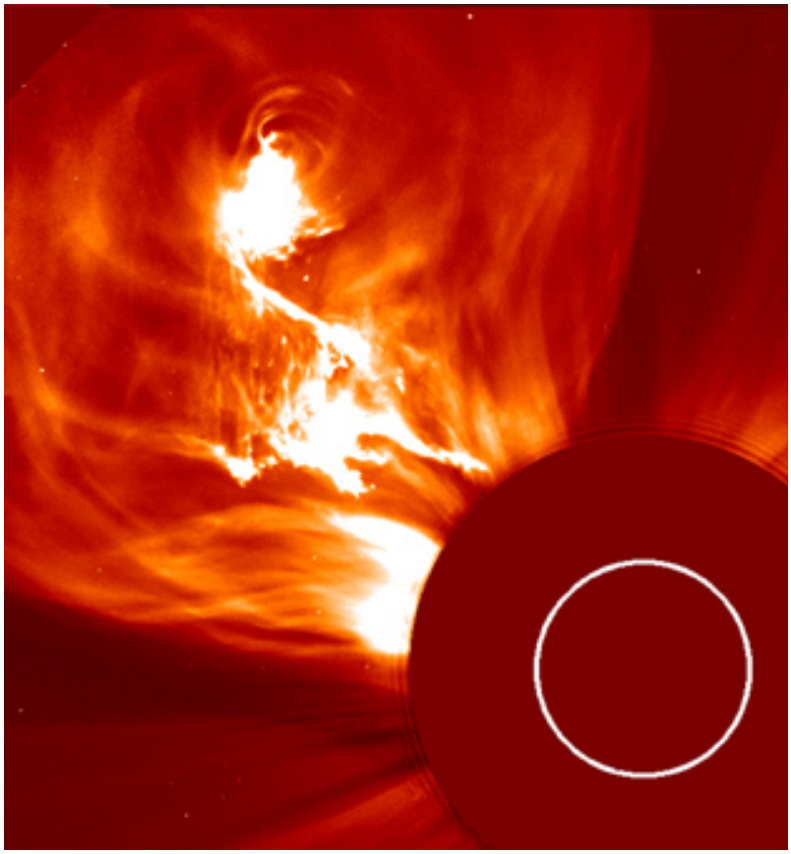
Inquiry – There are many possibilities, for example, how long did it take for the CME to reach Voyager in days? Hours? What was the speed of the CME as it traveled to Earth? How long after the flare did SOHO experience a radiation storm?

Solar Storm Timeline - Tabular

The sun is an active star that produces explosions of matter and energy. The space between the planets is filled with invisible clouds of gas that sometimes collide with Earth. Scientists call them Coronal Mass Ejections. They can travel at millions of miles per hour and carry several billion tons of gas called a plasma. When 'CMEs' collide with Earth, they produce the Northern Lights and magnetic storms.

In this exercise, you will examine one of these 'solar storm' events by examining a timeline of events that it caused.

The picture to the right was taken by the SOHO spacecraft showing a spectacular CME. The white circle is the size of the sun.



Solar Storm Timeline

Day	*Time*	*What Happened*
Tuesday	4:50 PM	Gas eruption on Sun
Thursday	3:36 AM	Plasma storm reaches Earth.
Thursday	5:20 AM	Storm at maximum intensity.
Thursday	5:35 AM	Auroral power at maximum.
Thursday	11:29 AM	Aurora power at minimum.
Thursday	2:45 PM	Space conditions normal

1) How much time passed between the solar gas eruption and its detection near Earth?

2) How long after the plasma storm reached Earth did the aurora reach their maximum power?

3) How long did the storm last near Earth from the time the plasma was detected, to the time when space conditions returned to normal?

<u>Extra for Experts!</u>
If the Earth is 150 million kilometers from the sun, how fast did the storm travel from the Sun in kilometers per hour? How long will the trip to Pluto take is Pluto is 40 times farther away from the sun than Earth?

Goal: Students will interpret a timeline table to extract information about a solar storm using time addition and subtraction skills.

Day	*Time*	*What Happened*
Tuesday	4:50 PM	Gas eruption on Sun
Thursday	3:36 AM	Plasma storm reaches Earth.
Thursday	5:20 AM	Storm at maximum intensity.
Thursday	5:35 AM	Auroral power at maximum.
Thursday	11:29 AM	Aurora power at minimum.
Thursday	2:45 PM	Space conditions normal

1) How much time passed between the solar gas eruption and its detection near Earth?
Answer: There are various ways to do this problem. You want to subtract the final time from the initial time so: (Tuesday 4:50 PM) – (Thursday, 3:36 AM) = (Thursday – Tuesday) + (3:36 AM – 4:50 PM) = 48 hrs – (4:50PM – 3:36AM) = 48h – 13h 14m = 34hours and 46minutes.

2) How long after the plasma storm reached Earth did the aurora reach their maximum power?
Answer: Storm arrived at 3:36 AM. Aurora at maximum at 5:35AM. Difference in time is 1 hour and 59 minutes.

3) How long did the storm last near Earth from the time the plasma was detected, to the time when space conditions returned to normal?
Answer: On Thursday, the storm started at 3:36 AM and ended at 2:45 PM, so the storm effects at Earth lasted from 03:36 to 14:45 so the difference is 14:45 – 03:36 = 11 hours and (45-36 =) 9 minutes.

Extra for Experts!
If the Earth is 150 million kilometers from the sun, how fast did the storm travel from the Sun in kilometers per hour? How long will the trip to Pluto take is Pluto is 40 times farther away from the sun than Earth?
Answer: The answer to Problem 1 is 34hours and 46minutes, which in decimal form is 34 + (46/60) = 34.8 hours with rounding. The speed is therefore 150 million km/34.8 hours or 4.3 million km/h. The trip to Pluto would take 40 x 34.8 hours = 1,392 hours or about 58 days. Note, the Space Shuttle is our fastest manned spacecraft and travels at about 27,000 km/h so it would take about 58 x (4.3 million/27,000) = 9,237 days to make this trip, which equals 25 years!!!! Of course, the Space Shuttle will be out of fuel and supplies within a week.

Solar flares

Solar flares are powerful explosions of energy and matter from the Sun's surface. One explosion, lasting only a few minutes, could power the entire United States for a full year. Astronauts have to be protected from solar flares because the most powerful ones can kill an astronaut if they were working outside their spacecraft.

In this exercise, you will learn how scientists classify flares, and how to decode them.

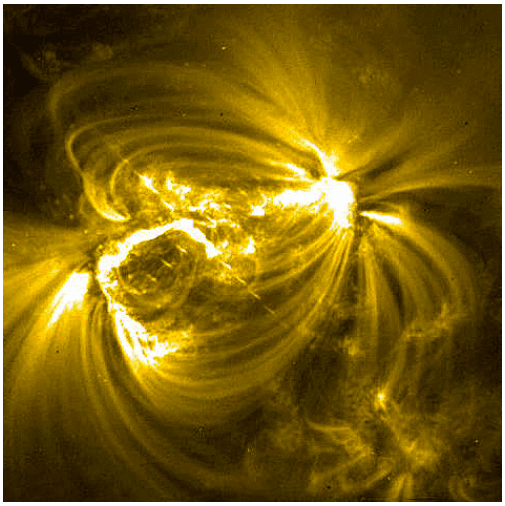


Image of Sun showing flare-like eruption.

Scientists create alphabetic and numerical scales to classify phenomena, and to assign names to specific events.

Simple equations can serve as codes.

Here's how to do it!

A solar flare scale uses three multipliers defined by the letter codes C = 1.0, M=10.0, X=1000.0.

A solar flare might be classified as M5.8 which means a brightness of (10.0) x (5.8) = 58.0.
A second solar flare might be classified as X15.6 which means (1000.0) x (15.6) = 15,600.0

The X15.6 flare is (15,600/58) = 269 times brighter than the M5.8 flare.

Now you try!

The GEOS satellite has an X-ray monitor that records daily solar flare activity. The table below shows the flares detected between January 11 and March 3, 2000.

Flare Codes for Major Events

Date	Code	Date	Code
1-11	M1.5	2-12	M1.7
1-12	M2.8	2-17	M2.5
1-18	M3.9	2-18	C2.7
1-22	M1.0	2-20	M2.4
1-24	C5.3	2-21	M1.8
1-25	C6.8	2-22	M1.2
2-3	C8.4	2-23	C6.8
2-4	M3.0	2-24	M1.1
2-5	X1.2	2-26	M1.0
2-6	C2.4	3-1	C6.9
2-8	M1.3	3-2	X1.1

1) What was the brightest flare detected during this time?

2) What was the faintest flare detected during this time?

3) How much brighter was the brightest flare than the faintest flare?

4) What percentage of the flares were brighter than M1.0?

Answer Key

Problem 1 - X1.2 on February 5 with a brightness of (1000) x 1.2 = 1,200

Problem 2 - C2.4 on February 6 with a brightness of (1.0) x 2.4 = 2.4

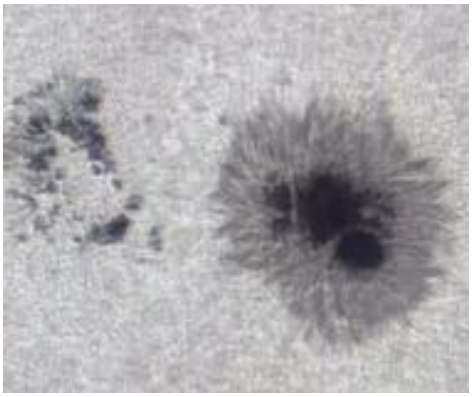
Problem 3 - 1200/2.4 = 500 times brighter

Problem 4 - There are a total of 22 flares in this table. There are 13 flares brighter than M1.0 but not equal to M1.0. The percentage is then (13/22)x100% = 59%

Sunspots and Solar Flares

Sunspots are some of the most interesting, and longest studied, phenomena on the sun's surface. The table below shows the areas of several sunspots observed between November 6 and January 19, 2004. In comparison, the surface area of Earth is '169' units on the sunspot scale. The table also shows the brightest flare seen from the vicinity of the sunspots. Flares are ranked by their brightness 'C', 'M' and 'X' with M-class flares being 10x more luminous that C-class flares, and X-class flares being 10x brighter than M-class flares.

Date	Spot #	Area	Flare
Nov 6	#696	820	M
Nov 7	#696	910	M
Nov 8	#696	650	X
Nov 10	#696	730	M
Nov 11	#696	470	X
Dec 2	#708	130	M
Dec 3	#708	150	M
Dec 9	#709	20	C
Dec 29	#713	150	M
Dec 30	#715	260	M
Dec 31	#715	350	M
Jan 1	#715	220	M
Jan 2	#715	180	X
Jan 4	#715	130	C
Jan 10	#719	100	M
Jan 14	#718	160	C
Jan 15	#720	1540	M
Jan 16	#720	1620	X
Jan 17	#720	1630	M
Jan 18	#720	1460	X
Jan 19	#720	1400	M



During the 75 day time period covered by this table, there were a total of (720-696=) 24 catalogued sunspots. The table shows only those cataloged sunspots that were active in producing flares during this time. Sunspot areas are in terms of millionths of the solar hemisphere area, so '1630' means 0.163% of the Sun's face. Earth's area = 169 millionths by comparison!

Sunspot and solar flare data from NOAA SWN data archive at http://www.sec.noaa.gov/Data/index.html

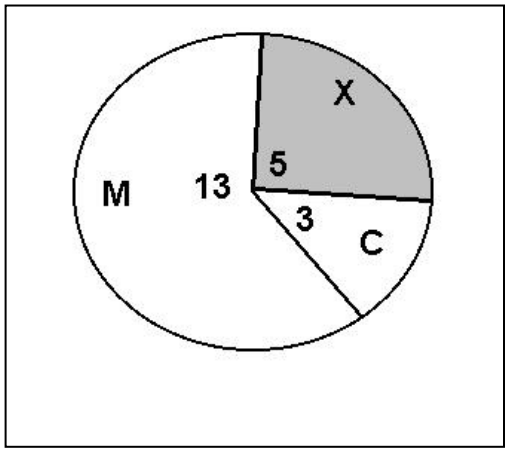
Problem 1 - Construct a pie chart for the X, M and C-class flare data. During this 75-day period, what percentage of flares are X-class?

Problem 2 - What percentage of sunspots produce X-class flares?

Problem 3 - What percentage of sunspots did not produce any flares during this time?

Problem 4 - What seems to be the minimum size for a sunspot that produces an X-class flare? An M-class flare? A C-class flare?

Problem 5 - If the area of Earth is '169' in the sunspot units used in the above tables, what are the maximum and minimum size of the sunspots compared to the area of Earth?



Note, there are 21 flares in the table.
X = 5 flares
M = 13 flares
C = 3 flares.
The pie chart angles are
21 = 360 degrees
X = (5/21)x360 = 86 degrees
M = (13/21)x360 = 223 degrees
C = (3/21)x360 = 51 degrees.
And to check: 223+86+51 = 360.

Problem 1: With a pie chart, what percentage of flares were X-type?
Answer: 5 out of 21 or (5/21)x100% = 24%

Problem 2: During this 75-day period, what percentage of flares are X-class flares?
Answer: There are 24 sunspots in the sample because the catalog numbers run from 720 to 698 as stated in the table caption (A 'reading to be informed' activity). There were three sunspots listed in the table that produced X-class flares: #696, #715, #720. The percentage is (3/24)x100% = 12.5% which may be rounded to 13%.

Problem 3: What percentage of sunspots did not produce flares during this time?
Answer: There were only 8 sunspots in the table that produced flares, so there were 16 out of 24 that did not produce any flares. This is (16/24)x100% = 67%. An important thing for students to note is that MOST sunspots do not produce any significant flares.

Problem 4: What seems to be the minimum size for a sunspot that produces an X-class flare? An M-class flare? A C-class flare?
Answer: Students may reasonably answer by saying that there doesn't seem to be any definite correlation for the X and M-class flares! For X-class flares, you can have them if the area is between 180 and 1620. For M-class flares, spots with areas from 130 to 1630 can have them. The two possibilities overlap. For C-class flares, they seem to be most common in the smaller spots from 20 – 130 in area, but the sample in the table is so small we cant really tell if this is a genuine correlation or not. Also, we have only shown in the table the largest flares on a given day, and smaller flares may also have occurred for many of these spots.

Problem: If the area of Earth is '169' in the sunspot units used in the above tables, what are the maximum and minimum size of the sunspots compared to the area of Earth?
Answer: The smallest spot size occurred for #709 with an equivalent size of (20/169)x100% = 11% of Earth's area. The largest spot was #720 with a size equal to (1630/169) = 9.6 times Earth's area.

Solar Flares and the Sunspot Cycle

Year	C-class	M-class	X-class
1996	76	4	1
1997	288	20	3
1998	1198	96	14
1999	1860	170	4
2000	2265	215	17
2001	3107	311	20
2002	2319	219	12
2003	1315	160	20
2004	912	122	12
2005	578	103	18
2006	150	10	4
2007	73	10	0
2008	8	1	0

Solar flares are the most dramatic explosions on the sun, which have been known for some time. An average-sized flare can release more energy in a few hours than thousands of hydrogen bombs exploding all at once.

Solar flares do not happen randomly, but like many other solar phenomena follow the rise and fall of the sunspot cycle.

The table to the left is a tally of the number of C, M and X-class flares identified during each year of the previous sunspot cycle.

Problem 1 - During the entire sunspot cycle, how many solar flares occur?

Problem 2 - What percentage of solar flares during an entire sunspot cycle are A) C-class? B) M-class? C) X-class?

Problem 3 - During a single week, how many flares of each type would the sun produce during A) Sunspot maximum in the year 2001? B) Sunspot minimum during the year 1996?

Problem 4 - During sunspot maximum, what is the average time in hours between flares for, A) C-class? B)M-class? and C) X-class? (Hint: 1 year = 8,760 hours)

Problem 5 - Do as many flares occur in the time before sunspot maximum (1996-2000) as after sunspot maximum (2002-2008) for A) C-class? B) M-class? C) X-class?

Problem 1 - During the entire sunspot cycle, how many solar flares occur?
Answer: The sum of all the flares in the table is **15,715**.

Problem 2 - What percentage of solar flares during an entire sunspot cycle are A) C-class? B) M-class? C) X-class?
Answer: A) C-class = 100% x (14149/15715) = **90%**
B) M-class = 100% x (1441/15715) = **9%**
C) X-class = 100% x (14149/15715) = **1%**

Problem 3 - During a single week, how many flares of each type would the sun produce during A) Sunspot maximum in the year 2001? B) Sunspot minimum during the year 1996?
Answer: A) C-type = 3107/52 = **60**
M-type = 311/52 = **6**
X-type = 20/52 = **0.4**

B) C-type = 76/52 = **1.4**
M-type = 4/52 = **0.08**
X-type = 1/52 = **0.02**

Problem 4 - During sunspot maximum, what is the average time in hours between flares for, A) C-class? B)M-class? and C) X-class? (Hint: 1 year = 8,760 hours)
Answer: C-class = 1 year/3107 flares x (8760 hours/year) = **2.8 hours/flare**.
M-class = 1 year/311 flares x (8760 hours/year) = **28 hours/flare**. (1 day)
X-class = 1 year/20 flares x (8760 hours/year) = **438 hours/flare**. (18 days)

Problem 5 - Do as many flares occur in the time before sunspot maximum (1996-2000) as after sunspot maximum (2002-2008) for A) C-class? B) M-class? C) X-class?

Answer: Between 1996-2000 there were 6,231 and between 2001-2008 there were 6,046 so although the numbers are nearly the same, there were slightly fewer flares after sunspot maximum.

Note: In statistics, the sampling error is n = $(6231)^{1/2}$ = 79, so the range of random variation for 1996-2000 is between 6231+79 and 6231-79 or [6151, 6310] for 2002-2008 we have $(6046)^{1/2}$ = 78, so the range of random variation for 2002-2008 is between 6046+78 and 6046-78 or [5968, 6124]. Because of this sampling uncertainty, the counts between each side of solar maximum are statistically similar. They do not differ by more than 3-sigma (3x78 = 234) which means they are the same to better than 98% certainty.

How Common are X-Class Solar Flares?

	CY	J	F	M	A	M	J	J	A	S	O	N	D
1996	1							1					
1997	2											3	
1998	3				2	2			5			5	
1999	4								2		1	1	
2000	5		1	3			4	3		1		5	
2001	6			1	8		1		1	1	4	2	3
2002	7				1	1		5	4		1		
2003	8			2		3	4				7	4	
2004	9							6	2		1	2	
2005	10	6						1		10			
2006	11												2

X-Class solar flares are among the most powerful, explosive events on the solar surface. They can cause short-wave radio interference, satellite malfunctions and can even cause the premature re-entry of satellites into the atmosphere.

The table above lists the number of X-class flares detected on the sun during the last sunspot cycle which lasted from 1996 to about 2008. The second column also gives the year from the start of the 11-year sunspot cycle in 1996. The counts are listed by year (rows) and by month (columns). Study this table and answer the following questions to learn more about how common these flares are.

Problem 1 – For the years and months considered, is the distribution of months with flares a uniform distribution? Explain.

Problem 2 - The sunspot cycle can be grouped into pre-maximum (1997,1998, 1999), maximum (2000,2001,2002) and post-maximum (2003,2004,2005). For each group, calculate A) The percentage of months with no flares; and B) The average number of weeks between flares.

Problem 3 – For each group, what is the median number of flares that occurs in the months that have flares?

Problem 4 – Taken as a whole, what is the average number of flares per month during the entire 11-year sunspot cycle?

Problem 5 – We are currently in Year-3 of the current sunspot cycle, which began in 2007. About how many X-class flares would you predict for this year using the tabulated flares from the previous sunspot cycle as a guide, and what is the average number of weeks between these flares for this year?

Problem 1 – For the years and months considered, is the distribution of months with flares a uniform distribution? Explain. Answer; If you shaded in all the months with flares you would see that most occur between 1999-2002 so the distribution is not random, and is not uniform.

Problem 2 - The sunspot cycle can be grouped into pre-maximum (1997,1998, 1999), maximum (2000,2001,2002) and post-maximum (2003,2004,2005). For each group, calculate A) the percentage of months with no flares, and B) The average number of weeks between flares. Answer: A) Pre-Maximum, N = 28 months so P = 100%x28/36 = 78% . Maximum; N = 17 months so P = 100% x 17/36 = 47%; post-maximum N= 24 months so P = 100% x 24/36 = 67%. B) pre-maximum N = 22 flares so T = 36 months/22 flares = 1.6 months. Maximum: N = 50 flares so T = 36 mo/50 = 0.7 months; post-maximum: N = 48 flares so T = 36 mo/48 = 0.8 months.

Problem 3 – For each group, what is the median number of flares that occurs in the months that have flares? Answer: Pre-maximum: 1,1,2,2,2,3,5,5 median = 3. Maximum: 1,1,1,1,1,1,1,1,1,2,3,3,3,4,4,4,5,8 median = 4; post-maximum 1,1,2,2,2,3,4,6,6,7 median = 6

Problem 4 – Taken as a whole, what is the average number of flares per month during the entire 11-year sunspot cycle? Answer: There were 122 flares detected during the 132 months of the sunspot cycle, so the average is 122 flares/132 months = 0.9 , which can be rounded to **1 flare/month**.

	CY	J	F	M	A	M	J	J	A	S	O	N	D	
1996	1							1						1
1997	2											3		3
1998	3				2	2			5			5		14
1999	4								2		1	1		4
2000	5		1	3			4	3		1		5		17
2001	6			1	8		1		1	1	4	2	3	21
2002	7				1	1		5	4		1			12
2003	8			2		3	4				7	4		20
2004	9							6	2		1	2		11
2005	10	6						1		10				17
2006	11												2	2
		6	1	6	11	6	9	16	14	12	14	22	5	

Problem 5 – We are in Year-3 of the current sunspot cycle, which began in 2007. About how many X-class flares would you predict for this year using the tabulated flares from the previous sunspot cycle as a guide, and what is the average number of weeks between these flares for this year? Answer: From the table we find for Year 3 that there were 14, X-class flares. Since there are 12 months in a year, this means that the average time between flares is about 14/12 = **1.2 months**.

Yr	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	5											5	5		4			4					
2											2	4					3						
3																							
4						6				19							2					4	
5				3									2										4
6													8			2			2				3
7									3				11		2	5			9				
8												3	8	3									3
9	4			6					9	7		4	3	3						5			
10		5		4													3						
11										2													
12																							
13						3																	

Note: The columns indicate the corresponding sunspot cycles from 1 to 23. The calendar year for 'Year 1' for each cycle is as follows:1755, 1765, 1775, 1785, 1798, 1810, 1823, 1833, 1843, 1856, 1867, 1878, 1889, 1901, 1913, 1923, 1933, 1944, 1954, 1964, 1975, 1986, 1996

The table above gives the intensity of Solar Proton Events (SPEs) detected since 1751 (Cycle 1) through the end of the last sunspot cycle in 2008 (Cycle 23). An SPE is an intense burst of high-energy protons from the sun that reach Earth traveling at nearly the speed of light. The data for Cycle 1 - 22 were obtained by Dr. Robert McCracken in a study of ice cores from Greenland and Antarctica, which recorded the atmospheric changes caused by the most intense SPEs during each year.

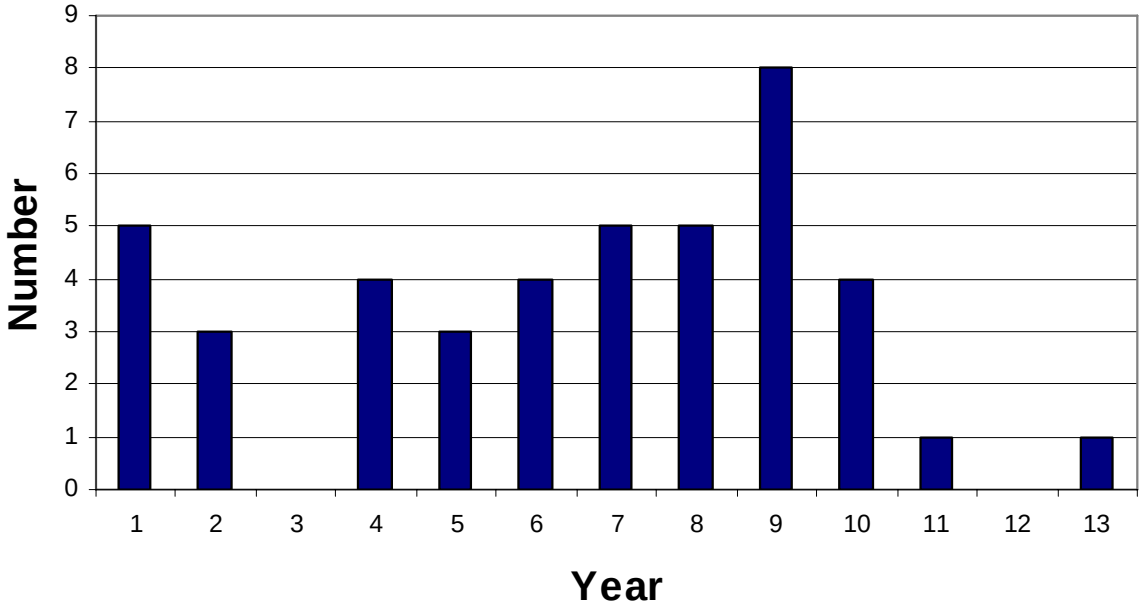
Although the ice core data was assembled chronologically as a continuous record of events since 1562, the above table folds the intensity of SPEs each year by the year of the sunspot cycle in which the SPE was recorded. The calendar year for 'Year 1' in the far-left column is given in the legend for the table.

Problem 1 - Create a histogram that tallies the number of SPEs occurring in each of the 13 years of the sunspot cycle. What is the total number of SPEs recorded for the 23 sunspot cycles?

Problem 2 - For each year in the average sunspot cycle, what is the percentage of SPEs that one expects to find?

Problem 3 - Which year of the sunspot cycle has the greatest historical percentage of SPEs?

Problem 1 - Create a histogram that tallies the number of SPEs occurring in each of the 13 years of the sunspot cycle. What is the total number of SPEs recorded for the 23 sunspot cycles?



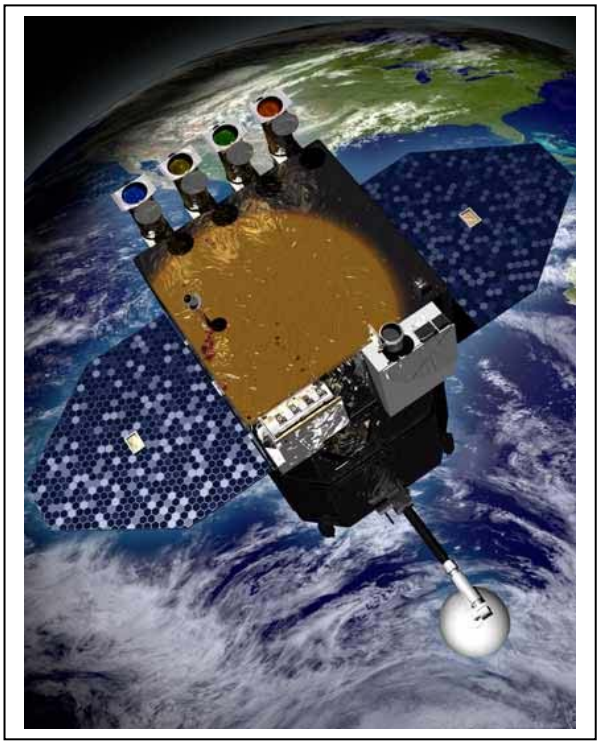
Answer: **Total = 43**

Problem 2 - For each year in the average sunspot cycle, what is the percentage of SPEs that one expects to find?

Answer:

Year	Number	Percent
1	5	12
2	3	7
3	0	0
4	4	9
5	3	7
6	4	9
7	5	12
8	5	12
9	8	19
10	4	9
11	1	2
12	0	0
13	1	2

Problem 3 - Which year of the sunspot cycle has the greatest historical percentage of SPEs?
Answer: **Year 9.**



The 15 instruments on NASA's latest solar observatory will usher in a new era of solar observation by providing scientists with 'High Definition'-quality viewing of the solar surface in nearly a dozen different wavelength bands.

One of the biggest challenges is how to handle all the data that the satellite will return to Earth, every hour of the day, for years at a time! It is no wonder that the design and construction of this data handling network has taken nearly 10 years to put together! To make sense of the rest of this story, here are some units and prefixes you need to recall (1 byte = 8 bits):

Kilo = 1 thousand
Mega = 1 million
Giga = 1 billion
Tera = 1 trillion
Peta = 1,000 trillion
Exa = 1 million trillion

Problem 1 - In 1982 an IBM PC desktop computer came equipped with a 25 megabyte hard drive (HD) and cost $6,000. In 2010, a $500 desktop comes equipped with a 2.5 gigabyte hard drive. By what factor do current hard drives have more storage space than older models?

Problem 2 - A 2.5 gigabyte hard drive is used to store music from iTunes. If one typical 4-minute, uncompressed, MPEG-4 song occupies 8 megbytes, about A) How many uncompressed songs can be stored on the HD? B) How many hours of music can be stored on the HD? (Note: music is actually stored in a compressed format so typically several thousand songs can be stored on a large HD)

Problem 3 - How long will it take to download 2 gigabytes of music from the iTunes store A) Using an old-style 1980's telephone modem with a bit rate of 56,000 bits/sec? B)With a modern fiber-optic cable with a bit rate of 16 megabits/sec?

Problem 4 - The SDO satellite's AIA cameras will generate 67 megabits/sec of data as they take 4096x4096-pixel images every 3/4 of a second. The other two instruments, the HMI and the EVE, will generate 62 megabits/sec of data. The satellite itself will also generate 20 megabits/sec of 'housekeeping' information to report on the health of the satellite. If a single DVD can store 5 gigabytes of information, how many DVDs-worth of data will be generated by the SDO: A) Each day? B) Each year?

Problem 5 - How many petabytes of data will SDO generate during its planned 5-year mission?

Problem 6 - It has been estimated that the total amount of audio, image and video information generated by all humans during the last million years through 2009 is about 50 exabytes including all spoken words (5 exabytes). How many DVDs does this equal?

Problem 1 - In 1982 an IBM PC desktop computer came equipped with a 25 megabyte hard drive (HD) and cost $6,000. In 2010, a $500 desktop comes equipped with a 2.5 gigabyte hard drive. By what factor do current hard drives have more storage space than older models? Answer: Take the ratio of the modern HD to the one in 1982 to get 2.5 billion / 25 million = 2,500,000,000/25,000,000 = 2,500/25 = **100 times.**

Problem 2 - A 2.5 gigabyte hard drive is used to store music from iTunes. If one typical 4-minute MPEG-4 song occupies 8 megbytes, about A) how many songs can be stored on the HD? B) How many hours of music can be stored on the HD?
Answer: A) Number of songs = 2.5 gigabytes/8 megabytes = 2,500 megabytes/8 megabytes = 312 B) Time = 312 songs x 4 minutes/song = 1,248 minutes = **20.8 hours.**

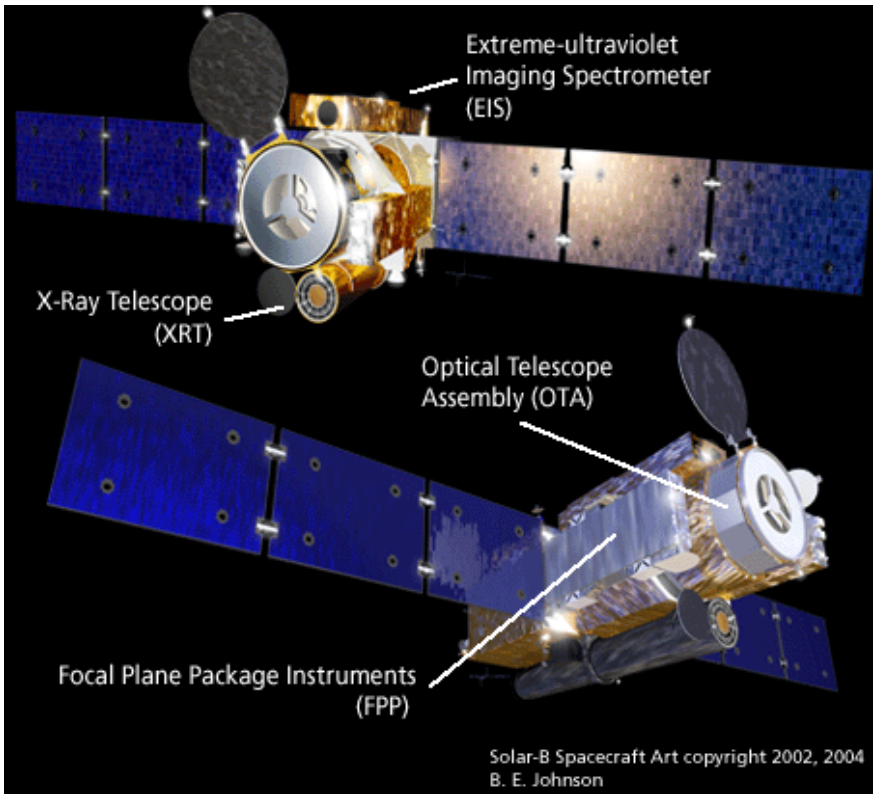
Problem 3 - How long will it take to download 2 gigabytes of music from the iTunes store A) Using an old-style 1980's telephone modem with a bit rate of 56,000 bits/sec? B)With a modern fiber-optic cable with a bit rate of 16 megabits/sec?
Answer; A) 2 gigabytes x 8 bits/1 byte = 16 gibabits. Then 16,000,000,000 bits x (1 second/56,000 bits) = 285,714 seconds or **79.4 hours**. B) 16,000,000,000 bits x (1 second/16,000,000 bits) = 1,000 seconds or about **17 minutes.**

Problem 4 - The SDO satellite's AIA cameras will generate 67 megabits/sec of data as they take 4096x4096-pixel images every 3/4 of a second. The other two instruments, the HMI and the EVE, will generate 62 megabits/sec of data. The satellite itself will also generate 20 megabits/sec of 'housekeeping' information to report on the health of the satellite. If a single DVD can store 5 gigabytes of information, how many DVDs-worth of data will be generated by the SDO each A) Day? B) Year? Answer: Adding up the data rates for the three instruments plus the satellite housekeeping we get 67+63+20 = 150 megabits/sec. A) In one day this is 150 megabits/sec x (86,400 sec/1day) = 12.96 terabites or since 1 byte = 8 bits we have 1.6 terabytes. This equals 1,600 gigabytes x (1 DVD/8 gigabytes) = **200 DVDs each day**. B) In one year this equals 365 days x 1.6 terabytes/day = 584 terabytes per year or 365 days/1 year x 200 DVDs/1 year = **73,000 DVDs/year.**

Problem 5 - How many petabytes of data will SDO generate during its planned 5-year mission? Answer: In 5 years it will generate 5 years x 584 terabytes/year = 2,920 terabytes. Since 1 petabyte = 1,000 terabytes, this becomes **2.9 petabytes**.

Problem 6 - It has been estimated that the total amount of audio, image and video information generated by all humans during the last million years through 2009 is about 50 exabytes including all spoken words (5 exabytes). How many DVDs does this equal? Answer: 1 exabyte = 1,000 petabytes = 1,000,000 terabytes = 1,000,000,000 gigabytes. So 50 exabytes equals 50 billion gigabytes. One DVD stores 5 gigabytes, so the total human information 'stream' would occupy 50 billion / 5 billon = **10 billion DVDs.**

Hinode Satellite Power



The Hinode satellite weighs approximately 700 kg (dry) and carries 170 kg of gas for its steering thrusters, which help to maintain the satellite in a polar, sun-synchronous orbit for up to two years. The satellite has two solar panels (blue) that produce all of the spacecraft's power. The panels are 4 meters long and 1 meter wide, and are covered on both sides by solar cells.

Problem 1 - What is the total area of the solar panels covered by solar cells in square centimeters?

Problem 2 - If a solar cell produces 0.03 watts of power for each square centimeter of area, what is the total power produced by the solar panels when facing the sun? Can the satellite supply enough power to operate the experiments which require 1,150 watts?

Problem 3 - Suppose engineers decided to cover the surface of the cylindrical satellite body with solar cells instead. If the satellite is 4 meters long and a diameter of 1 meter, how much power could it produce if only half of the area was in sunlight at a time? Can the satellite supply enough power to keep the experiments running, which require 1,150 watts?

Answer Key:

Problem 1 - What is the total area of the solar panels covered by solar cells in square centimeters?

Answer: The surface area of a single panel is 4 meters x 1 meter = 4 square meter per side. There are two sides, so the total area of one panel is 8 square meters. There are two solar panels, so the total surface area covered by solar cells is 16 square meters. Converting this to square centimeters:

$$16 \text{ square meters} \times (10{,}000 \text{ cm}^2/\text{m}^2) = \mathbf{160{,}000\ cm^2}$$

Problem 2 - If a solar cell produces 0.03 watts of power for each square centimeter of area, what is the total power produced by the solar panels when facing the sun? Can the satellite supply enough power to operate the experiments which require 1,150 watts?

Answer: Only half of the solar cells can be fully illuminated at a time, so the total exposed area is 80,000 cm^2. The power produced is then:

$$\text{Power} = 80{,}000 \text{ cm}^2 \times 0.03 \text{ watts/cm}^2 = \mathbf{2{,}400\ watts.}$$

Yes, the satellite solar panels can keep the experiments running, with 2400-1150 = 1,250 watts to spare!

Problem 3 - Suppose engineers decided to cover the surface of the cylindrical satellite body with solar cells instead. If the satellite is 4 meters long and a diameter of 1 meter, how much power could it produce if only half of the area was in sunlight at a time? Can the satellite supply enough power to keep the experiments running, which require 1,150 watts?

Answer - Surface area of a cylinder = Area of 2 circular end caps + area of side of cylinder

$$= 2\pi R^2 + 2\pi R h$$

$$S = 2 \times (3.14)\ (0.5 \text{ meters})^2 + 2 \times (3.14)\ (0.5 \text{ meters})\ (4 \text{ meters})$$
$$= 1.57 \text{ square meters} + 12.56 \text{ square meters}$$
$$= 14.13 \text{ square meters.}$$

Only half of the solar cells can be illuminated, so the usable area is 7.06 square meters or 70,600 square centimeters. The power produced is 70600 x 0.03 = **2,100 watts.**

Yes..the satellite can keep the experiments running with this solar cell configuration.

Sunspot Size and the Sunspot Cycle

Sunspots are some of the most interesting, and longest studied, phenomena on the sun's surface. They were known to ancient Chinese observers over 2000 years ago. Below is a list of the largest sunspots seen since 1859. Their sizes are given in terms of the area of the solar hemisphere facing earth. For example, '3600' means 3600 millionths of the solar area or (3600/1000000). On this scale, the area of Earth is '169'. All of these spots were large enough to be seen with the naked eye when proper (and safe!) viewing glasses were used.

Date	Size	Earths
February 10, 1917	3600	**21.3**
January 25, 1926	3700	
January 18, 1938	3650	
February 6, 1946	5250	
July 27, 1946	4700	
March 10, 1947	4650	
April 7, 1947	6150	
May 16, 1951	4850	
Nov. 14, 1970	3500	
August 23, 1971	3500	
October 30, 1972	4120	
Nov. 11, 1980	3820	
July 28, 1981	3800	

Date	Size	Earths
October 14, 1981	4180	
October 19, 1981	4500	
February 10, 1982	3800	
June 18, 1982	4400	
July 15, 1982	4900	
April 28, 1984	5400	
May 13, 1984	3700	
March 13, 1989	5230	
September 5, 1989	3500	

Years of Peak Sunspot Activity:

Sunspot Cycle	Peak Year
14	1906
15	1917
16	1928
17	1937
18	1947
19	1958
20	1968
21	1979
22	1989

Question 1: The peaks of the 11-year sunspot cycle during the 20th century occurred during the years shown to the left. On average, how close to sunspot maximum do the largest spots occur?

Question 2: From this sample, do more of these spots happen in the 5 years before sunspot maximum, or within 5 years after sunspot maximum?

Question 3: Convert the sunspot areas into an equivalent area of the Earth. (Note the first one has been done as an example). What is the average large spot size in terms of Earth?

Date	Size	Area In Earths	Nearest Sunspot Max. Year	Difference In Years
February 10, 1917	3600	21.3	1917	0
January 25, 1926	3700	21.9	1928	-2
January 18, 1938	3650	21.6	1937	+1
February 6, 1946	5250	31.1	1947	-1
July 27, 1946	4700	27.8	1947	-1
March 10, 1947	4650	27.5	1947	0
April 7, 1947	6150	36.4	1947	0
May 16, 1951	4850	28.7	1951	0
Nov. 14, 1970	3500	20.7	1968	+2
August 23, 1971	3500	20.7	1968	+3
October 30, 1972	4120	24.4	1968	+4
Nov. 11, 1980	3820	22.6	1979	+1
July 28, 1981	3800	22.5	1979	+2
October 14, 1981	4180	24.7	1979	+2
October 19, 1981	4500	26.6	1979	+2
February 10, 1982	3800	22.5	1979	+3
June 18, 1982	4400	26.0	1979	+3
July 15, 1982	4900	29.0	1979	+3
April 28, 1984	5400	32.0	1989	-5
May 13, 1984	3700	21.9	1989	-5
March 13, 1989	5230	31.0	1989	0
September 5, 1989	3500	20.7	1989	0

Question 1: On average, how close to sunspot maximum do the largest spots occur? **Answer**: Find the average of the differences in last column = 0 years. So, the largest sunspots occur, on average, close to the peak of the sunspot cycle.

Question 2: For this sample, do more of these spots happen in the 5 years before sunspot maximum, or within 5 years after sunspot maximum? **Answer**: More happen after the peak. Five happen before the peak (negative differences) and 11 happen after the peak (Positive differences).

Question 3: What is the average large spot size in terms of Earth? **Answer:** Average = (561.6/22) = 25.5 Earth Areas.

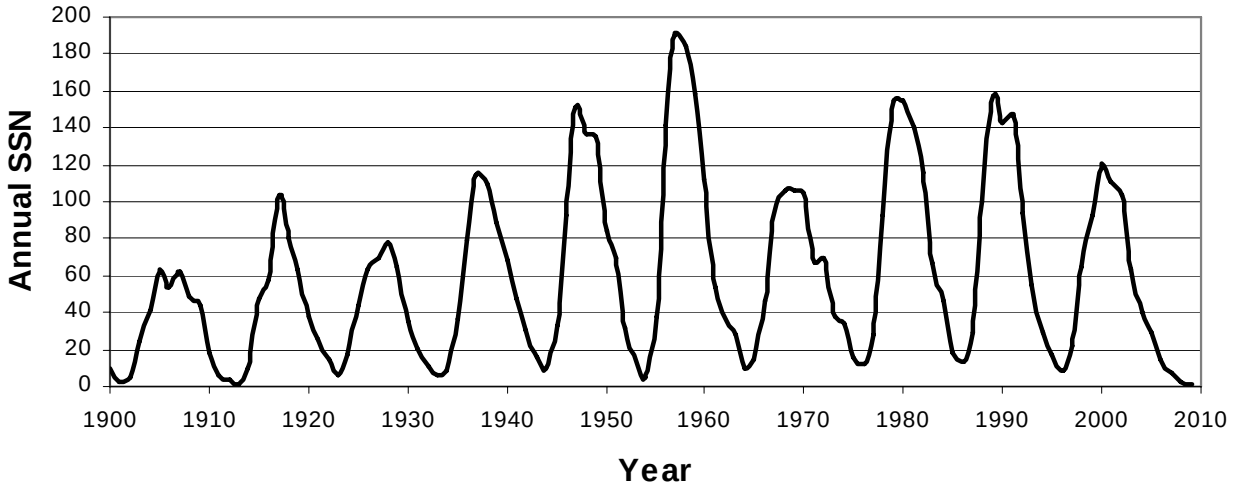
Table of Sunspot Numbers Since 1900

Year	SSN	Year	SSN	Year	SSN	Year	SSN
1900	9	1928	78	1956	142	1984	46
1901	3	1929	65	1957	190	1985	18
1902	5	1930	36	1958	185	1986	13
1903	24	1931	21	1959	159	1987	29
1904	42	1932	11	1960	112	1988	100
1905	63	1933	6	1961	54	1989	158
1906	54	1934	9	1962	38	1990	142
1907	62	1935	36	1963	28	1991	146
1908	49	1936	80	1964	10	1992	94
1909	44	1937	114	1965	15	1993	55
1910	19	1938	110	1966	47	1994	30
1911	6	1939	89	1967	94	1995	18
1912	4	1940	68	1968	106	1996	9
1913	1	1941	47	1969	106	1997	21
1914	10	1942	31	1970	105	1998	64
1915	47	1943	16	1971	67	1999	93
1916	57	1944	10	1972	69	2000	120
1917	104	1945	33	1973	38	2001	111
1918	81	1946	93	1974	34	2002	104
1919	64	1947	152	1975	15	2003	64
1920	38	1948	136	1976	13	2004	40
1921	26	1949	135	1977	27	2005	30
1922	14	1950	84	1978	93	2006	15
1923	6	1951	69	1979	155	2007	8
1924	17	1952	31	1980	155	2008	3
1925	44	1953	14	1981	140	2009	2
1926	64	1954	4	1982	116		
1927	69	1955	38	1983	67		

In 1843, the German astronomer Samuel Schwab plotted the number of sunspots on a graph and discovered that the number of sunspots on the sun was not the same every year, but goes through a cyclical increase and decrease over time. The table above gives the average number of sunspots (SSN) detected each year since 1900.

Problem 1 - Graph the data in the table over the domain [1900, 2009] and range [0,200]. How many sunspot cycles can you count over this time interval?

Problem 2 - What is the average period of the sunspot cycle during this time interval?



Problem 1 - Graph the data in the table over the domain [1900, 2009] and range [0,200]. How many sunspot cycles can you count over this time interval?

Answer: **There are 10 complete cycles**.

Problem 2 - What is the average period of the sunspot cycle during this time interval?

Answer: The time interval is 109 years during which 10 cycles occur for an average period of **10.9 years**.

Note: Additional sunspot data can be found at

Spaceweather.com **http://www.spaceweather.com/java/archive.html**

NASA Marshall Spaceflight Center
http://solarscience.msfc.nasa.gov/SunspotCycle.shtml

NOAA Space Weather Prediction Center

http://www.swpc.noaa.gov/ftpmenu/warehouse.html
Select year from list, select file 'DSD.txt' and open. Column 5 gives average SSN for given day.

Are you ready for Sunspot Cycle 24?

In 2007, the best estimates and observations suggested that we had just entered sunspot minimum, and that the next solar activity cycle might begin during the first few months of 2008. In May, 2007, solar physicist Dr. William Pesnell at the NASA, Goddard Spaceflight Center tabulated all of the current predictions for when the next sunspot cycle (2008-2019) will reach its peak. These predictions, reported by many other solar scientists are shown in the table below.

Current Predictions for the Next Sunspot Maximum

Author	Prediction Year	Spots	Year	Method Used
Horstman	2005	185	2010.5	Last 5 cycles
Thompson	2006	180		Precurser
Tsirulnik	1997	180	2014	Global Max
Podladchikova	2006	174		Integral SSN
Dikpati	2006	167		Dynamo Model
Hathaway	2006	160		AA Index
Pesnell	2006	160	2010.6	Cycle 24 = Cycle22
Maris & Onicia	2006	145	2009.9	Neural Network Forecast
Hathaway	2004	145	2010	Meridional Circulation
Gholipour	2005	145	2011.5	Spectral Analysis
Chopra & Davis	2006	140	2012.5	Disturbed Day Analysis
Kennewell	2006	130		H-alpha synoptic charts
Tritakis	2006	133	2009.5	Statistics of Rz
Tlatov	2006	130		H-alpha Charts
Nevanlinna	2007	124		AA at solar minimum
Kim	2004	122	2010.9	Cycle parameter study
Pesnell	2006	120	2010	Cycle 24 = Cycle 23
Tlatov	2006	115		Unipolar region size
Tlatov	2006	115		Large Scale magnetic field
Prochasta	2006	119		Average of Cycles 1 to 23
De Meyer	2003	110		Transfer function model
Euler & Smith	2006	122	2011.2	McNish-Lincoln Model
Hiremath	2007	110	2012	Autoregressive Model
Tlatov	2006	110		Magnetic Moments
Lantos	2006	108	2011	Even/Odd cycle pattern
Kane	1999	105	2010.5	Spectral Components
Pesnell	2006	101	2012.6	Linear Prediction
Wang	2002	101	2012.3	Solar Cycle Statistics
Roth	2006	89	2011.1	Moving averages
Duhau	2003	87		Sunspot Maxima and AA
Baranovski	2006	80	2012	Non-Linear Dynamo model
Schatten	2005	80	2012	Polar Field Precurser
Choudhuri	2007	80		Flux Transport Dynamo
Javariah	2007	74		Low-Lat. Spot Groups
Svalgaard	2005	70		Polar magnetic field
Kontor	2006	70	2012.9	Statistical extrapolation
Badalyan	2001	50	2010.5	Coronal Line
Cliverd	2006	38		Atmospheric Radiocarbon
Maris	2004	50		Flare energy during Cycle 23

Problem 1: What is the average year for the predicted sunspot maximum?
Problem 2: What is the average prediction for the total number of sunspots during the next sunspot maximum?
Problem 3: Which scientist has offered the most predictions? Do they show any trends?
Problem 4: What is the average prediction for the total sunspots during each prediction year from 2003 to 2006?
Problem 5: As we get closer to sunspot minimum in 2008, have the predictions for the peak sunspots become larger, smaller, or remain unchanged on average?
Problem 6: Which methods give the most different prediction for the peak sunspot number, compared to the average of the predictions made during 2006?

Answer Key:

Problem 1: What is the average year for the predicted sunspot maximum?
Answer: There are 21 predictions, with an average year of 2011.3.
This corresponds to about March, 2011.

Problem 2: What is the average prediction for the total number of sunspots during the next sunspot maximum?
Answer: The average of the 39 estimates in column 3 is 116 sunspots at sunspot maximum.

Problem 3: Which scientist has offered the most predictions? Do they show any trends?
Answer: Tlatov has offered 4 predictions, all made in the year 2006. The predicted numbers were 130, 115, 115 and 110. There does not seem to be a significant trend towards larger or smaller predictions by this scientist. The median value is 115 and the mode is also 115.

Problem 4: What is the average prediction for the total sunspots during each prediction year from 2003 to 2006?
Answer: Group the predictions according to the prediction year and then find the average for that year.

2003: 110, 87 average = 98

2004: 145, 122, 50 average= 106

2005: 185,145,80,70 average= 120

2006: 180,174,167,160,160,145,140,130,133,130,120,
115,115,119,122,110,108,101,89,80,70,38 **average= 123**

Problem 5: As we get closer to sunspot minimum in 2008, have the predictions for the peak sunspots become larger, smaller, or remain unchanged on average?
Answer: Based on the answer to problem 4, it appears that the predictions have tended to get larger, increasing from about 98 to 123 between 2003 and 2006.

Problem 6: Which methods give the most different prediction for the peak sunspot number, compared to the average of the predictions made during 2006?
Answer: Cliverd's Atmospheric Radiocarbon Method (38 spots), Badalyan's Coronal Line Method (50 spots), and Maris's Flare Energy during Cycle 23 (50 spots) seem to be the farthest from the average predictions that have been made by other forecasting methods.

Solar Storms: Odds, Fractions and Percentages

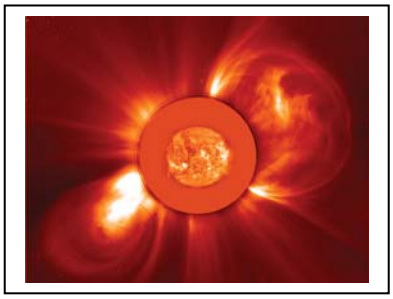
One of the most basic activities that scientists perform with their data is to look for correlations between different kinds of events or measurements in order to see if a pattern exists that could suggest that some new 'law' of nature might be operating. Many different observations of the Sun and Earth provide information on some basic phenomena that are frequently observed. The question is whether these phenomena are related to each other in some way. Can we use the sighting of one phenomenon as a prediction of whether another kind of phenomenon will happen?

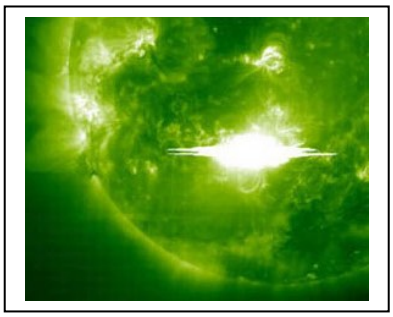
During most of the previous sunspot cycle (January-1996 to June-2006), astronomers detected 11,031 coronal mass ejections, (CME: Top image) of these 1186 were 'halo' events. Half of these were directed towards Earth.

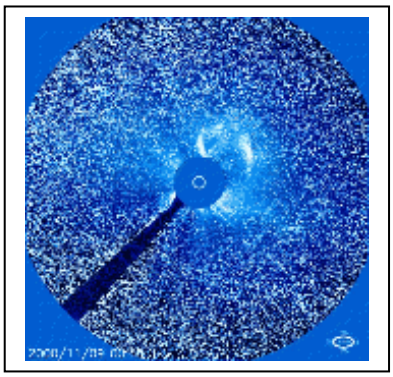
During the same period of time, 95 solar proton events (streaks in te bottom image were caused by a single event) were recorded by the GOES satellite network orbiting Earth. Of these SPEs, 61 coincided with Halo CME events.

Solar flares (middle image) were also recorded by the GOES satellites. During this time period, 21,886 flares were detected, of which 122 were X-class flares. Of the X-class flares, 96 coincided with Halo CMEs, and 22 X-class flares also coincided with 22 combined SPE+Halo CME events. There were 6 X-flares associated with SPEs but not associated with Halo CMEs. A total of 28 SPEs were not associated with either Halo CMEs or with X-class solar flares.

From this statistical information, construct a Venn Diagram to interrelate the numbers in the above findings based on resent NASA satellite observations, then answer the questions below.







1 - What are the odds that a CME is directed towards Earth?

2 - What fraction of the time does the sun produce X-class flares?

3 - How many X-class flares are not involved with CMEs or SPEs?

4 - If a satellite spotted both a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur?

5 - What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both?

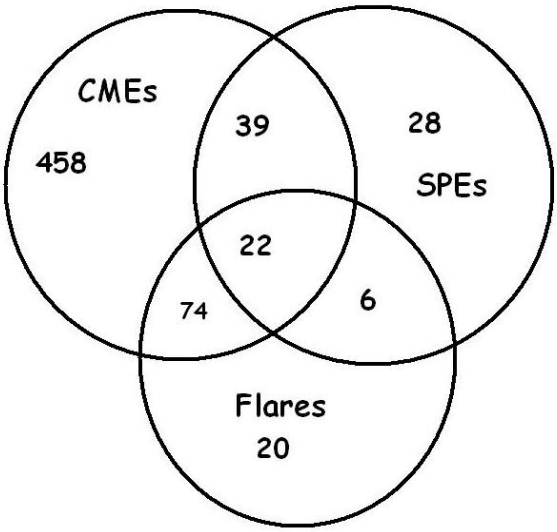
6 - If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed?

7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed?

8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?

9 - If you observed 100 CMEs, X-class flares and SPEs, how many times might you expect to see all three phenomena?

Answer Key



Venn Diagram Construction.

1. There are 593 Halo CMEs directed to Earth so 593 = 74 with flares + 39 with SPEs + 22 both SPEs and Flares + 458 with no SPEs or Flares..

2. There are 95 SPEs. 95 = 39 with CMEs + 6 with flares + 22 with both flares and CMEs + 28 with no flares or CMEs

3. There are 122 X-class flares. 122 = 74 With CMEs only + 6 with SPEs only + 22 both CMEs and SPEs + 20 with no CMEs or SPEs.

1 - What are the odds that a CME is directed towards Earth? 593/11031 = 0.054 **odds = 1 in 19**

2 - What fraction of the time does the sun produce X-class flares? 122/21886 = **0.006**

3 - How many X-class flares are not involved with CMEs or SPEs? 122 - 74 - 22 - 6 = **20.**

4 - If a satellite spotted BOTH a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur? 22/(74+22) = **0.23**

5 - What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both?
100% x (39+22+6 / 95) = **70.1 %**

6 - If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed?
39+22+74 / 593 = 0.227 so the odds are 1/0.227 or about **1 in 4.**

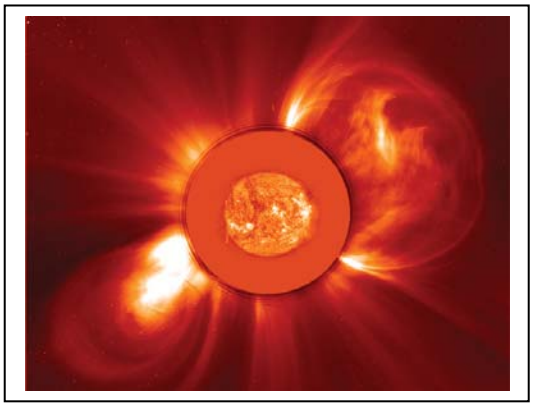
7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed?
(6+22)/95 = 0.295 or 1 out of 3 times for X-flares
(39+22)/95 = 0.642 or 2 out of 3 for Halo CMEs
It is more likely to detect an SPE if a Halo CME occurs by 2 to 1.

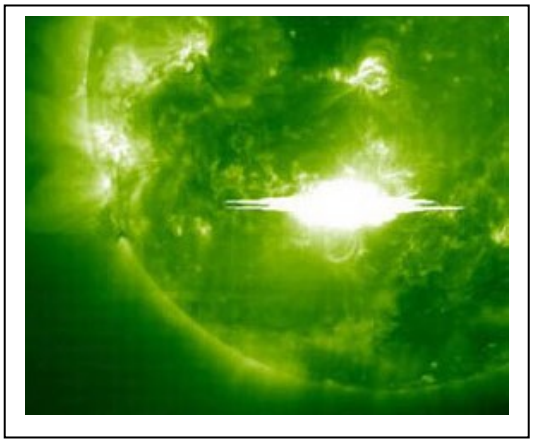
8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?
39+6 / 95 = 0.50 so the odds are 1/0.50 or **2 to 1 .**

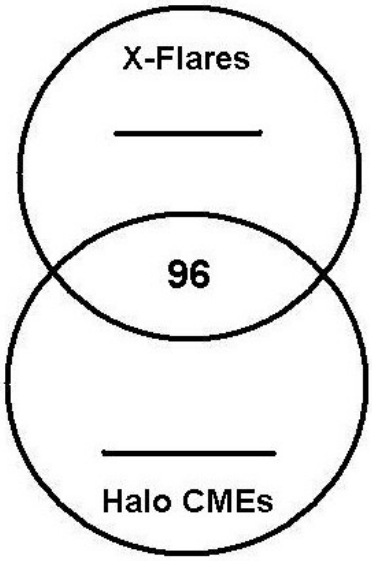
9 - If you observed 100 CMEs, X-class flares and SPEs, how many times might you expect to see all three phenomena?

100 x 22/(95+122+593) = **3 times**

Studying Solar Storms with Venn Diagrams







Solar storms come in two varieties:

Coronal Mass Ejections (CMEs) are clouds of gas ejected from the sun that can reach Earth and cause the Aurora Borealis (Northern Lights). These clouds can travel at over 2 million kilometers/hr, and carry billions of tons of matter in the form of charged particles (called a plasma). The picture to the left shows one of many CMEs witnessed by the SOHO satellite.

Solar Flares are intense bursts of X-ray energy that can cause short-wave radio interference on Earth. The picture to the left shows a powerful X-ray flare seen by the SOHO satellite on October 28, 2003.

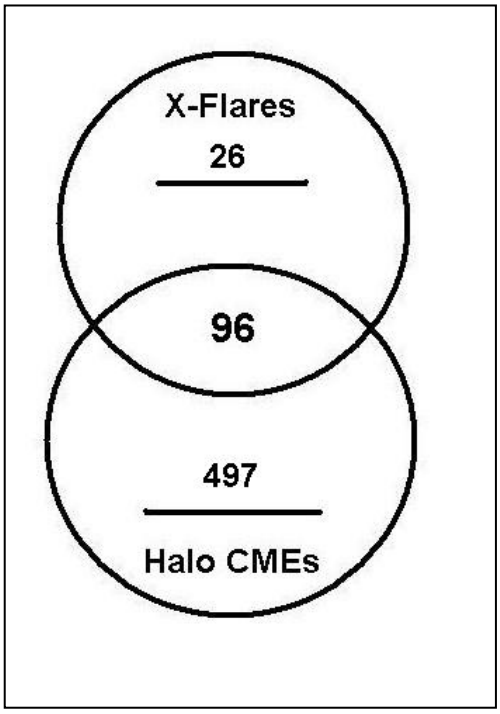
Between 1996 and 2006, astronomers detected 11,031 coronal mass ejections (CMEs), and of these, 593 were directed towards Earth. These are called 'Halo CMEs' because the ejected gas surrounds the sun's disk on all sides and looks, like a halo around the sun. During these same years, astronomers also witnessed 122 solar flares that were extremely intense X-flares. Of these X-flares, 96 happened at the same time as the Halo CMEs.

Problem 1 - From this statistical information, fill-in the missing numbers in the circular Venn Diagram to the left.

Problem 2 – What percentage of X-Flares also happened at the same time as a Halo CME?

Problem 3 – What percentage of Halo CMEs happened at the same time as an X-Flare?

Problem 4 – What percentage of all CMEs detected between 1996 and 2006 produced X-Flares?



Answer 1 - The total number of Halo CMEs is 593 and the total number of X-Flares is 122. The intersecting area of the two circles in the Venn Diagram shows the 96 events in which a Halo CME and X-Flare are BOTH seen together. The areas of the circles not in the intersection represent all of the X-flares that are not spotted with Halo CMEs (top ring) and all of the Halo CMEs that are not spotted with X-Flares (bottom ring).

The missing number in the X-Flare ring is just 122 – 96 = 26, and for the Halo CMes we have 593 – 96 = 497.

Answer 2 – The total number of X-Flares is 122 and of these only 96 occurred with a Halo CME, so the fraction of X-Flares is just 96/122 = 0.79. In terms of percentage, this represents 79 %.

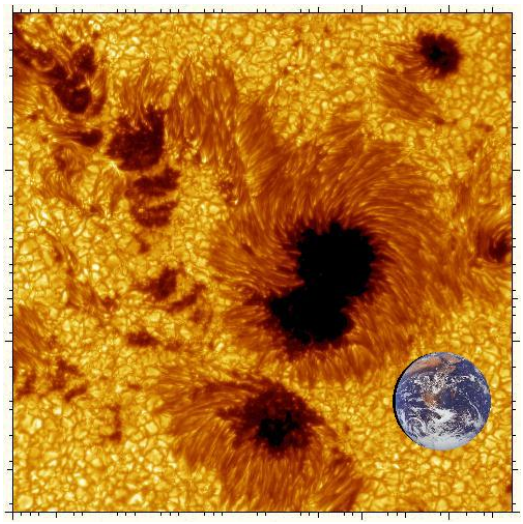
Answer 3 – The total number of Halo CMEs is 593 and of these only 96 occurred with an X-Flare, so the fraction of Halo CMEs is just 96/593 = 0.16. In terms of percentage, this represents 16 %.

Answer 4 – There were 11,031 CMEs detected, and of these only 96 coincided with X-Flares, so the fraction is 96/11031 = 0.0087. In terms of percentage, this represents 0.87 % or less than 1 % of all CMEs.

Space Weather Indicators

Sunspots are a sign that the Sun is in a stormy state. Sometimes these storms can affect Earth and cause all kinds of problems such as satellite damage and electrical power outages. They can even harm astronauts working in space.

Scientists use many different kinds of measurements to track this stormy activity. In this exercise, you will learn how to use some of them!



This sunspot is as big as Earth!

Looking at sequences of numbers can help you identify unusual events that depart from the average trend.

Here's how to do it!

An astronomer counts sunspots for 5 days and gets the following sequence:

149, 136, 198, 152, 145

Maximum = 198
Minimum = 136
Mean = (149+136+198+152+145)/5 = 156
Median = 149

Find the maximum, minimum, mean and median of each sequence.

1) Number of Sunspots

241	240	243	229	268	335	342	401	325	290	276	232	214

2) Number of Solar Flares

5	7	13	8	9	14	9	13	16	6	14	15

3) Aurora Power (measured in billions of watts!)

171.2	122.2	219.4	107.9	86.2	112.4	76.2	39.8	153.9

Answer Key

Problem 1 - Maximum= 401, minimum=214;
Ordered = 214, 229 ,232 ,240, 241, 243 ,268, 276, 290, 325, 335 ,342, 401
Median = 268
Mean = 264

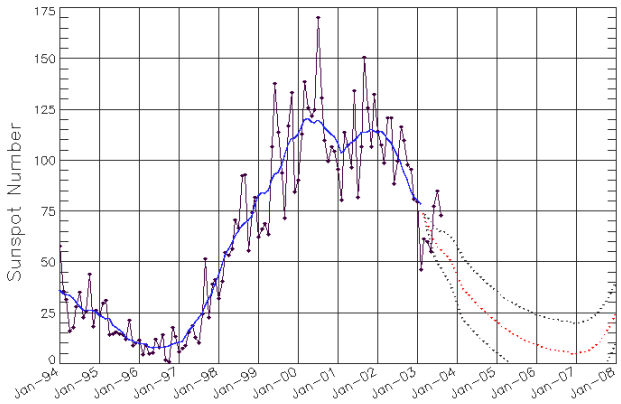
Problem 2 - Maximum= 16, minimum= 5
Ordered = 5, 6, 7, 8, 9, 9, 13, 13, 14, 14, 15
Median = 9
Mean = 10

Problem 3 - Maximum = 219.4; minimum=39.8
Ordered = 39.8 ,76.2 ,86.2 ,107.9 ,112.4, 122.2, 153.9, 171.2, 219.4
Median = 112.4
Mean = 121.0

Sunspot cycles

The Sun is an active star that goes through cycles of high and low activity. Scientists mark these changes by counting sunspots. The numbers of spots increase and decrease about every 11 years in what scientists call the Sunspot Cycle.

This activity will let you investigate how many years typically elapse between the sunspot cycles. Is the cycle really, exactly 11-years long?



The sunspot cycle between 1994 and 2008

Scientists study many phenomena that run in cycles. The Sun provides a number of such 'natural rhythms' in the solar system.

Sequences of numbers often have maximum and minimum values that re-occur periodically.

Here's how to do it!

Consider the following measurements taken every 5 minutes:

100, 200, 300, 200, 100, 200, 300, 200

1. **There are two maxima (value '300').**
2. **The maxima are separated by 4 intervals.**
3. **The cycle has a period of 4 x 5 = 20 minutes.**
4. **The pairs of minima (value = 100) are also separated by this same period of time.**

Now you try!

Sunspot Numbers

Solar Maximum | Solar Minimum

Year	Number	Year	Number
2000	125	1996	9
1990	146	1986	14
1980	154	1976	13
1969	106	1964	10
1957	190	1954	4
1947	152	1944	10
1937	114	1933	6
1928	78	1923	6
1917	104	1913	1
1905	63	1901	3
1893	85	1889	6
1883	64	1879	3
1870	170	1867	7

This table gives the sunspot numbers for pairs of maximums and minimums in the sunspot cycle.

1) From the solar maximum data, calculate the number of years between each pair of maxima.

2) From the solar minimum data, calculate the number of years between each pair of minima.

3) What is the average time between solar maxima?

4) What is the average time between solar minima?

5) Combining the answers to #3 and #4, what is the average sunspot cycle length?

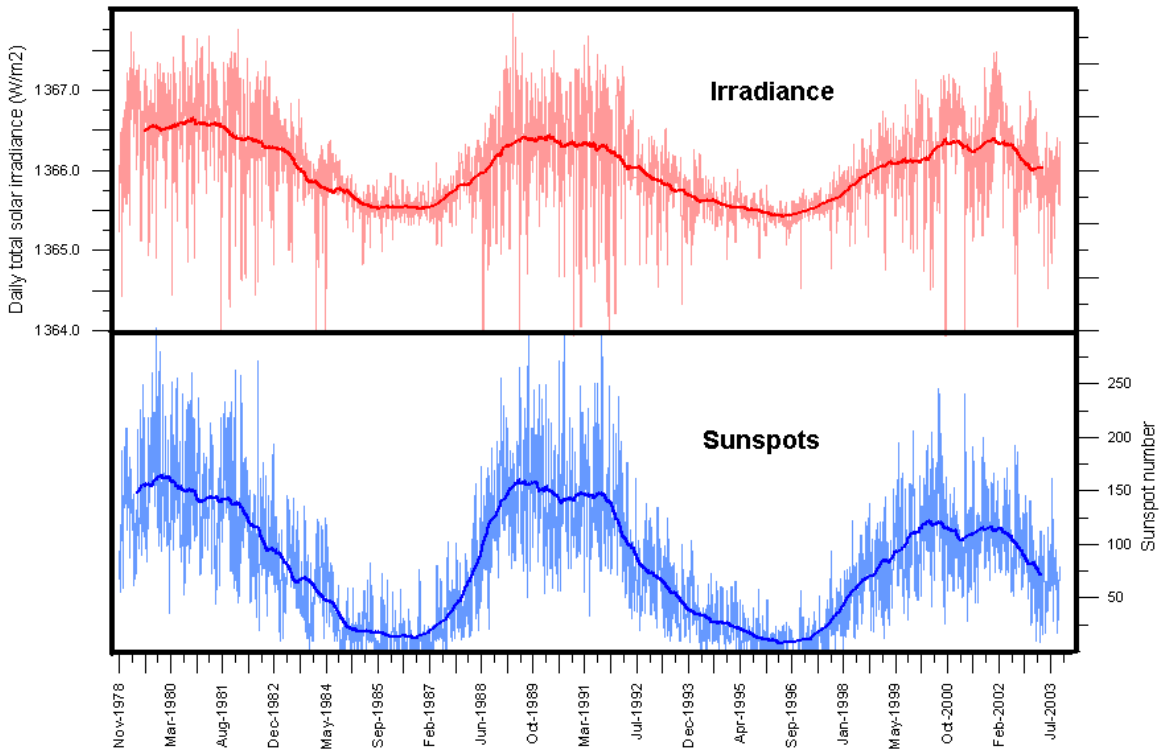
Answer Key

Problem 1 - Maxima Table		Problem 2 - Minima Table	
Year	Difference	Year	Difference
2000		1996	
1990	10	1986	10
1980	10	1976	10
1969	11	1964	12
1957	12	1954	10
1947	10	1944	10
1937	10	1933	11
1928	9	1923	10
1917	11	1913	10
1905	12	1901	12
1893	12	1889	12
1883	10	1879	10
1870	13	1867	12

Problem 3 - Average time = 130/12 = 10.8 years between sunspot maxima

Problem 4 - Average time = 129/12 = 10.8 years between minima

Problem 5 - Average length = (10.8 + 10.8)/2 = 10.8 years.



Irradiance (also called insolation) is a measure of the amount of sunlight power that falls upon one square meter of exposed surface, usually measured at the 'top' of Earth's atmosphere. This energy increases and decreases with the season and with your latitude on Earth, being lower in the winter and higher in the summer, and also lower at the poles and higher at the equator. But the sun's energy output also changes during the sunspot cycle!

The figure above shows the solar irradiance and sunspot number since January 1979 according to NOAA's National Geophysical Data Center (NGDC). The thin lines indicate the daily irradiance (red) and sunspot number (blue), while the thick lines indicate the running annual average for these two parameters. The total variation in solar irradiance is about 1.3 watts per square meter during one sunspot cycle. This is a small change compared to the 100s of watts we experience during seasonal and latitude differences, but it may have an impact on our climate. The solar irradiance data obtained by the ACRIM satellite, measures the total number of watts of sunlight that strike Earth's upper atmosphere before being absorbed by the atmosphere and ground.

Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003?

Problem 2 - What appears to be the relationship between sunspot number and solar irradiance?

Answer Key

Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003?

Answer: Draw a horizontal line across the upper graph that is mid-way between the highest and lowest points on the curve. An approximate answer would be **1366.3 watts per square meter.**

Problem 2 - What appears to be the relationship between sunspot number and solar irradiance?

Answer: When there are a lot of sunspots on the sun (called sunspot maximum) the amount of solar radiation is higher than when there are fewer sunspots. The solar irradiance changes follow the 11-year sunspot cycle.

Sunspots and Satellite Re-entry

Satellite technology is everywhere! Right now, there are over 1587 working satellites orbiting Earth. They represent over $160 billion in assets to the world's economy. In the United States alone, satellites and the many services they provide produce over $225 billion every year. But satellites do not work forever. Typically they have to be replaced every 10 to 15 years as new services are created, and better technology is developed. Satellites in the lowest orbits, called Low Earth Orbit (LEO) orbit between 300 to 1000 kilometers above the ground. Because Earth's atmosphere extends hundreds of kilometers into space, LEO satellites eventually experience enough frictional drag from the atmosphere that at altitudes below 300 km, they fall back to Earth and burn up. The table below gives the number of LEO satellites that re-entered Earth's atmosphere, and the average sunspot number, for each year since 1969.

Year	Sunspots	Satellites
2004	43	19
2003	66	31
2002	109	38
2001	123	41
2000	124	37
1999	96	25
1998	62	30
1997	20	21
1996	8	22
1995	18	20
1994	31	17
1993	54	28
1992	93	41
1991	144	40
1990	145	30
1989	162	45
1988	101	33
1987	29	13
1986	11	16
1985	16	17
1984	43	14
1983	65	28
1982	115	19
1981	146	32
1980	149	41
1979	145	42
1978	87	33
1977	26	18
1976	12	16
1975	14	15
1974	32	21
1973	37	14
1972	67	12
1971	66	19
1970	107	25
1969	105	26

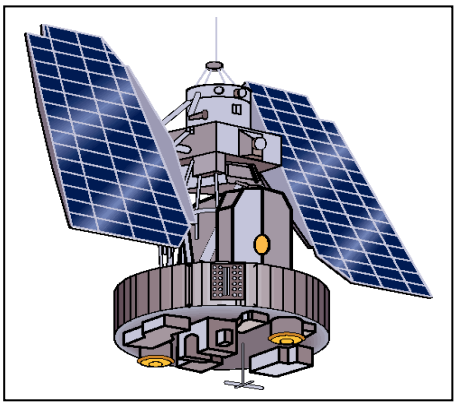


Figure: A typical weather satellite

Problem 1: On the same graph, plot the number of sunspots and decayed satellites (vertical axis) for each year (horizontal axis). During what years did the peaks in the sunspots occur?

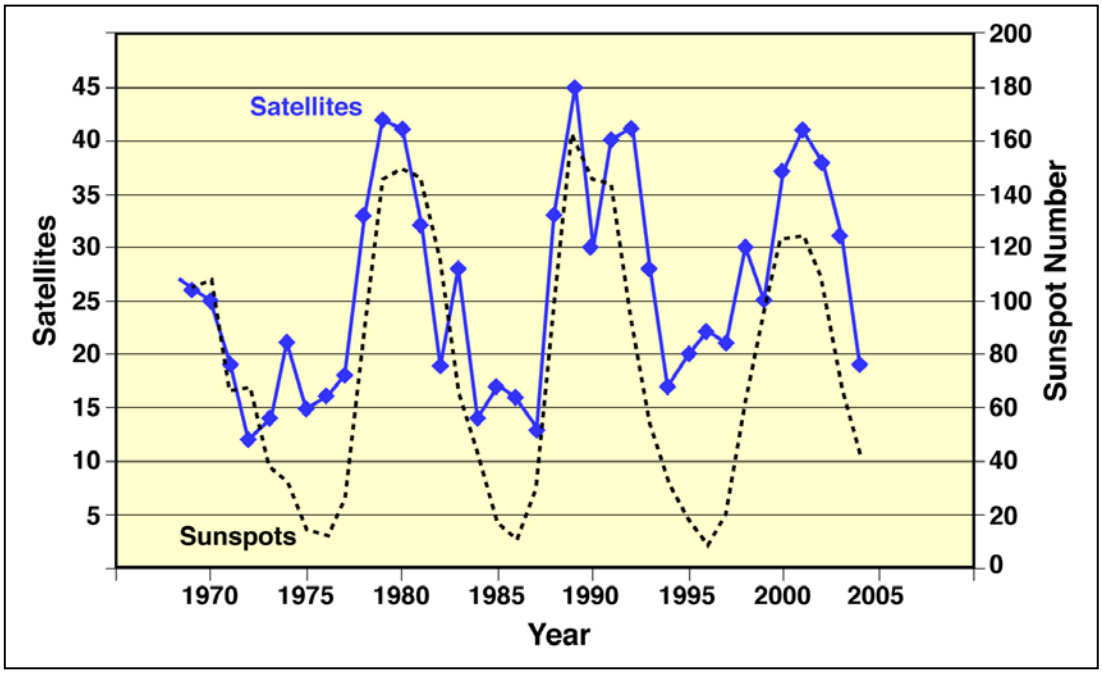
Problem 2: When did the peaks in the satellite re-entries occur?

Problem 3: Is there a correlation between the two sets of data?

Problem 4: If you are a satellite operator, should you be concerned about the sunspot cycle?

Problem 5: Do some research on the topic of how the sun affects Earth. Can you come up with at least two ways that the sun could affect a satellite's orbit?

Problem 6: Can you list some different ways that you rely on satellites, or satellite technology, during a typical week?



Problem 1: On the same graph, plot the number of sunspots (divided by 4) and decayed satellites for each year. During what years did the peaks in the sunspots occur? **Answer:** From the graph or the table, the 'sunspot maximum' years were 2000, 1989, 1980 and 1970.

Problem 2: When did the peaks in the satellite re-entries occur? **Answer:** The major peaks occurred during the years 2001, 1989, 1979 and 1969.

Problem 3: Is there a correlation between the two sets of data? **Answer:** A scientist analyzing the two plots 'by eye' would be impressed that there were increases in the satellite decays that occurred within a year or so of the sunspot maxima years. This is more easy to see if you subtract the overall 'trend' line which is increasing from about 10 satellites in 1970 to 20 satellites in 2004. What remains is a pretty convincing correlation between sunspots numbers and satellite re-entries.

Problem 4: If you were a satellite operator, should you be concerned about the sunspot cycle? **Answer:** Yes, because for LEO satellites there seems to be a good correlation between satellite re-entries near the times of sunspot maxima.

Problem 5: Do some research on the topic of how the sun affects Earth. Can you come up with at least two ways that the sun could affect a satellite's orbit? **Answer:** The answers may vary, but as a guide, space physicists generally believe that during sunspot maxima, the sun's produces more X-rays and ultraviolet light, which heat Earth's upper atmosphere. This causes the atmosphere to expand into space. LEO satellites then experience more friction with the atmosphere, causing their orbits to decay and eventually causing the satellite to burn-up. There are also more 'solar storms', flares and CMEs during sunspot maximum, than during sunspot minimum. These storms affect satellites in space, causing loss of data or operation, and can also cause electrical blackouts and other power problems.

Problem 6: Can you list some different ways that you rely on satellites, or satellite technology, during a typical week? **Answer:** Satellite TV, ATM banking transactions, credit card purchases, paying for gas at the gas pump, weather forecasts, GPS positions from your automobile, news reports from overseas, airline traffic management, tsunami reports in the Pacific Basin, long distance telephone calls, internet connections to pages overseas.

Satellite Failures and Outages During Cycle 23

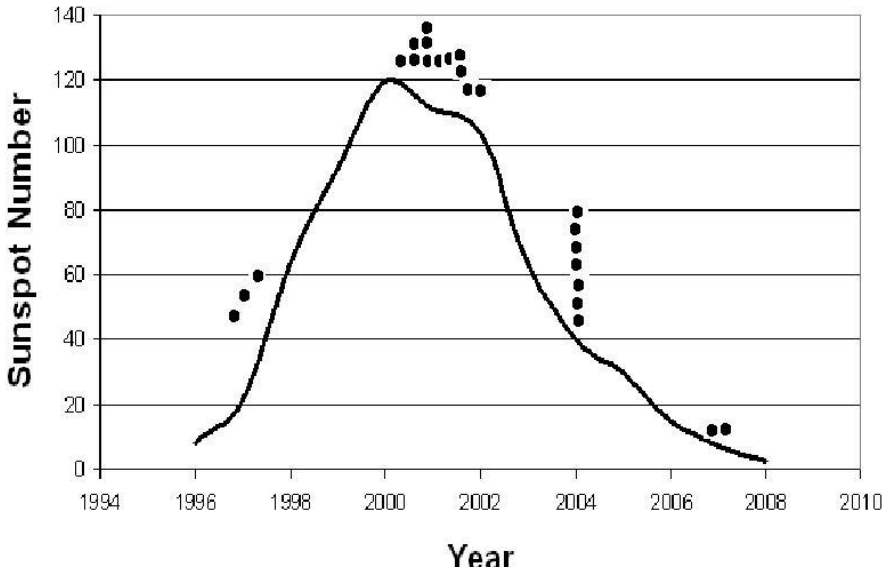
Event Date	AP*	SSN	SPE	Flare	Satellite	Condition
1-11-1997	------	7	---------	--------	Telstar-401	Satellite failure - Pointing/attitude
4-11-1997	------	23	72	--------	Tempo-2	Transponder loss
9-20-1997	46	14	---------	M1.7	Adeos-1	Main computer malfunction - GCR
3-21-2000	------	150	---------	M1.8	Hotbird-2	Loss of service for 9 hours
4-9-2000	137	108	55	M3.1	Brazilsat-A2	Loses TWTA
4-28-2000	------	124	--------	M7.8	Turksat-1C	ESD causes 55-minute service loss
7-15-2000	192	148	24,000	X5.7	GOES-8	Electron sensor problems- power
7-15-2000	192	148	24,000	X5.7	GOES-10	Data corruption
7-15-2000	192	148	24,000	M1.3	ASCA	Satellite Failure and reentry
8-27-2000	45	113	--------	--------	Solidaridad-1	Backup SCP fails
10-31-2000	41	111	---------	M4.4	Echostar-4	26/44 transponders lost
11-4-2000	78	130	---------	-------	Insat-2B	Service outage
9-6-2001	------	141	---------	M2.2	PAS-7	25% power loss
10-23-2001	105	143	24	X1.2	Echostar-6	Solar array string loss announced
12-7-2001	------	138	---------	M1.0	Arabsat-3A	Loss of transponders
10-25-2003	42	88	---------	--------	Adeos-2	Satellite Failure - Solar array malfunction
10-28-2003	252	165	29,500	X17.2	DRTS	Enters Safe Mode
10-28-2003	252	165	29,500	X17.2	FEDSAT	Magnetometer data corrupted
10-28-2003	252	165	29,500	X17.2	GOES-8	X-ray sensor disabled
10-30-2003	252	167	29,500	X10.0	INMARSAT	CPU outages and attitude errors
10-30-2003	252	167	29,500	X10.0	KODAMA	Goes into Safe Mode
10-30-2003	252	167	29,500	X10.0	DMSP F14	One microwave sounder damaged
9-23-2006	-----	8	---------	--------	Meteosat-8	SEU
12-5-2006	120	20	1,980	X9.0	GOES-13	X-ray imager damaged

The table gives a short list of the publicly-admitted satellite failures and outages during sunspot cycle 23 (1996-2008).

Problem 1 - The annual sunspot counts for Cycle 23 between 1996 and 2008 (inclusive) were as follows: [9,21,64,93,120,111,104,64,40,30,15,8,3]. How do the outages compare in time with the sunspot cycle? (Hint: Graph the sunspot cycle and then indicate on the graph when the outages occurred)

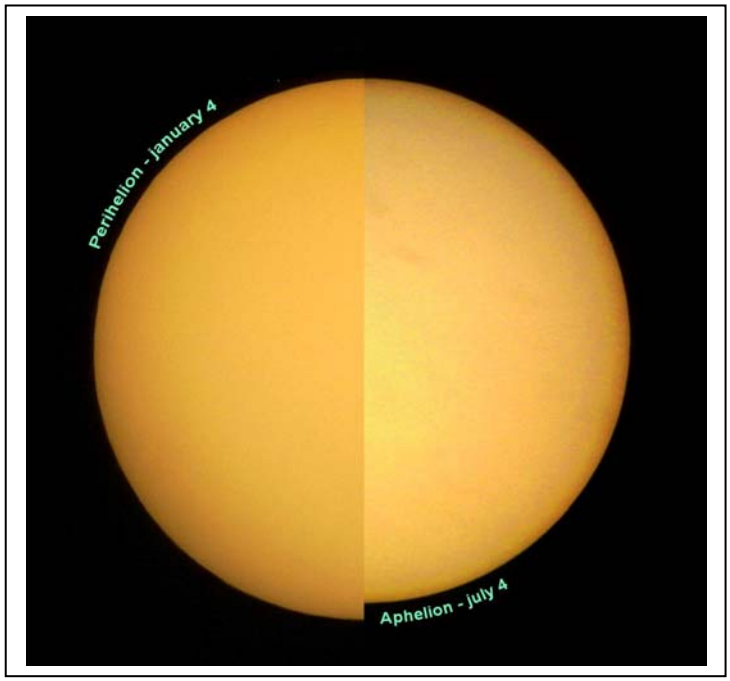
Problem 2 - Assuming that this list is complete, what is the probability that a satellite outage/failure will occur during a solar proton event?

Problem 1 - The annual sunspot counts for Cycle 23 between 1996 and 2008 (inclusive) were as follows: [9,21,64,93,120,111,104,64,40,30,15,8,3]. How do the outages compare in time with the sunspot cycle? (Hint: Graph the sunspot cycle and then indicate on the graph when the outages occurred) Answer: **The graph below shows the sunspot cycle (line) and dots representing the approximate dates of the outages. The outages can come at any time in the cycle, but are most common close to the peak of the cycle.**



Problem 2 - Assuming that this list is complete, what is the probability that a satellite outage/failure will occur during a solar proton event?

Answer: The total number of satellite outages is 24, of which 13 were reported during SPE events so the probability is 100% x 13/24 = **54%**.



Earth's orbit is not a perfect circle centered on the sun, but an ellipse! Because of this, in January, Earth is slightly closer to the sun than in July. This means that the sun will actually appear to have a bigger disk in the sky in January than in July…but the difference is impossible to see with the eye, even if you could do so safely!

The figure above shows the sun's disk taken by the SOHO satellite. The left side shows the disk on January 4 and the right side shows the disk on July 4, 2009. As you can see, the diameter of the sun appears to change slightly between these two months.

Problem 1 - What is the average diameter of the Sun, in millimeters, in this figure?

Problem 2 - By what percentage did the diameter of the Sun change between January and July compared to its average diameter?

Problem 3 - If the average distance to the Sun from Earth is 149,600,000 kilometers, how much closer is Earth to the Sun in July compared to January?

Problem 1 - What is the average diameter of the Sun, in millimeters, in this figure?
Answer: Using a millimeter ruler, and measuring vertically along the join between the two images, the left-hand, January, image is 72 millimeters in diameter, while the right-hand image is 69 millimeters in diameter. The average of these two is (72 + 69)/2 = **70.5 millimeters.**

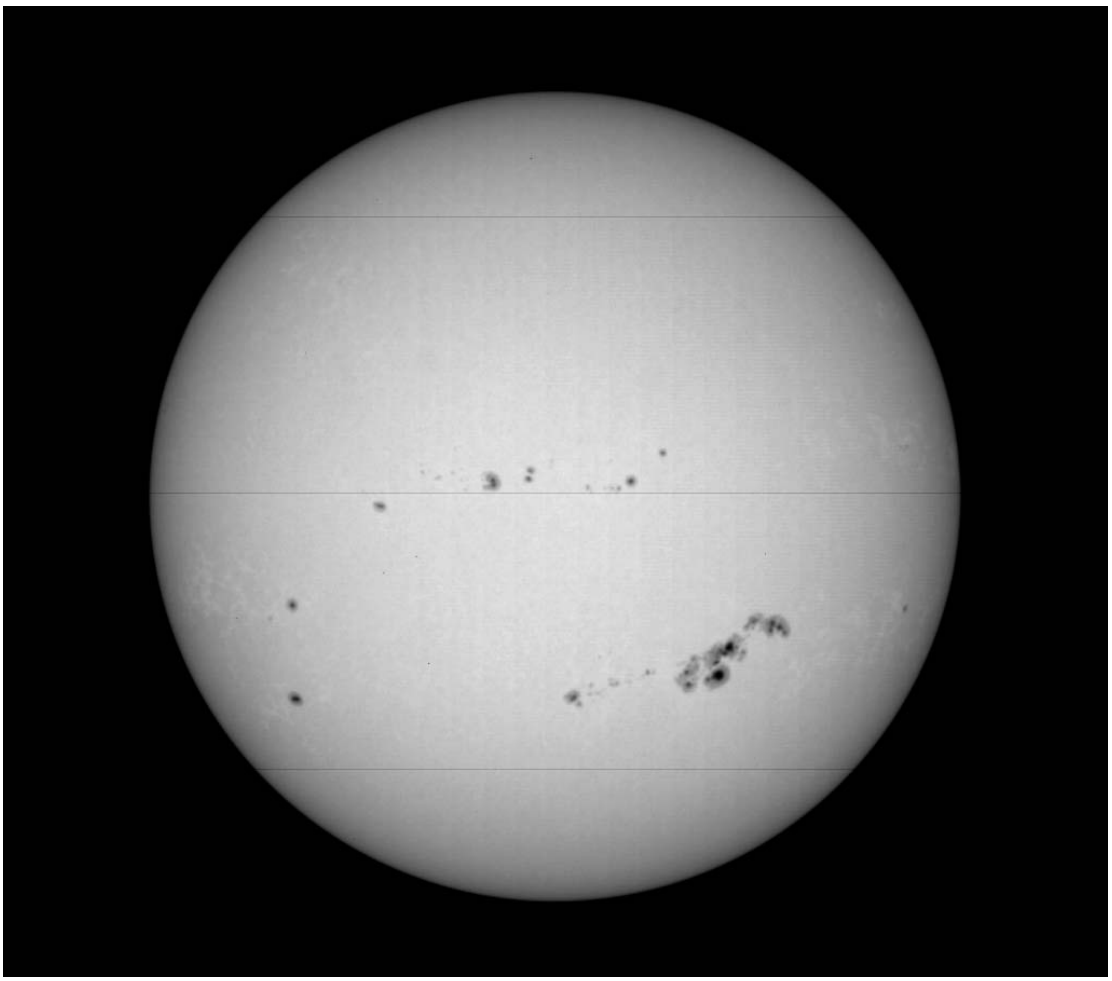
Problem 2 - By what percentage did the diameter of the Sun change between January and July compared to its average diameter?

Answer: In January the moon was larger then the average diameter by 100% x (72 - 70.5)/70.5 = **2.1 %**. In July it was smaller then the average diameter by 100% x (70.5 - 69)/70.5 = 2.1%.

Problem 3 - If the average distance to the Sun from Earth is 149,600,000 kilometers, how much closer is Earth to the Sun in July compared to January?

Answer: The diameter of the sun appeared to change by 2.1% + 2.1% = 4.2% between January and June. Because the apparent size of an object is inversely related to its distance (i.e. the farther away it is the smaller it appears), this 4.2% change in apparent size occurred because of a 4.2% change in the distance between Earth and the Sun, so since 0.042 x 149,600,000 km = 6,280,000 kilometers, the change in the Sun's apparent diameter reflects the 6,280,000 kilometer change in earth's distance between January and June. The Earth is **6,280,000 kilometers closer to the Sun in July than in January.**

The Sun in Visible Light



This picture shows the Sun in white light as seen by the Global Oscillation Network Group (GONG) instrument in Big Bear, California on March 30, 2001. The image was obtained at a wavelength of 500 nm, or a frequency of 600 teraHertz (6.0×10^{14} Hz). The large sunspot is more than 140,000 kilometers across; about 22 times the diameter of Earth. (Image courtesy: NSO/AURA/NSF).

Visible light images of the sun have been used for hundreds of years to study its sunspots and other features, since the advent of the telescope by Galileo in 1609. Ancient Chinese astrologers viewed the sun near sunset and often saw large sunspots. Recently, the intense study of sunspots, their cycles of activity, and other features such as prominences and coronal mass ejections, have led to a comprehensive understanding of solar activity and 'space weather'.

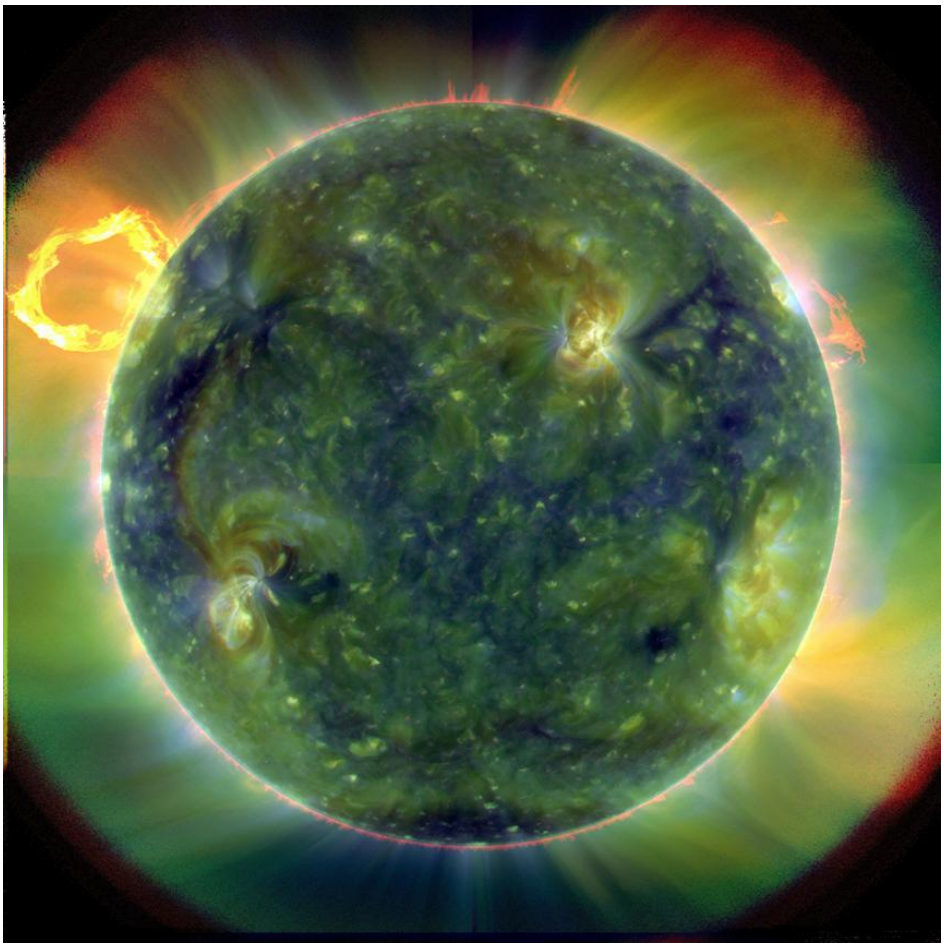
Problem - If the solar diameter is 0.5 degrees and the human eye can see features about 2 arcminutes across, how large would a naked-eye sunspot be on the scale of the image above? (Note: 1 degree = 60 arcminutes)

Problem - If the solar diameter is 0.5 degrees and the human eye can see features about 2 arcminutes across, how large would a naked-eye sunspot be on the scale of the image above?

Answer: Since 1 degree = 60 arc minutes, 0.5 degrees is 30 arcminutes. For a sun disk diameter of about 90 mm, the proportion would be: 90 mm x (2 arcminutes/30 arcminutes) = **6 millimeters**. That is about as large as the main clump of spots in the largest group in the image to the lower right.

The brightness of the sun at visible wavelengths is about

$$F = 2 \times 10^{-21} \text{ watts/meter}^2$$



On April 21, 2010 NASA's Solar Dynamics Observatory released its much-awaited 'First Light' images of the sun. The image above shows a full-disk, multi-wavelength, extreme ultraviolet image of the sun taken by SDO on March 30, 2010. False colors trace different gas temperatures. Black indicates very low temperatures near 10,000 K close to the solar surface (photosphere). Reds are relatively cool plasma (a gas consisting of atoms stripped of some of their electrons) heated to 60,000 Kelvin (100,000 F); blues, greens and white are hotter plasma with temperatures greater than 1 million Kelvin (2,000,000 F).

Problem 1 – The radius of the sun is 690,000 kilometers. Using a millimeter ruler, what is the scale of this image in kilometers/millimeter?

Problem 2 – What are the smallest features you can find on this image, and how large are they in kilometers, and in comparison to Earth if the radius of Earth is 6378 kilometers?

Problem 3 – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

Problem 1 – The radius of the sun is 690,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Answer: The diameter of the Sun is 98 millimeters, so the scale is 1,380,000 km/98 mm = **14,000 km/mm**.

Problem 2 – What are the smallest features you can find on this image, and how large are they in kilometers, and in comparison to Earth if the radius of Earth is 6378 kilometers?

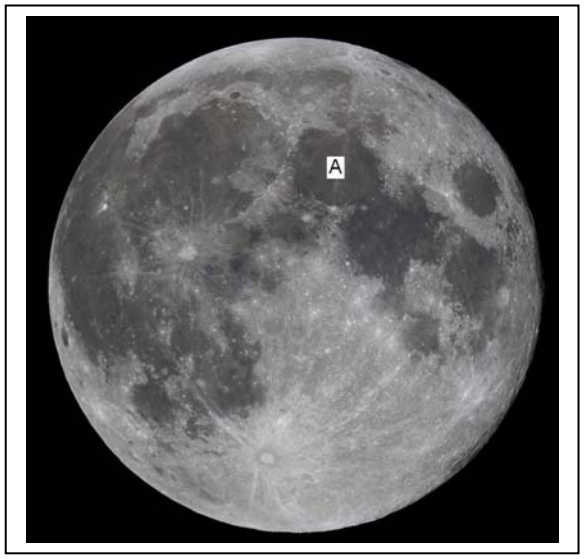
Answer: **Students should see numerous bright points speckling the surface, the smallest of these are about 0.5 mm across or 7,000 km. This is slightly larger than ½ the diameter of Earth.**

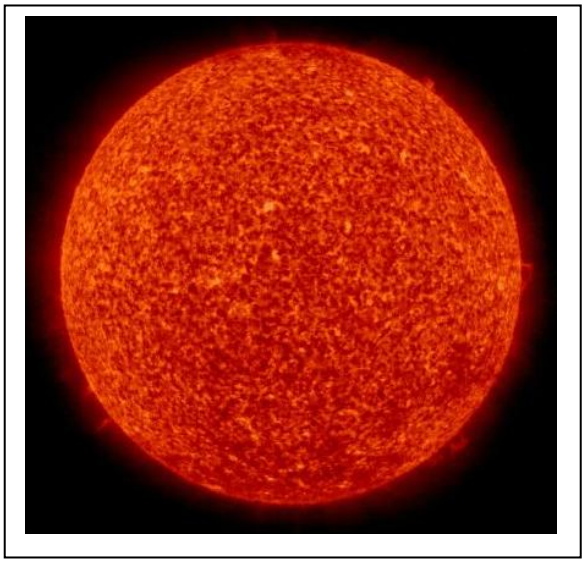
Problem 3 – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

Answer: **There are large irregular blotches all across the disk of the sun that are dark blue-black. These are regions where there is little of the hot coronal gas and only the 'cold' photosphere can be seen. The hottest gas seems to reside in the corona, and in the very small point-like 'microflare' regions that are generally no larger than the size of Earth**.

Note: Microflares were first observed, clearly, by the Hinode satellite between 2007-2009. Some solar physicists believe that these microflares, which erupt violently, are ejecting hot plasma that eventually ends up in the corona to replenish it. Because the corona never disappears, these microflares happen all the time no matter what part of the sunspot cycle is occurring.

Getting an Angle on the Sun and Moon





The Sun (Diameter = 1,400,000 km) and Moon (Diameter = 3,476 km) have very different physical diameters in kilometers, but in the sky they can appear to be nearly the same size. Astronomers use the angular measure of arcseconds (asec) to measure the apparent sizes of most astronomical objects. (1 degree equals 60 arcminutes, and 1 arcminute equals 60 arcseconds). The photos above show the Sun and Moon at a time when their angular diameters were both about 1,865 arcseconds.

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter?

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon?

Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon?

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle?

Problem 5 - What is the area of Mare Serenitatis in square kilometers?

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun?

Answer Key

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter? Answer: Moon diameter = 65 mm and sun diameter = 61 mm so the lunar image scale is 1,865 asec/65mm = **28.7 asec/mm** and the solar scale is 1865 asec/61 mm = **30.6 asec/mm.**

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon? Answer: the smallest feature is about 0.5 mm or 0.5 x 28.7 asec/mm = **14.4 asec for the Moon** and 0.5 x 30.6 asec/mm = **15.3 asec for the Sun.**

Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon? Answer: The diameter of the mare is 1 centimeter, so the radius is 5 mm or 5 mm x 28.7 asec/mm = 143.5 asec. Assuming a circle, the area is $A = \pi \times (143.5\ asec)^2$ = **64,700 $asec^2$**.

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle? Answer: The angular scale at the sun would correspond to 400 x 1.9 km = **760 kilometers per arcsecond**.

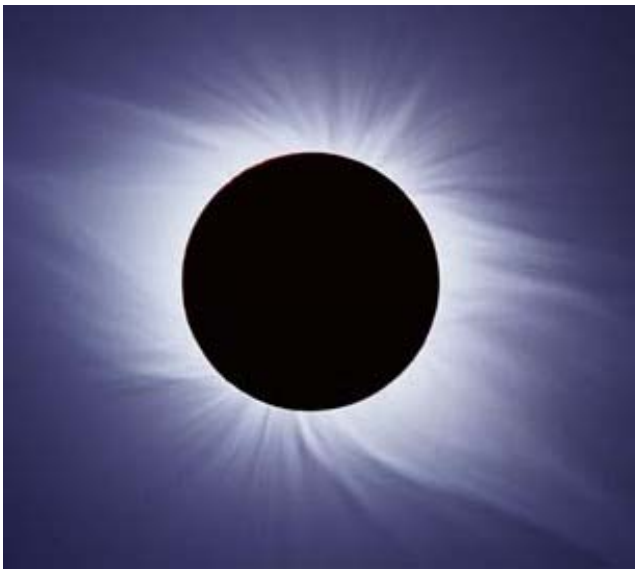
Problem 5 - What is the area of Mare Serenitatis in square kilometers? Answer: We have to convert from square arcseconds to square kilometers using a two-step unit conversion 'ladder'.

$$64{,}700\ asec^2 \times (1.9\ km/asec) \times (1.9\ km/asec) = \mathbf{233{,}600\ km^2.}$$

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun? Answer: The angular area is 400-times further away, so we have to use the scaling of 760 kilometers/asec deduced in Problem 4. The unit conversion for the solar area becomes:

$$64{,}700\ asec^2 \times (760\ km/asec) \times (760\ km/asec) = \mathbf{37{,}400{,}000{,}000\ km^2.}$$

The Last Total Solar Eclipse...Ever!



Total solar eclipses happen because the angular size of the moon is almost exactly the same as the sun's, despite their vastly different distances and sizes.

The moon has been steadily pulling away from earth over the span of billions of years. There will eventually come a time when these two angular sizes no longer match up. The moon will be too small to cause a total solar eclipse.

When will that happen?

Image courtesy Fred Espenak
http://sunearth.gsfc.nasa.gov/eclipse/eclipse.html

Problem 1 - The minimum distance to the moon, called the perigee, is 356,400 kilometers. At that distance, the angular size of the moon from the surface of Earth is 0.559 degrees. Suppose you doubled the distance to the moon. What would its new angular size be, as seen from the surface of Earth?

Problem 2 - Suppose you increased the moon's distance by 50,000 kilometers. What would the angular size now be?

Problem 3 - The smallest angular size of the sun occurs near the summer solstice at a distance of 152 million kilometers, when the sun has an angular diameter of 0.525 degrees. How far away, in kilometers, does the moon have to be to match the sun's apparent diameter?

Problem 4 - How much further away from Earth will the moon be at that time?

Problem 5 - The moon is moving away from Earth at a rate of 3.8 centimeters per year. How many years will it take to move 3.8 kilometer further away?

Problem 6 - How many years will it take to move the distance from your answer to Problem 4?

Problem 7 - When will the last Total Solar Eclipse be sighted in the future?

Answer Key:

Problem 1 - The minimum distance to the moon, called the perigee, is 356,400 kilometers. At that distance, the angular size of the moon from the surface of Earth is 0.559 degrees. Suppose you doubled the distance to the moon. What would its new angular size be, as seen from the surface of Earth?

Answer: Because objects appear smaller the farther away they are, if you double the distance, the moon will appear half its former size, or 0.559/2 = 0.279 degrees across.

Problem 2 - Suppose you increased the moon's distance by 50,000 kilometers. What would the angular size now be?

Answer: The distance is now 356,400 kilometers + 50,000 kilometers = 406,400 kilometers. The distance has increased by 406,400/356,400 = 1.14, so that means that the angular size has been reduced to 0.559 / 1.14 = 0.49 degrees.

Problem 3 - The smallest angular size of the sun occurs near the summer solstice at a distance of 152 million kilometers, when the sun has an angular diameter of 0.525 degrees. How far away, in kilometers, does the moon have to be to match the sun's apparent diameter?

Answer: 0.559/0.525 = 1.06 times further away from Earth or 356,400 km x 1.06 = 377,800 kilometers.

Problem 4 - How much further away from Earth will the moon be at that time?

Answer: 377,800 kilometers - 356,400 kilometers = 21,400 kilometers.

Problem 5 - The moon is moving away from Earth at a rate of 3.8 centimeters per year. How many years will it take to move 3 kilometer further away?

Answer: (380,000 centimeters) / (3.8 centimeters / year) = 100,000 years.

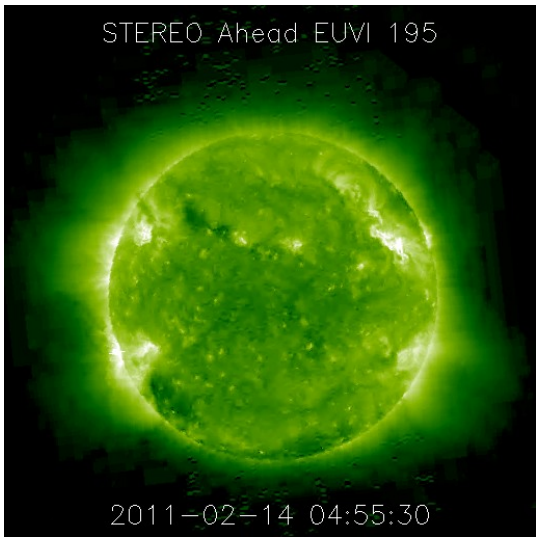
Problem 6 - How many years will it take to move the distance from your answer to Problem 4?

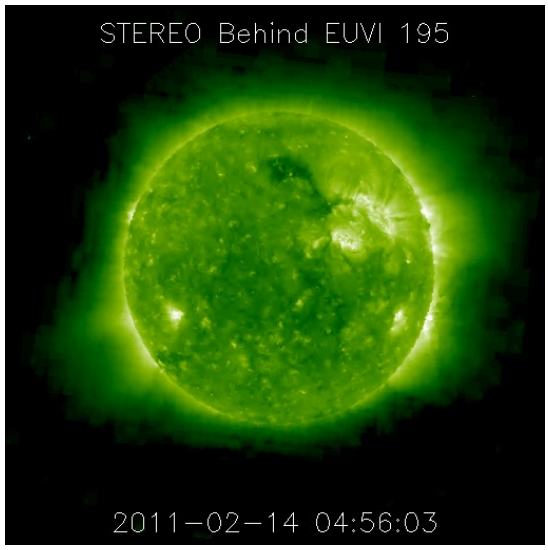
Answer: (21,400 kilometers / 3.8 kilometers) x 100,000 years = 713 million years.

Problem 7 - When will the last Total Solar Eclipse be sighted in the future?

Answer: About 713 million years from now.

STEREO Spacecraft give 360° Solar View





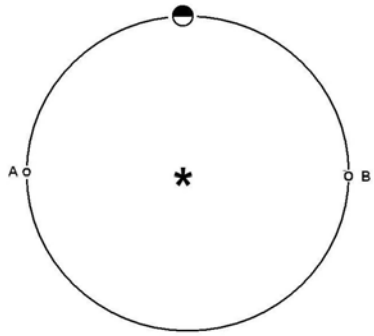
This remarkable pair of images of the sun was taken on February 14, 2011 by NASA's STEREO A (left) and B (right) spacecraft. The spacecraft are close to Earth's orbit, however as viewed looking down on Earth's orbit with the sun at the center, STEREO-A is 90-degrees counter-clockwise of Earth's orbit location, and STEREO-B is 90-degrees clockwise. This means that when the two images above are combined, the entire 360-degree span of the solar surface can be seen at the same time, making this an historical moment for Humanity.

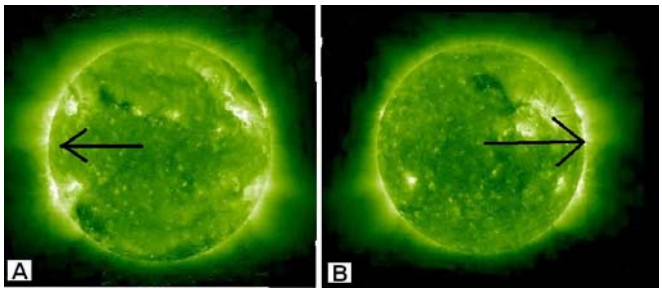
Problem 1 – From the information in the text, draw a diagram that shows the location of the sun, Earth's orbital path (assume it is a circle, whose plane passes through the equator of the sun) and the locations of the STEREO spacecraft. In each of the two images, draw an arrow that points in the direction of Earth.

Problem 2 – The images were taken in the light of iron atoms heated to 1.1 million degrees. The bright (white) features are solar 'active regions' that correspond to the locations of sunspots. At the time the images were taken, what active regions in the above images: A) Could be observed from Earth? B) Could not be observed from Earth?

Problem 3 – Relative to the location of Earth in its orbit, the STEREO-A and B spacecraft move 22 degrees farther from Earth each year. On your diagram from Problem 1, about what will be the positions of the spacecraft along Earth's orbit in February of A) 2013? B) 2015? C) 2017? and D) 2022?

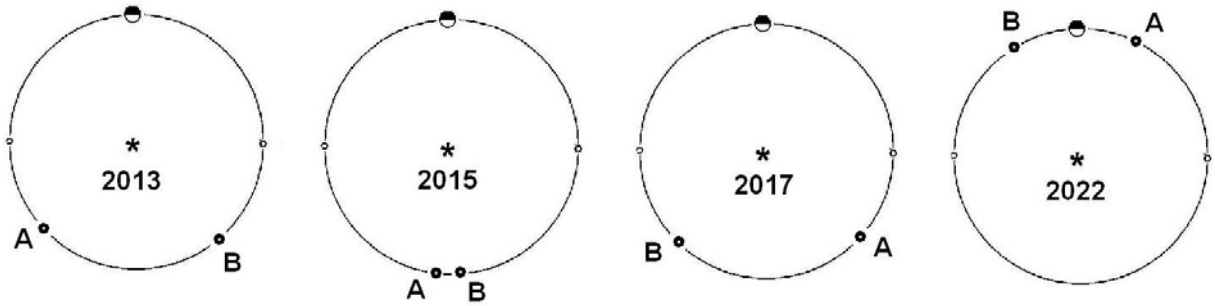
Problem 1 – From the information in the text, draw a diagram that shows the location of the sun, Earth's orbital path (assume it is a circle whose plane passes through the equator of the sun) and the locations of the STEREO spacecraft. In each of the two images, draw an arrow that points in the direction of Earth.



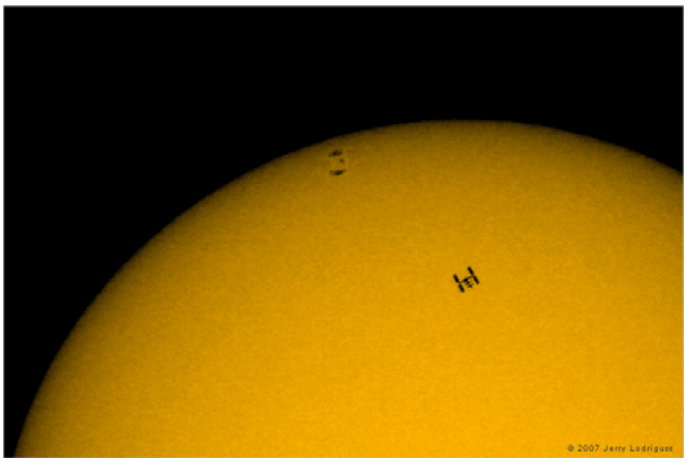


Problem 2 – The images were taken in the light of iron atoms heated to 1.1 million degrees. The bright (white) features are solar 'active regions' that correspond to the locations of sunspots. At the time the images were taken, what active regions in the above images: A) Could be observed from Earth? B) Could not be observed from Earth? **Answer A) Features on the left half of Image A and the right half of Image B. B) Features on the right half of Image A and the left half of Image B.**

Problem 3 – Relative to the location of Earth in its orbit, the STEREO-A and B spacecraft move 22 degrees farther from Earth each year. On your diagram from Problem 1, about what will be the positions of the spacecraft along Earth's orbit in February of A) 2013? B) 2015? C) 2017? and D) 2022? Answer: **If the satellites move 22 degrees along Earth's orbit each year, then for A) 2013, Spacecraft A will be (2013 - 2011) x 22 = 44 degrees counter-clockwise of its February 2011 position. Spacecraft B will be 44 degrees clockwise of its February 2011 position. B) For 2015, Spacecraft A = (2015 - 2011) x 22 = 88 degrees CCW; Spacecraft B = (2015 - 2011) x 22 = 88 degrees CW C) For 2017, Spacecraft A = (2017 - 2011) x 22 = 132 degrees CCW; Spacecraft B = (2017 - 2011) x 22 = 132 degrees CW; For 2022, Spacecraft A = (2022 - 2011) x 22 = 242 degrees CCW; Spacecraft B = (2022 - 2011) x 22 = 242 degrees CW. See diagram below**: - Angles approximate at this scale



Note: The spacecraft were launched in October 2006, so the time for the spacecraft to drift back to earth's vicinity will be 360/22 = 16 1/3 years after October 2006 or about February 2023.



The relationship between the distance to an object, **R**, the objects size, **L**, and the angle that it subtends at that distance, θ, is given by:

$$\theta = 57.29 \frac{L}{R} \text{ degrees}$$

$$\theta = 3{,}438 \frac{L}{R} \text{ arcminutes}$$

$$\theta = 206{,}265 \frac{L}{R} \text{ arcseconds}$$

To use these formulae, the units for length, L, and distance, R, must be identical.

Problem 1 - You spot your friend (L = 2 meters) at a distance of 100 meters. What is her angular size in arcminutes?

Problem 2 - The sun is located 150 million kilometers from Earth and has a radius of 696.000 kilometers. What is its angular diameter in arcminutes?

Problem 3 - How far away, in meters, would a dime (1 centimeter) have to be so that its angular size is exactly one arcsecond?

Problem 4 - The spectacular photo above was taken by Jerry Lodriguss (Copyright 2007, http://www.astropix.com/HTML/SHOW_DIG/055.HTM) and shows the International Space Station streaking across the disk of the sun. If the ISS was located 379 kilometers from the camera, and the ISS measured 73 meters across, what was its angular size in arcseconds?

Problem 5 - The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) what was the angle, in arcminutes, that it moved through in one second as seen from the location of the camera? B) What was its angular speed in arcminutes/second?

Problem 6 - Given the diameter of the sun in arcminutes (Problem 2), and the ISS angular speed (Problem 5) how long, in seconds, did it take the ISS to travel across the face of the sun?

Answer Key

Problem 1 - Answer: Angle = 3,438 x (2 meters/100 meters) = 68.8 arcminutes.

Problem 2 - Answer: 3,438 x (696,000/150 million) = 15.9 arcminutes in radius, so the diameter is 2 x 15.9 = 31.8 arcminutes.

Problem 3 - Answer: From the second formula R = 3438 * L/A = 3438 * 1 cm/1 arcsecond so R = 3,438 centimeters or a distance of 34.4 meters.

Problem 4 - Answer: From the third formula, Angle = 206,265 * (73 meters/379,000 meters) = 39.7 arcseconds.

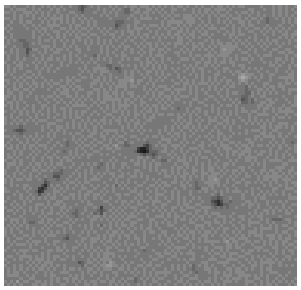
Problem 5 - Answer: The orbital speed of the space station is 7.4 kilometers/second. If its distance traveled in 1 second is 7.4 kilometers, A) The ISS traveled L = 7.4 kilometers so from the second formula Angle = 3,438 * (7.4 km/379 km) = 67 arcminutes. B) The angular speed is just 67 arcminutes per second.

Problem 6 - Answer: The time required is T = 31.8 arcminutes/ (67 arcminutes/sec) = 0.47 seconds.

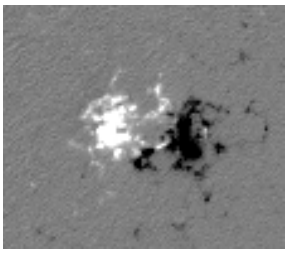
The spectacular photo by Jerry Lodriguss had to be taken with careful planning beforehand. He had to know, to the second, when the sun and ISS would be in the right configuration in the sky as viewed from his exact geographic location. Here's an example of the photography considerations in his own words:

" I considered trying to monitor the transit visually with a remote release in my hand and just firing (the camera) when I saw the ISS in my guidescope. Then I worked out the numbers. I know that my reaction time is 0.19 seconds. This is actually quite good, but I make my living shooting sports where this is critical, so I better be good at it. I also know that the Canon 1D Mark IIn has a shutter lag of 55 milliseconds. Adding these together, plus a little bit of a fudge factor, the best I could hope for was about 1/4 of a second from when I saw it to when the shutter opened. Since the entire duration of the transit was only 1/2 of a second, in theory, I could capture the ISS at about the center of the disk if I fired as soon as I saw it start to cross. This was not much of a margin for error. I could easily blink and miss the whole thing… Out of 49 frames that the Mark IIn recorded, the ISS is visible in exactly one frame."

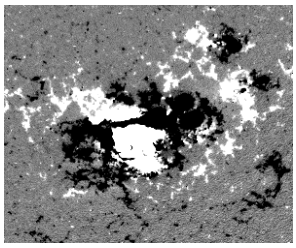
The Magnetic Classification of Sunspots



September 29, 2008
Alpha-Pores



June 2, 2009
Beta - Simple spot



October 29, 2003
Gamma - Complex spot

Solar flares are produced by magnetic releases of energy in sunspots. The more complex the magnetic field, the more likely a solar flare will result. Sunspots are magnetically classified according to the alpha, beta, gamma, and delta 'Mt Wilson' scheme:

α ….. A group having only one polarity.

β…… A group of magnetic polarities, with a simple division between them.

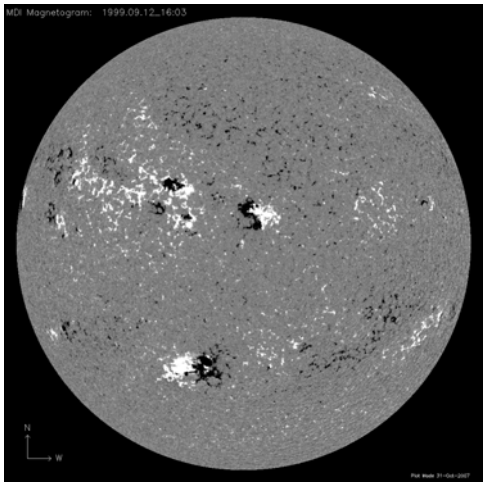
β–γ …A bipolar group in which no continuous line can be drawn separating spots of opposite polarities.

γ…….. A complex active region not classifiable as a bipolar group.

δ……. A complex magnetic configuration consisting of opposite polarity umbrae within the same penumbra.

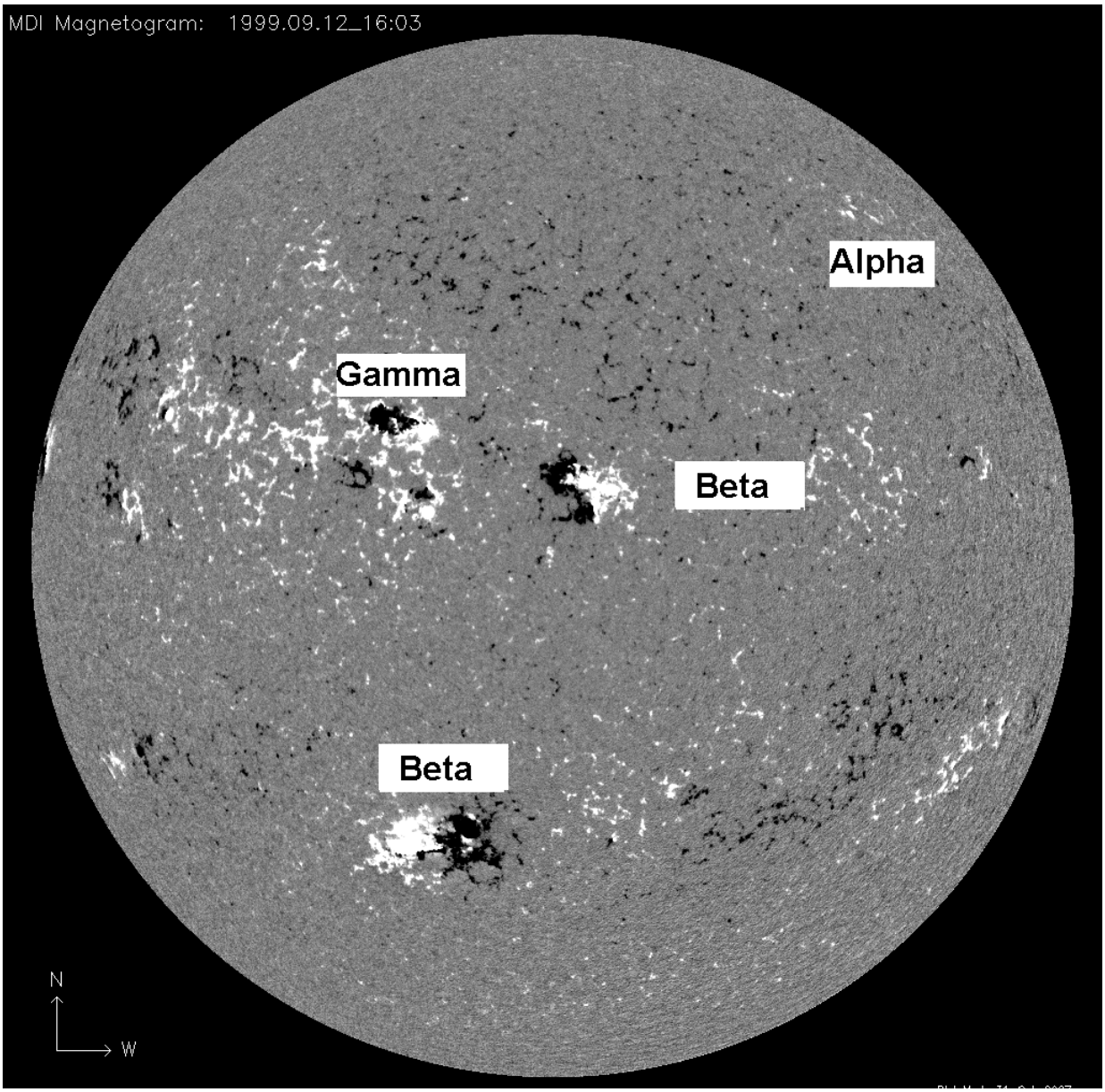
The images to the left are from the SOHO/MDI instrument in which North polarities are white and South polarities are black. Solar flares are more common in sunspots with complex magnetic classifications such as β–γ, γ or δ.

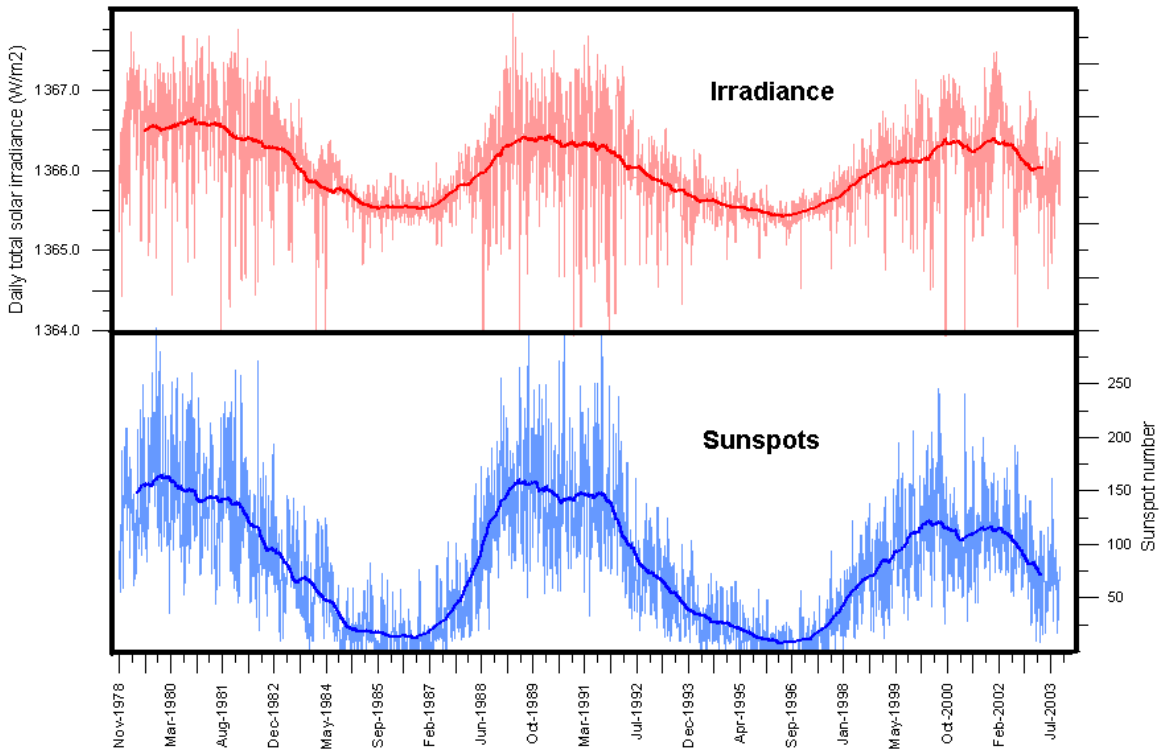
Problem 1 - What classifications would you assign to the sunspots in the bottom image obtained on September 12, 1999?



The SOHO/MDI instrument website has a complete archive of full-sun magnetograms at **http://soi.stanford.edu/production/mag_gifs.html**. Classifications can be found in the Solar Region Summaries at the Space Weather Prediction Center:
http://www.swpc.noaa.gov/ftpmenu/forecasts/SRS.html

Problem 1 - What classifications would you assign to the sunspots in the bottom image obtained on September 12, 1999?
Answer: The image below has the suggested classifications for the major active regions. Assignment of a classification to the more complex spots can be difficult for non-professionals, but students should be able to distinguish the major classes, α, β and γ.





Irradiance (also called insolation) is a measure of the amount of sunlight power that falls upon one square meter of exposed surface, usually measured at the 'top' of Earth's atmosphere. This energy increases and decreases with the season and with your latitude on Earth, being lower in the winter and higher in the summer, and also lower at the poles and higher at the equator. But the sun's energy output also changes during the sunspot cycle!

The figure above shows the solar irradiance and sunspot number since January 1979 according to NOAA's National Geophysical Data Center (NGDC). The thin lines indicate the daily irradiance (red) and sunspot number (blue), while the thick lines indicate the running annual average for these two parameters. The total variation in solar irradiance is about 1.3 watts per square meter during one sunspot cycle. This is a small change compared to the 100s of watts we experience during seasonal and latitude differences, but it may have an impact on our climate. The solar irradiance data obtained by the ACRIM satellite, measures the total number of watts of sunlight that strike Earth's upper atmosphere before being absorbed by the atmosphere and ground.

Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003?
Problem 2 - What appears to be the relationship between sunspot number and solar irradiance?
Problem 3 - A homeowner built a solar electricity (photovoltaic) system on his roof in 1985 that produced 3,000 kilowatts-hours of electricity that year. Assuming that the amount of ground-level solar power is similar to the ACRIM measurements, about how much power did his system generate in 1989?

Answer Key

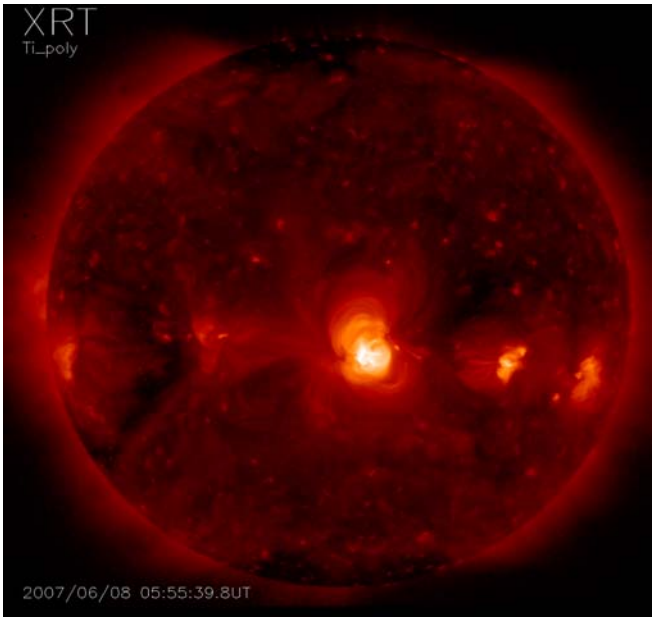
Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003? Answer: Draw a horizontal line across the upper graph that is mid-way between the highest and lowest points on the curve. An approximate answer would be 1366.3 watts per square meter.

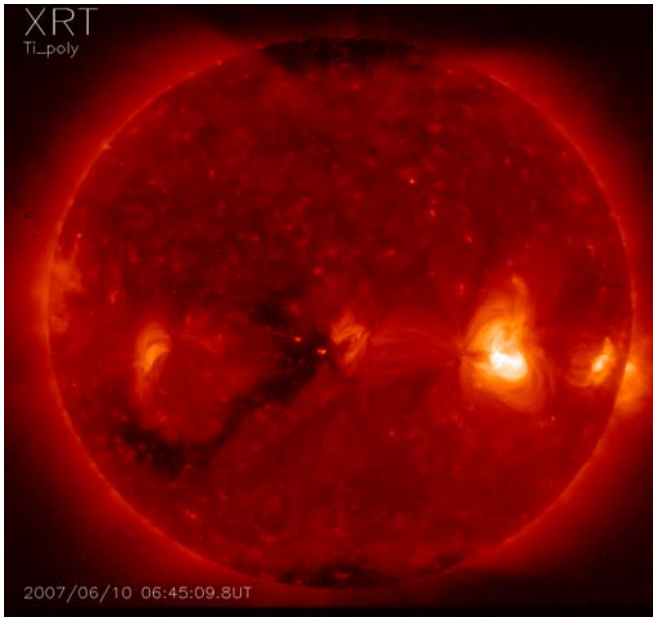
Problem 2 - What appears to be the relationship between sunspot number and solar irradiance? Answer: When there are a lot of sunspots on the sun (called sunspot maximum) the amount of solar radiation is higher than when there are fewer sunspots. The solar irradiance changes follow the 11-year sunspot cycle.

Problem 3 - A homeowner built a solar electricity (photovoltaic) system on his roof in 1985 that produced 3,000 kilowatts-hours of electricity that year. Assuming that the amount of ground-level solar power is similar to the ACRIM measurements, about how much power did his system generate in 1989?

Answer: In 1985, the amount of insolation was about 1365.5 watts per square meter when the photovoltaic system was built. Because of changes in the sunspot cycle, in 1989 the insolation increased to about 1366.5 watts per square meter. This insolation change was a factor of 1366.5/1365.5 = 1.0007. That means that by scaling, if the system was generating 3,000 kilowatt-hours of electricity in 1985, it will have generated 1.0007 x 3,000 kWh = 2 kWh more in 1989 during sunspot maximum! That is equal to running one 60-watt bulb for about 1 day (actually 33 hours).

How fast does the sun spin?





The sun, like many other celestial bodies, spins around on an axis that passes through its center. The rotation of the sun, together with the turbulent motion of the sun's outer surface, work together to create magnetic forces. These forces give rise to sunspots, prominences, solar flares and ejections of matter from the solar surface.

Astronomers can study the rotation of stars in the sky by using an instrument called a spectroscope. What they have discovered is that the speed of a star's rotation depends on its age and its mass. Young stars rotate faster than old stars, and massive stars tend to rotate faster than low-mass stars. Large stars like supergiants, rotate hardly at all because they are so enormous they reach almost to the orbit of Jupiter. On the other hand, very compact neutron stars rotate 30 times each second and are only 40 kilometers across.

The X-ray telescope on the Hinode satellite creates movies of the rotating sun, and makes it easy to see this motion. A sequence of these images is shown on the left taken on June 8, 2007 (Left); June 10 2007 (Right) at around 06:00 UT.

Although the sun is a sphere, it appears as a flat disk in these pictures when in fact the center of the sun is bulging out of the page at you! We are going to neglect this distortion and estimate how many days it takes the sun to spin once around on its axis.

The radius of the sun is 696,000 kilometers.

Problem 1 - Using the information provided in the images, calculate the speed of the sun's rotation in kilometers/sec and in miles/hour.

Problem 2 – About how many days does it take to rotate once at the equator?

Inquiry Question: What geometric factor produces the largest uncertainty in your estimate, and can you come up with a method to minimize it to get a more accurate rotation period?

Answer Key:

Problem 1 - Using the information provided in the images, calculate the speed of the sun's rotation in kilometers/sec and in miles/hour.

First, from the diameter of the sun's disk, calculate the image scale of each picture in kilometers per millimeter.

Diameter= 76 mm. so radius = 38 mm. Scale = (696,000 km)/38 mm = 18,400 km/mm

Then, find the center of the sun disk, and using this as a reference, place the millimeter ruler parallel to the sun's equator, measure the distance to the very bright 'active region' to the right of the center in each picture. Convert the millimeter measure into kilometers using the image scale.

Picture 1: June 8 distance = 4 mm d = 4 mm (18,400 km/mm) = 74,000 km
Picture 2; June 10 distance = 22 mm d = 22 mm (18,400 km/mm) = 404,000 km

Calculate the average distance traveled between June 8 and June 10.

Distance = (404,000 - 74,000) = 330,000 km

Divide this distance by the number of elapsed days (2 days)…….. 165,000 km/day

Convert this to kilometers per hour…….. 6,875 km/hour

Convert this to kilometers per second…….. 1.9 km/sec

Convert this to miles per hour ……. 4,400 miles/hour

Problem 2 – About how many days does it take to rotate once at the equator?

The circumference of the sun is 2 π (696,000 km) = 4,400,000 kilometers.

The equatorial speed is 66,000 km/day so the number of days equals
4,400,000/165,000 = **26.6 days.**

Inquiry Question:
Because the sun is a sphere, measuring the distance of the spot from the center of the sun on June 10 gives a distorted linear measure due to foreshortening.

The sun has rotated about 20 degrees during the 2 days, so that means a full rotation would take about (365/20) x 2 days = 36.5 days which is closer to the equatorial speed of the sun of 35 days.

The Speed of a Coronal Mass Ejection

The last few days from November 6 - 9, 2004 were very active days for solar and auroral events. A major sunspot group, AR 0696, with a complex magnetic field produced several Coronal Mass Ejections (CME) and flares during this time. (A CME is billion-ton cloud of gas ejected by the sun at millions of kilometers per hour). The latest ones were an X2.0-class solar flare on Nov 7, 16:06 UT and a CME. On November 9 and 12:00 UT, the beginnings of a major geomagnetic storm started. The NOAA Space Weather Bulletin announced that:

The Geomagnetic field is expected to be at unsettled to major storm levels on 09 November due to the arrival of a CME associated with the X2.0 flare observed on 07 November. Unsettled to minor storm levels are expected on 10 November. Quiet to active levels are expected on 11 November.

Many observers as far south as Texas and Oklahoma reported seeing beautiful aurora on Sunday night from an earlier CME/flare combination on Saturday, November 6th. In the space provided below, calculate the speed of the CME as it traveled to Earth between November 7th - 9th assuming that the distance to Earth is 93 million miles, or 147 million kilometers.

Problem 1 - How long did it take for the CME to arrive?

Problem 2 - What is the speed of the CME in miles per hour?

Problem 3 - What is the speed of the CME in kilometers per hour?

Problem 4 - What is the speed of the CME in miles per second?

Problem 5 - What is the speed of the CME in kilometers per second?

The distance to the Earth is 93 million miles or 147 million kilometers.
Start time = November 7 at 16:06 UT
Arrival time = November 9 at 12:00 UT

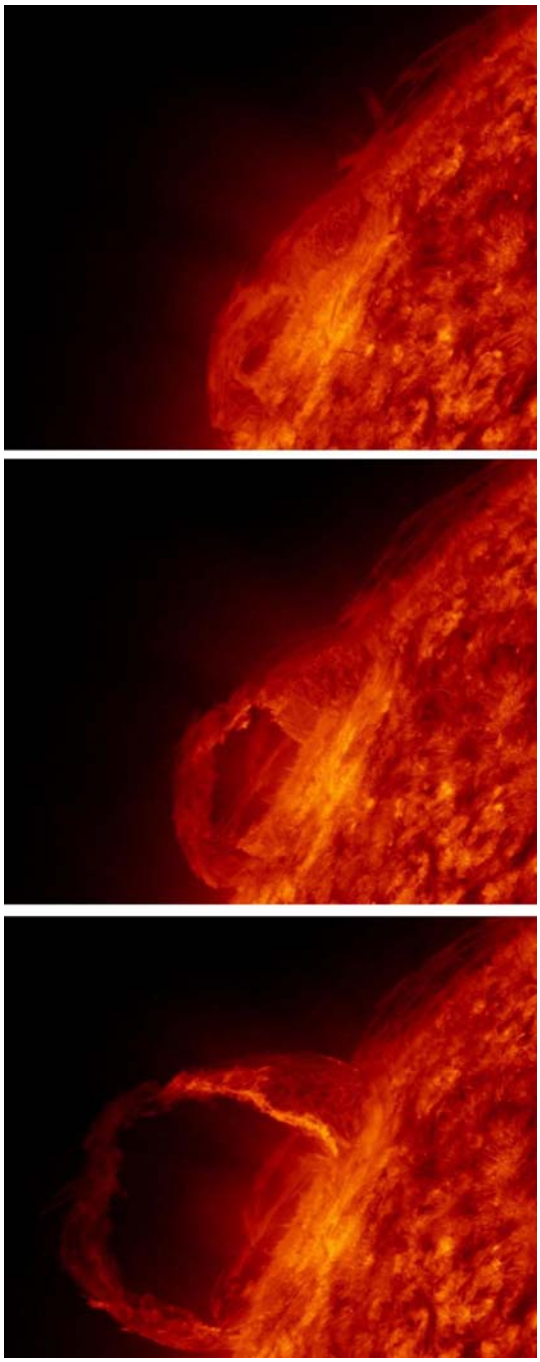
Problem 1 - How long did it take for the CME to arrive?
Answer: November 7 at 16:06 UT to November 8 at 16:06 UT is 24 hours.
From Nov 8 at 16:06 to Nov 9 at 12:00 UT is
(24:00 - 16:06) + 12:00
= 7:54 + 12:00
= **19 hours and 54 minutes**.
Total time = 24 hours + 19 hours and 54 minutes = 43 hours and 54 minutes.

Problem 2 - What is the speed of the CME in miles per hour?
Answer: In decimal units, the travel time from question 1 equals 43.9 hours.
The distance is 93 million miles so the speed is 93 million miles/43.9hours
or **2.1 million miles per hour**.

Problem 3 - What is the speed of the CME in kilometers per hour?
Answer: Use the conversion that 1.0 miles = 1.6 kilometers, then
2.1 million miles/hour x 1.6 km/mile = **3.4 million kilometers per hour**

Problem 4 - What is the speed of the CME in miles per second?
Answer: Convert hours to seconds by
1 hour x 60 minutes/hour x 60 seconds/minute = 3,600 seconds.
Then from question 2: 2.1 million miles/hour divided by 3600 seconds/hour
= **583 miles/second.**

Problem 5 - What is the speed of the CME in kilometers per second?
Answer: From question 3 and the conversion of 1 hour = 3600 seconds:
3.4 million kilometers / second divided by 3600 seconds/hour
= **944 kilometers/sec**.



On April 21, 2010 NASA's Solar Dynamics Observatory released its much-awaited 'First Light' images of the Sun. Among them was a sequence of images taken on March 30, showing an eruptive prominence ejecting millions of tons of plasma into space. A plasma is a gas consisting of atoms stripped of some of their electrons.

The three images to the left show selected scenes from the first 'high definition' movie of this event. The top image was taken at 17:50:49, the middle image at 18:02:09 and the bottom image at 18:13:29.

Problem 1 – The width of the image is 300,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Problem 2 – If the Earth were represented by a disk the size of a penny (10 millimeters), on this same scale how big was the loop of the eruptive prominence in the bottom image if the radius of Earth is 6,378 kilometers?

Problem 3 – What was the average speed of the prominence in A) kilometers/second? B) Kilometers/hour? C) Miles/hour?

For additional views of this prominence, see the NASA/SDO movies at:
http://svs.gsfc.nasa.gov/vis/a000000/a003600/a003693/index.html

or to read the Press Release:
http://www.nasa.gov/mission_pages/sdo/news/first-light.html

Problem 1 – The width of the image is 300,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Answer: The width is 70 millimeters so the scale is 300,000 km/70 mm **= 4,300 km/mm**

Problem 2 – If the Earth were represented by a disk the size of a penny (10 millimeters), on this same scale how big was the loop of the eruptive prominence in the bottom image if the radius of Earth is 6,378 kilometers?

Answer: **The diameter of the loop is about 35 millimeters or 35mm x 4300km/mm = 150,000 km. The diameter of Earth is 13,000 km, so the loop is 12 times the diameter of Earth. At the scale of the penny, 13 penny/Earth's can fit across a scaled drawing of the loop.**

Problem 3 – What was the average speed of the prominence in A) kilometers/second? B) Kilometers/hour? C) Miles/hour?

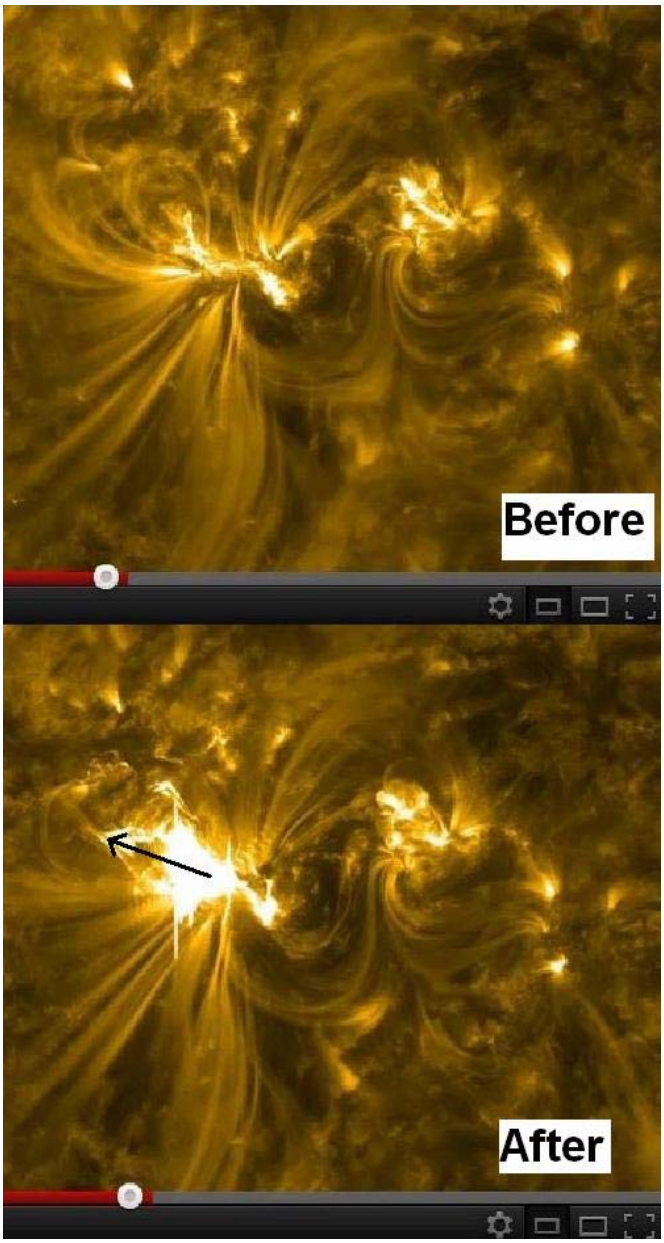
Answer: Speed = distance traveled / time elapsed.
In the bottom image, draw a straight line from the lower right corner THROUGH the peak of the coronal loop. Now draw this same line at the same angle on the other two images. With a millimeter ruler, measure the distance along the line from the lower right corner to the edge of the loop along the line. Example:
Top: 47 mm ;
Middle: 52 mm,
Bottom: 67 mm.
The loop has moved 67mm – 47mm = 20 millimeters. At the scale of the image this equals 20mm x 4,300 km/mm so D = 86,000 km.

The time between the bottom and top images is 18:13:29 – 17:50:49 or 22minutes and 40 seconds or 1360 seconds.

A) The average speed of the loop is then S = 86,000 km/1360 sec = **63 km/sec**.

B) 63 km/sec x 3600 sec/hr = **227,000 km/hour**.

C) 227,000 km/hr x 0.62 miles/km = **140,000 miles/hour**.



On March 6, 2012 Active Region 1429 produced a spectacular X5.4 solar flare shown in the pair of images taken by the NASA Solar Dynamics Observatory. The time between the two images is 2 seconds, and the width of each image is 318,000 kilometers.

Problem 1 - The arrowed line indicates how far the million-degree gas produced by the flare traveled in the time interval between the images. What was the speed of the gas, called a plasma, in kilometers/sec?

Problem 2 - By carefully looking at the two images, what other features can you find that changed their position in the time between the images?

Problem 3 - What would you estimate the speeds to be for the features that you identified in Problem 2?

The explosion can be seen in the movie located on *YouTube* at
http://www.youtube.com/watch?v=4xKRBkBBEP0

The above 'high definition' images of the spectacular March 6 solar flare were taken by the NASA Solar Dynamics Observatory (SDO) located in geosynchronous orbit around Earth. Within a few minutes, this flare produced more energy than a thousand hydrogen bombs going off all at once. This energy caused gasses nearby to be heated to millions of degrees, and produced a blast-wave that traveled across the face of the sun at over 4 million km/hour. In the movie above, watch for magnetic loops and filaments to be disturbed by the explosion as the blast wave passes by them.

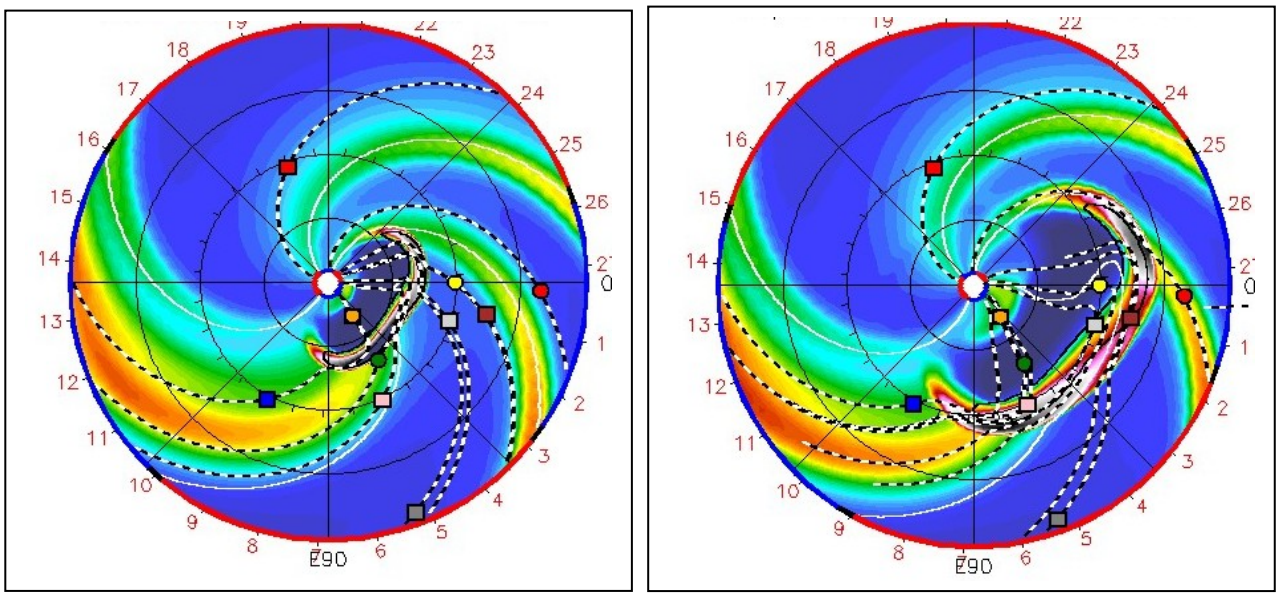
Problem 1 - The arrowed line indicates how far the million-degree gas produced by the flare traveled in the time interval between the images. What was the speed of the gas, called a plasma, in kilometers/sec?

Answer: Students should compare the length of the arrowed line to the width of the image and solve the proportion to get the physical distance that the plasma traveled. Example, for standard printing onto an 81/2 x 11 page, the line width is about 14 millimeters long, and the width of the image is about 84 millimeters so the proportion for the true length of the line in kilometers is just

$$\frac{14}{84} = \frac{X}{318,000km}$$ so X = 53,000 km

The speed of the flare is simply the distance traveled (53,000 km) divided by the time between the two images (2 seconds) so V = 53,000/2 = **26,500 km/sec**.

The actual speed of the plasma will be much less than this because some of the brightening you see in the second image is because gases trapped in the magnetic field loops were heated in place and brightened rapidly without much movement. As one region dims and another brightens we have the appearance of something moving between the two locations when in fact there was little of no actual movement.



This pair of computed images shows the spiral pattern of the solar wind inside the orbit of Mars. It was created by the NASA Goddard, Coordinated Community Modeling Center to show the condition of the solar wind just after the March 6, 2012 solar storm. The images correspond to March 8 (00:00) and March 9 (00:00).

The planets are indicated at their correct positions by circles (yellow=earth; red=mars; orange=mercury and green=venus). Also shown are the positions of the STEREO A and B spacecraft (red and blue squares) and the Spitzer Space Telescope (pink square). The black concentric circles are drawn at intervals of 75 million kilometers.

The pinwheel pattern is formed from the high-speed gas streams leaving the sun through coronal holes. The crescent shaped cloud is the coronal mass ejection (CME) from the sun, which caused brilliant aurora on Earth.

Problem 1 - About what was the speed, in km/h, of the CME when it reached Earth?

Problem 2 - The dark cavity behind the CME represents a very low density region of space. How do you think this was created? Where did the gas go that once filled the cavity?

Problem 3 - At the orbit of Earth, about how fast do the high-speed gas streams sweep past the Earth in kilometers/hour?

Problem 4 - Assuming that the CME does not slow down, on what date will it arrive at Neptune, which is located 4.5 billion kilometers from the sun?

Problem 1 - About what was the speed, in km/h, of the CME when it reached Earth?

Answer: Students can estimate from the scaled figures that between the two days the crescent-shaped CMRE moves about one division or 75 million kilometers. This took 1 day or 24 hours, so the speed was 75 million km/24 h = **about 3 million km/h**.

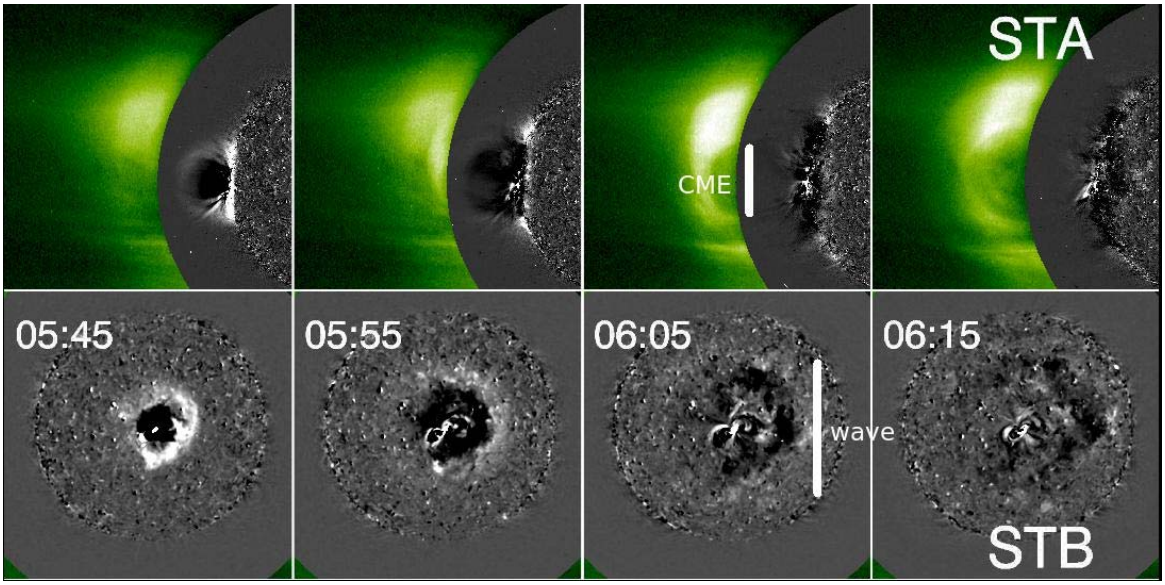
Problem 2 - The dark cavity behind the CME represents a very low density region of space. How do you think this was created? Where did the gas go that once filled the cavity?
Answer: **The CME traveled through the gas and swept it up like a snow-plow in front of the CME.**

Problem 3 - At the orbit of Earth, about how fast do the high-speed gas streams sweep past the Earth in kilometers/hour?

Answer: It will be a challenge to find a feature in the spiral gas streams that can be tracked between the two days, but a reasonable answer would be that the pattern rotated by about 25 million km in 1 day, so the speed is about **1 million km/h**.

Problem 4 - Assuming that the CME does not slow down, on what date will it arrive at Neptune, which is located 4.5 billion kilometers from the sun?

Answer: The CME travels about 3 million km/h so it traveled about 4500 million km during the transit time, so T = 4500/3 = 1500 hours which equals 62.5 days. When this is added to the launch date of March 6 in the first frame, we get a date of about **May 7.**



A solar tsunami that occurred in February 13, 2009 has recently been identified in the data from NASA's STEREO satellites. It was spotted rushing across the Sun's surface. The blast hurled a billion-ton Coronal Mass Ejection (CME) into space and sent a tsunami racing along the sun's surface. STEREO recorded the wave from two positions separated by 90 degrees, giving researchers a spectacular view of the event. Satellite A (STA) provided a side-view of the CME, while Satellite B (STB) viewed the CME from directly above. The technical name for the 'tsunami' is a "fast-mode magnetohydrodynamic wave" – or "MHD wave" for short. The one STEREO saw raced outward at 560,000 mph (250 km/s) packing as much energy as 2,400 megatons of TNT.

Problem 1 - In the lower strip of images, the sun's disk is defined by the mottled circular area, which has a physical radius of 696,000 kilometers. Use a millimeter ruler to determine the scale of these images in kilometers/mm.

Problem 2 - The white circular ring defines the outer edge of the expanding MHD wave. How many kilometers did the ring expand between 05:45 and 06:15? (Note '05:45' means 5:45 o'clock Universal Time).

Problem 3 - From your answers to Problem 1 and 2, what was the approximate speed of this MHD wave in kilometers/sec?

Problem 4 - Kinetic Energy is defined by the equation K.E. = 1/2 m V^2 where m is the mass of the object in kilograms, and V is its speed in meters/sec. Suppose the mass of the CME was about 1 million metric tons, use your answer to Problem 3 to calculate the K.E., which will be in units of Joules.

Problem 5 - If 1 kiloton of TNT has the explosive energy of 4.1×10^{12} Joules, how many megatons of TNT does the kinetic energy of the tsunami represent?

Problem 1 - In the lower strip of images, the sun's disk is defined by the mottled circular area, which has a physical radius of 696,000 kilometers. Use a millimeter ruler to determine the scale of these images in kilometers/mm.

Answer: The diameter is 31 millimeters ,which corresponds to 2 x 696,000 km or 1,392,000 km. The scale is then 1,392,000 km/31mm = 45,000 km/mm.

Problem 2 - The white circular ring defines the outer edge of the expanding MHD wave. How many kilometers did the ring expand between 05:45 and 06:15? (Note '05:45' means 5:45 o'clock Universal Time).

Answer: From the scale of 45,000 km/mm, the difference in the ring radii is 12mm - 5 mm = 7mm which corresponds to 7 mm x (45,000 km/1 mm) = 315,000 kilometers. Students answers may vary depending on where they defined the outer edge of the ring.

Problem 3 - From your answers to Problem 1 and 2, what was the approximate speed of this MHD wave in kilometers/sec?

Answer: The time difference is 06:15 - 05:45 = 30 minutes. The speed was about 315,000 km / 30 minutes = 11,000 kilometers/minute, which is 11,000 km/minute x (1 minute/60 seconds) = 180 kilometers/sec.

Problem 4 - Kinetic Energy is defined by the equation K.E. = 1/2 m V^2 where m is the mass of the object in kilograms, and V is its speed in meters/sec. Suppose the mass of the CME was about 1 million metric tons, use your answer to Problem 3 to calculate the K.E., which will be in units of Joules.

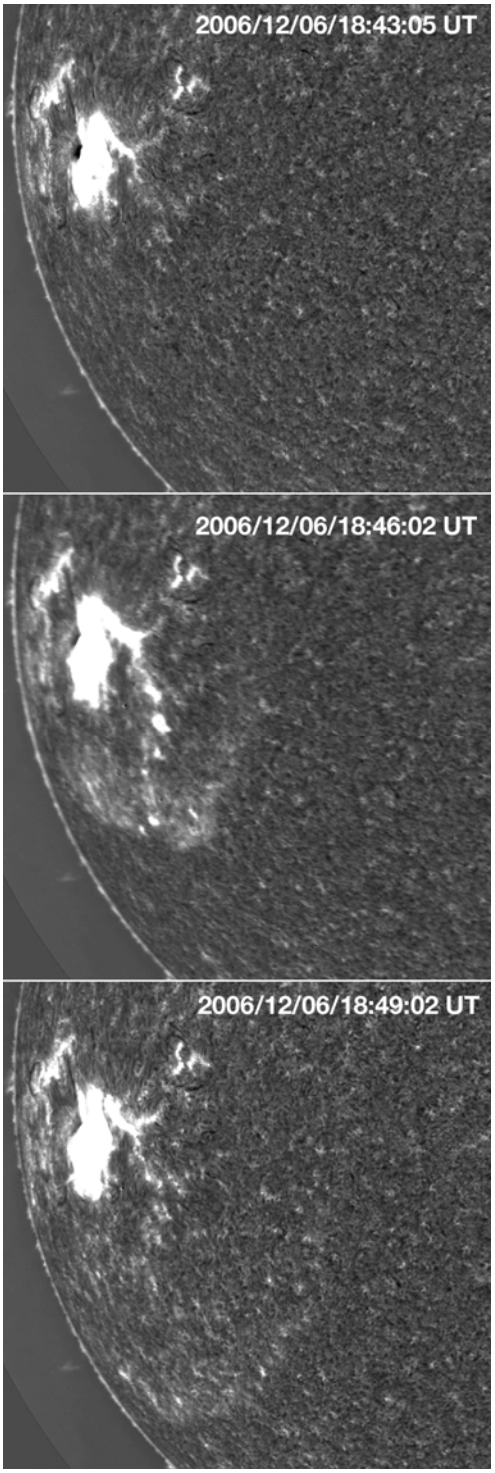
Answer: The mass of the CME was 1 billion metric tons. There are 1,000 kilograms in 1 metric ton, so the mass was 1.0×10^{12} kilograms. The speed is 180 km/sec which is 180,000 meters/sec. The kinetic energy is then about $0.5 \times 1.0 \times 10^{12} \times (180{,}000)^2$ = **1.6×10^{22} Joules.**

Problem 5 - If 1 kiloton of TNT has the explosive energy of 4.1×10^{12} Joules, how many megatons of TNT does the kinetic energy of the tsunami represent?

Amswer: 1.6×10^{22} Joules x (1 kiloton TNT/4.1×10^{12} Joules) = 3.9×10^{9} kilotons TNT. Since 1 megaton = 1,000 kilotons, we have an explosive yield of **3,900,000 megatons TNT**. (Note; this answer differs from the STEREO estimate because the speed is approximate, and does not include the curvature of the sun).

Teacher Note: Additional information, and movies of the event, can be found at the STEREO website: **http://stereo.gsfc.nasa.gov/news/SolarTsunami.shtml.** Also published in the Astrophysical Journal Letters (*ApJ* **700** L182-L186)

Measuring the Speed of a Solar Tsunami!



Moments after a major class X-6 solar flare erupted at 18:43:59 Universal Time on December 6, 2006, the National Solar Observatory's new Optical Solar Patrol Camera captured a movie of a shock wave 'tsunami' emerging from Sunspot 930 and traveling across the solar surface. The three images to the left show the progress of this Morton Wave. The moving solar gasses can easily be seen. You can watch the entire movie and see it more clearly (http://image.gsfc.nasa.gov/poetry/weekly/MortonWave.mpeg).

Note: because the event is seen near the solar limb, there is quite a bit of fore-shortening so the motion will appear slower than what the images suggest.

Problem 1: From the portion of the sun's edge shown in the images, complete the solar 'circle'. What is the radius of the sun's disk in millimeters?

Problem 2: Given that the physical radius of the sun is 696,000 kilometers, what is the scale of each image in kilometers/millimeter?

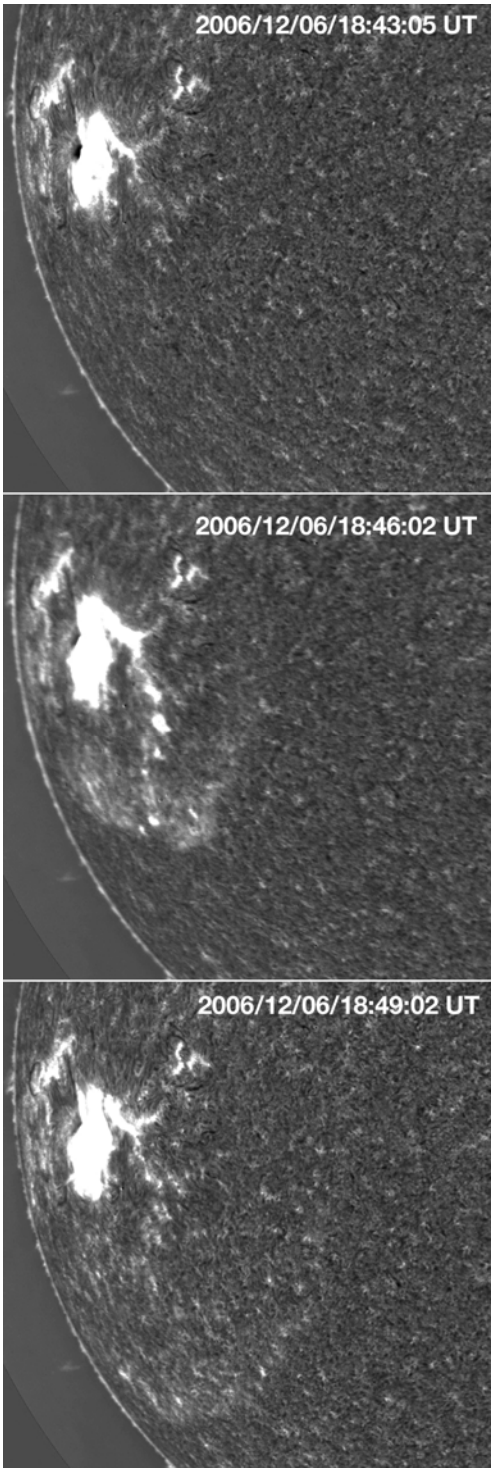
Problem 3: Select a spot near the center of the sunspot (large white spot in the image), and a location on the leading edge of the shock wave. What is the distance in kilometers from the center of the sunspot, to the leading edge of the shock wave in each image?

Problem 4: The images were taken at 18:43:05, 18:47:03 and 18:50:11 Universal Time. How much elapsed time has occurred between these images?

Problem 5: From your answers to Problem 3 and 4, what was the speed of the Morton Wave in kilometers per hour between the three images? B) did the wave accelerate or decelerate as it expanded?

Problem 6: The speed of the Space Shuttle is 44,000 kilometers/hour. The speed of a passenger jet is 900 kilometers/hour. Would the Morton Wave have overtaken the passenger jet? The Space Shuttle?

Answer Key:



Problem 1: From the portion of the sun's edge shown in the images, complete the solar 'circle'. What is the radius of the sun's disk in millimeters?

Answer: About 158 millimeters using a regular dessert plate as a guide.

Problem 2: Given that the physical radius of the sun is 696,000 kilometers, what is the scale of each image in kilometers/millimeter?

Answer: 696,000/158 = 4,405 kilometers/millimeter

Problem 3: What is the distance in kilometers from the center of the sunspot, to the leading edge of the shock wave in each image?

Answer:

Image 2 = 27 mm = 27x4405 = 119,000 km
Image 3 = 38 mm = 167,000 km

Problem 4: The images were taken at 18:43:05, 18:47:03 and 18:50:11 Universal Time. How much elapsed time has occurred between these images?

Answer: Image 1 - Image 2 = 3 minutes 58 seconds
Image 2 - Image 3 = 3 minutes 8 seconds

Problem 5: From your answers to Problem 3 and 4, A) what was the speed of the Morton Wave in kilometers per hour between the three images?

Answer:

V12 = 119,000 km/3.9 min x (60 min/1 hr)
= 1.8 million kilometers/hour

V23 = 167,000/3.1 min x (60 min/1 hr)
= 3.2 million kilometers/hour

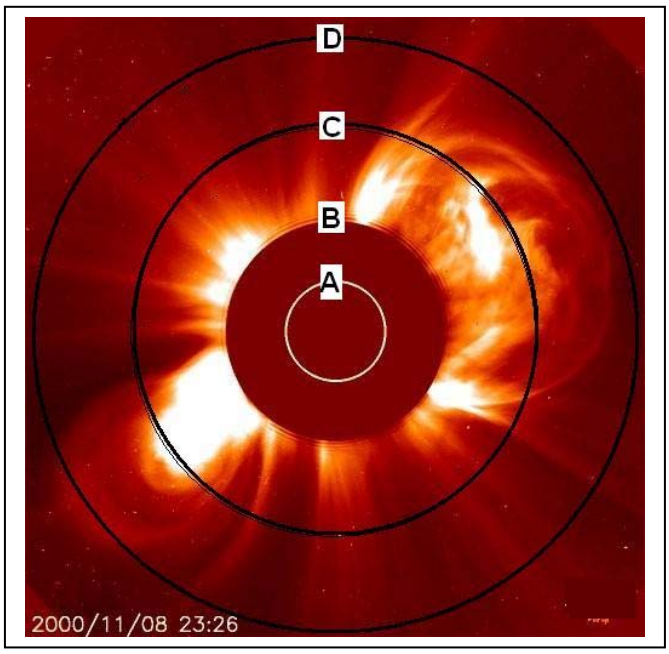
B) Did the speed of the wave accelerate or decelerate?

Answer: Because V23 > V12 the wave accelerated.

Problem 6: The speed of the Space Shuttle is 44,000 kilometers/hour. The speed of a passenger jet is 900 kilometers/hour. Would the Morton Wave have overtaken the passenger jet? The Space Shuttle?

Answer: It would easily have overtaken the Space Shuttle! Because of fore-shortening, the actual speed of the wave was even higher than the estimates from the images, so the speed could have been well over 4 million km/hr.

Estimating Halo CME Speeds



Frames	Cadence (seconds)					
	10	15	20	**25**	30	35
1	2333	1556	1167	**933**	778	667
2	1167	778	583	**467**	389	333
3	778	519	389	**311**	259	222
4	583	389	292	**233**	194	167
5	467	311	233	**187**	156	133

The SOHO satellite LASCO instrument takes images of the sun every 24 seconds in order to detect the movement of coronal mass ejections such as the one shown in the image to the left. The circles indicate the scale of the image in multiples of the solar radius (690,000 kilometers). By counting the number of consecutive frames required for the CME to move between circles B and D, the speed of the CME can be estimated.

The table above gives the speed of the CME in kilometers/sec, based on the time between the frames (cadence) and the number of frames required for the CME to move from Circle B to D.

Problem 1 - An astronomer watches the CME on one satellite which has a cadence of 15 seconds between frames, and it moves from B to D in 5 frames. A second satellite has a cadence of 35 seconds. How many frames will the second satellite require to see the CME move the same distance?

Problem 2 - Using this method, an astronomer wants to observe very fast CMEs with speeds between 934 and 4,666 km/sec. What must the cadence be for these measurements to be possible?

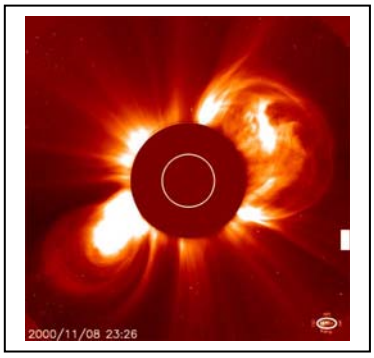
Problem 1 - An astronomer watches the CME on one satellite which has a cadence of 15 seconds between frames, and it moves from B to D in 5 frames. A second satellite has a cadence of 35 seconds. How many frames will the second satellite require to see the CME move the same distance?

Answer: From the first satellite and the table, the speed is 311 km/sec. If the cadence of the second satellite is **35 seconds**, it will see the CME move the same distance in about 2 frames.

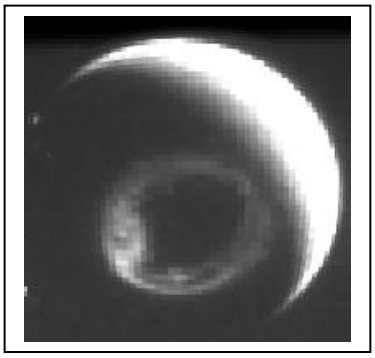
Problem 2 - Using this method, an astronomer wants to observe very fast CMEs with speeds between 934 and 4,666 km/sec. What must the cadence be for these measurements to be possible?

Answer: **The speeds are twice as fast as the range of speeds measurable with a cadence of 10 seconds/frame. Because for a given number of frames the speed is linear with the cadence, reducing the cadence to 5 seconds/frame the required speeds can be measured within an interval of 5 frames.**

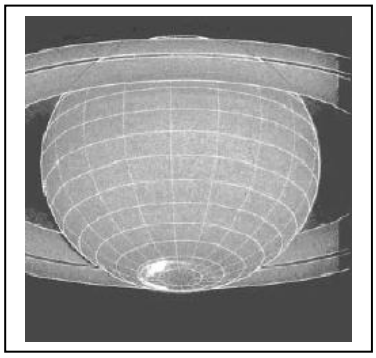
An Interplanetary Shock Wave



Sun - CME



Earth - Aurora



Saturn - Aurora

On November 8, 2000 the sun ejected a billion-ton cloud of gas called a coronal mass ejection or CME. On November 12, the CME collided with Earth and produced a brilliant aurora detected from space by the IMAGE satellite.

On December 8, the Hubble Space Telescope detected an aurora on Saturn. During the period from November to December, 2000, Earth, Jupiter and Saturn were almost lined-up with each other. Assuming that the three planets were located on a straight line drawn from the sun to Saturn, with distances from the sun of 150 million, 778 million and 1.43 billion kilometers respectively, answer the questions below:

Problem 1 – How many days did the disturbance take to reach Earth and Saturn?

Problem 2 – What was the average speed of the CME in its journey between the Sun and Earth in millions of km per hour?

Problem 3 – What was the average speed of the CME in its journey between Earth and Saturn in millions of km per hour?

Problem 4 – Did the CME accelerate or decelerate as it traveled from the Sun to Saturn?

Problem 5 – How long would the disturbance have taken to reach Jupiter as it passed Earth's orbit?

Problem 6 – On what date would you have expected to see aurora on Jupiter?

On November 8, 2000 the sun ejected a billion-ton cloud of gas called a coronal mass ejection or CME. On November 12, the CME collided with Earth and produced a brilliant aurora detected from space by the IMAGE satellite. On December 8, the Hubble Space telescope detected an aurora on Saturn. During the period from November to December, 2000, Earth, Jupiter and Saturn were almost lined-up with each other. Assuming that the three planets were located on a straight line drawn from the sun to Saturn, with distances from the sun of 150 million, 778 million and 1.43 billion kilometers respectively, answer the questions below:

1 – How many days did the disturbance take to reach Earth and Saturn?
Answer: Earth = 4 days; Saturn = 30 days.

2 – What was the average speed of the CME in its journey between the Sun and Earth in millions of km per hour? **Answer:** Sun to Earth = 150 million km. Time = 4 days x 24 hrs = 96 hrs so Speed = 150 million km/96hr = **1.5 million km/hr.**

3 – What was the average speed of the CME in its journey between Earth and Saturn in millions of km per hour? **Answer:** Distance = 1,430 – 150 = 1,280 million km. Time = 30 days x 24h = 720 hrs so Speed = 1,280 million km/720hrs = **1.8 million km/hr.**

4 – Did the CME accelerate or decelerate as it traveled from the Sun to Saturn? **Answer: The CME accelerated from 1.5 million km/hr to 1.8 million km/hr.**

5 – How long would the disturbance have taken to reach Jupiter as it passed Earth's orbit?
Answer: Jupiter is located 778 million km from the Sun or (778 – 150 =) 628 million km from Earth. Because the CME is accelerating, it is important that students realize that it is more accurate to use the average speed of the CME between Earth and Saturn which is (1.8 + 1.5)/2 = 1.7 million km/hr. The travel time to Jupiter is then 628/1.7 = **369 hours.**

6 – On what date would you have expected to see aurora on Jupiter? **Answer:** Add 369 hours (~ 15 days) to the date of arrival at Earth to get **November 23.** According to radio observations of Jupiter, the actual date of the aurora was November 20. Note: If we had used the Sun-Earth average speed of 1.5 million km/hr to get a travel time of 628/1.5 = 418 hours, the arrival date would have been November 29, which is 9 days later than the actual storm. This points out that the CME was accelerating after passing Earth, and its speed was between 1.5 and 1.8 million km/hr.

For more details about this interesting research, read the article by Renee Prange et al. "An Interplanetary Shock Traced by Planetary Auroral Storms from the Sun to Saturn" published in the journal Nature on November 4, 2004, vol. 432, p. 78. Also visit the Physics Web online article "Saturn gets a shock" at http://www.physicsweb.org/articles/news/8/11/2/1

Estimating Halo CME Arrival Times

Speed	Distance from the Sun in Astronomical Units										
(km/s)	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
300	139	153	167	181	194	208	222	236	250	264	278
400	104	115	125	135	146	156	167	177	188	198	208
500	83	92	100	108	117	125	133	142	150	158	167
600	69	76	83	90	97	104	111	118	125	132	139
700	60	65	71	77	83	89	95	101	107	113	119
800	52	57	63	68	73	78	83	89	94	99	104
900	46	51	56	60	65	69	74	79	83	88	93
1000	42	46	50	54	58	63	67	71	75	79	83
1100	38	42	45	49	53	57	61	64	68	72	76
1200	35	38	42	45	49	52	56	59	63	66	69
1300	32	35	38	42	45	48	51	54	58	61	64
1400	30	33	36	39	42	45	48	51	54	57	60
1500	28	31	33	36	39	42	44	47	50	53	56
1600	26	29	31	34	36	39	42	44	47	49	52
1700	25	27	29	32	34	37	39	42	44	47	49
1800	23	25	28	30	32	35	37	39	42	44	46
1900	22	24	26	29	31	33	35	37	39	42	44
2000	21	23	25	27	29	31	33	35	38	40	42
2100	20	22	24	26	28	30	32	34	36	38	40
2200	19	21	23	25	27	28	30	32	34	36	38
2300	18	20	22	24	25	27	29	31	33	34	36
2400	17	19	21	23	24	26	28	30	31	33	35
2500	17	18	20	22	23	25	27	28	30	32	33
2600	16	18	19	21	22	24	26	27	29	30	32
2700	15	17	19	20	22	23	25	26	28	29	31
2800	15	16	18	19	21	22	24	25	27	28	30
2900	14	16	17	19	20	22	23	24	26	27	29
3000	14	15	17	18	19	21	22	24	25	26	28

The shaded values in the table above give the arrival times (in hours) to the various distances in Astronomical Units, given the speed of the coronal mass ejection (CME) indicated in the first column. The distance from Earth to the sun (150 million km) is defined to be exactly 1 Astronomical Unit (1.0 AU). A 'Halo' CME is an explosive expulsion of heated gas from the sun directed towards Earth.

Problem 1 - Suppose that the planet Earth and Mars are in opposition, which means that a straight line can be drawn from the center of the sun directly through the centers of Earth and Mars. Astronomers at Earth detect a CME leaving the sun at a speed of 1500 km/sec. When will this CME arrive at Earth and how long afterwards will it arrive at Mars located at a distance of 1.5 AU?

Problem 2 - A CME leaves the sun at a speed of 900 km/sec on July 4, 2015 at 13:00 UT. On what dates and times will it arrive at A) Earth? B) Mars? C) An asteroid located at 1.9 AU?

Problem 1 - Suppose that the planet Earth and Mars are in opposition, which means that a straight line can be drawn from the center of the sun directly through the centers of Earth and Mars. Astronomers at Earth detect a CME leaving the sun at a speed of 1500 km/sec. When will this CME arrive at Earth (1.0 AU) and how long afterwards will it arrive at Mars located at a distance of 1.5 AU?

Answer: From the table, look at the row for '1500' in Column 1 and under '1.0' AU the time for arrival at Earth is about **26 hours**. In the column for Mars at '1.5' the arrival time is 39 hours, so it takes **13 additional hours for the CME to arrive at Mars**.

Problem 2 - A CME leaves the sun at a speed of 900 km/sec on July 4, 2015 at 13:00 UT. On what dates and times will it arrive at A) Earth? B) Mars? C) An asteroid located at 1.9 AU?

Answer: A) The transit time to Earth is 46 hours so adding 46 hours to the given date and time we get an additional 1 day and 22 hours so July 4, 2015 at 13:00 + 1 day and 22:00 gives July 5, 2015 at 35:00 which becomes **July 6, 2015 at 11:00 UT**.

B) The transit time is 69 hours or 2 days + 21 hours, which gives July 6, 2015 and 34:00 or **July 7, 2015 at 10:00 UT**.

C) The transit time is 88 hours or 3 days and 16:00 which becomes July 7, 2015 at 29:00 or **July 8, 2015 at 05:00 UT**.

Histogram of Halo CME Speeds: 1996-2008

Speed (km/s)	N	Speed (km/s)	N	Speed (km/s)	N
0	1	1000	20	2000	9
100	6	1100	22	2100	3
200	8	1200	14	2200	4
300	19	1300	21	2300	3
400	38	1400	13	2400	4
500	33	1500	14	2500	4
600	33	1600	11	2600	2
700	25	1700	12	2700	0
800	30	1800	9	2800	1
900	30	1900	7		

During the past sunspot cycle, there were nearly 2,000 events, called coronal mass ejections (CMEs) in which the sun ejected huge clouds of plasma into space. When directed at earth, these clouds can arrive within a few days and cause disturbances in Earth's magnetic field, called geomagnetic storms.

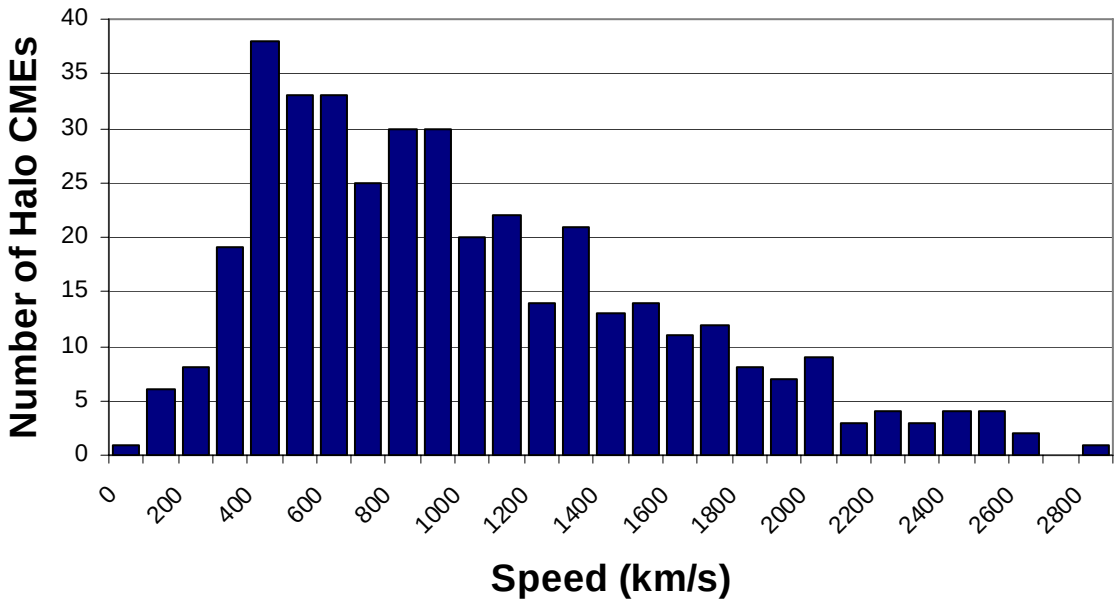
The arrival time is determined by the transit speed. The table to the left gives the speeds for a sample of Earth-directed CMEs.

Problem 1 - Create a histogram of the frequency data in the table.

Problem 2 - For this distribution of CME speeds, what are the A) mean speed? B) median speed?

Problem 3 - If the distance to the sun from Earth is 150 million kilometers, what is the average transit time, in days, represented by the histogram of CME speeds?

Problem 1 - Create a histogram of the frequency data in the table.



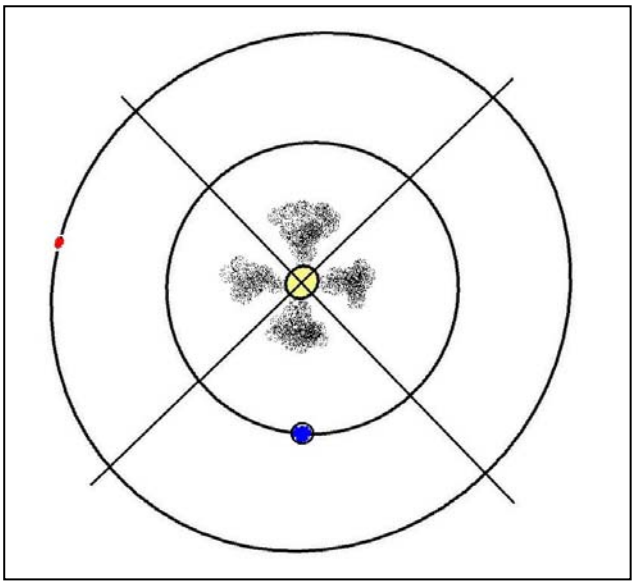
Problem 2 - For this distribution of CME speeds, what are the A) mean speed? B) median speed?

Answer: A) From the table, the average speed is found by the sun of the products of the speeds and Ns divided by the total number of tabulated CMEs which equals 395 events, so Va = (388700/395) so **Va = 984 km/sec**.
B) The median speed is the speed for which half of the events are below and half are above. From the table, N = 395/2 = 197, so **Vm = 800 km/sec**.

Problem 3 - If the distance to the sun from Earth is 150 million kilometers, what is the average transit time, in days, represented by the histogram of CME speeds?

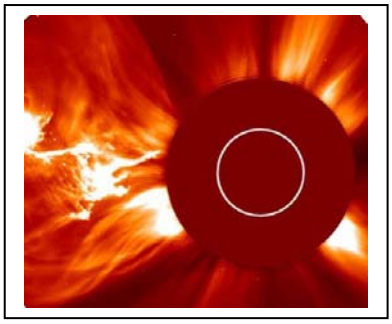
Answer: T = 150 million km / 984 km/s so T = 152,632 seconds which is about 42.4 hours or **T = 1.8 days**.

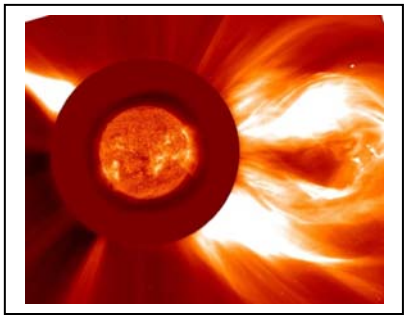
Predicting the Direction of Motion of a CME

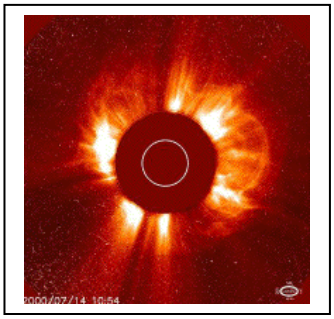


Although we cannot accurately determine the direction of travel of a coronal mass ejection (CME), images taken from one vantage point can often be used to learn the approximate direction of travel.

The diagram shows four CMEs ejected by the sun in 4 quadrants. The lower quadrant includes Earth. CMEs ejected into this quadrant often look like halos of gas (lower left image) as do CMEs ejected from the far-side of the sun (top quadrant). CMEs in the other two quadrants look like the remaining two images.







Problem 1 - The red dot in the diagram is the planet Mars. Which of the CMEs shown in the views above may be directed towards an eventual arrival at Mars?

Problem 2 - Suppose the above images were taken by a solar observatory orbiting Mars. Which of the CMEs might eventually collide with Earth - the blue dot in the diagram?

Problem 3 - Halo CMEs, such as the one in the right-hand image, are moving either towards the observer or directly away, having been ejected either from active regions on the near-side of the sun or the far-side. Radio bursts are often detected just before a CME is ejected. If a radio burst was not detected from a halo CME, is it headed towards Earth or away from Earth?

Problem 1 - The red dot in the diagram is the planet Mars. Which of the CMEs shown in the view above may be directed towards an eventual arrival at Mars?

Answer: **The first CME on the far-left is most likely headed in the general direction of Mars.**

Problem 2 - Suppose the above images were taken by a solar observatory orbiting Mars. Which of the CMEs might eventually collide with Earth - the blue dot in the diagram?

Answer: **From the location of Mars, the first CME on the far-left would be headed directly away from Earth. The CME on the far-right is headed towards Mars, and so the middle CME is headed in the general direction of Earth.**

Problem 3 - Halo CMEs, such as the one in the right-hand image, are moving either towards the observer or directly away, having been ejected either from active regions on the near-side of the sun or the far-side. Radio bursts are often detected just before a CME is ejected. If a radio burst was not detected from a halo CME, is it headed towards Earth or away from Earth?

Answer: **Because radio waves cannot pass through the sun, halo CMEs that are not accompanied by radio bursts are probably far-side events traveling directly away from Earth, while halo CMEs with radio bursts are on the near-side of the sun and may be headed towards Earth.**

Fastest Halo CME Speeds by Year and Month

Table of Fast CMEs between 1996 - 2006

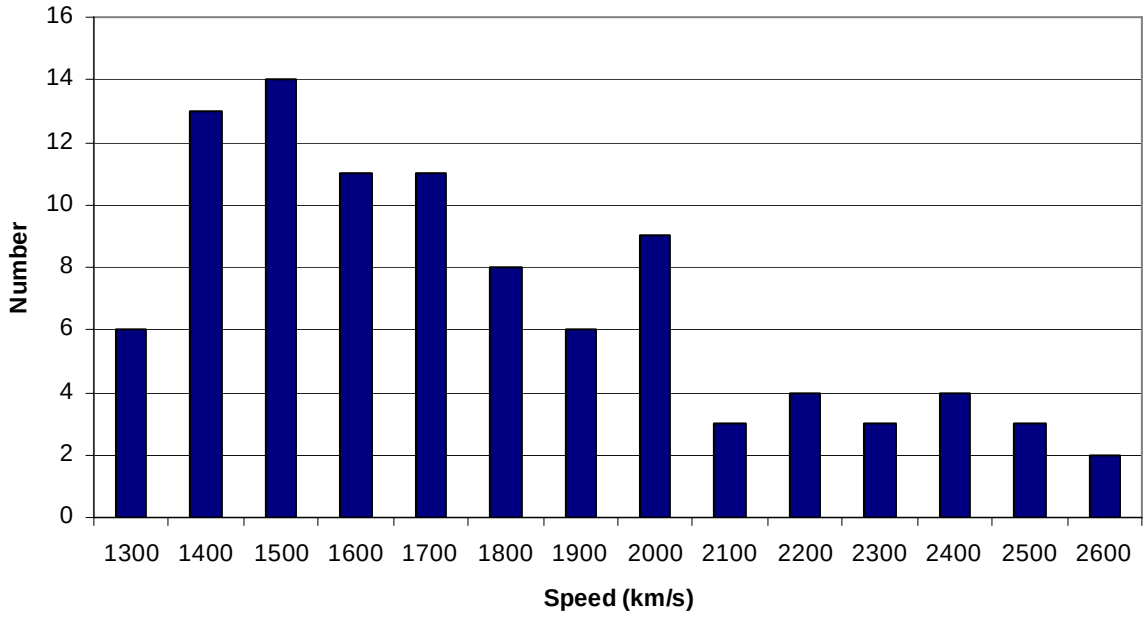
Year	Month	Speed	Year	Month	Speed	Year	Month	Speed	Year	Month	Speed
2004	11	3387	2002	7	2047	1999	6	1772	2001	10	1537
2005	1	2861	2003	11	2036	2004	11	1759	2005	7	1527
2003	11	2657	2003	10	2029	2002	3	1750	2003	11	1523
2000	5	2604	2005	1	2020	1998	12	1749	2001	1	1507
2003	11	2598	2003	11	2008	2002	9	1748	2001	12	1506
2005	1	2547	2004	11	2000	2001	6	1701	1998	11	1505
2005	7	2528	1998	3	1992	2002	10	1694	1999	4	1495
2000	11	2519	2005	7	1968	1999	5	1691	2002	1	1492
2001	4	2465	2002	7	1941	2005	5	1689	2003	10	1484
2003	10	2459	2005	8	1929	2005	6	1679	2001	4	1475
2001	4	2411	2005	9	1922	2000	7	1674	2005	7	1458
2001	9	2402	2002	8	1913	2005	9	1672	2004	7	1444
2002	4	2393	2005	9	1893	2003	11	1661	2001	11	1443
2005	8	2378	2005	9	1866	2005	7	1660	2001	11	1437
2005	9	2326	2002	11	1838	2002	7	1636	2001	8	1433
2002	7	2285	2003	5	1835	1998	4	1618	2002	3	1429
2005	9	2257	2003	6	1813	2005	8	1600	2005	7	1423
2003	11	2237	2001	11	1810	2000	5	1594	2001	6	1407
2001	12	2216	2005	8	1808	2002	8	1585	2001	10	1405
2002	7	2191	1998	6	1802	1999	5	1584	1998	3	1397
2002	10	2115	1998	11	1798	2001	8	1575	2001	4	1390
2005	7	2115	2002	1	1794	2002	5	1557	1999	7	1389
2005	1	2094	2005	7	1787	1997	11	1556	1998	4	1385
2003	6	2053	2000	5	1781	2000	9	1550	2005	9	1384
2005	1	2049	2001	12	1773	2005	7	1540	2005	2	1380

Coronal Mass Ejections (CMEs) are billion-ton clouds of gas that can leave the solar surface at many different speeds as shown in the table above. This table gives the speeds of the 100-fastest CMEs seen during the last sunspot cycle between 1996-2008.

Problem 1 - Create a histogram of the CME speeds using bins that are 100 km/sec wide over the domain of speeds [1300, 2600].

Problem 2 - What is the: A) Average speed? B) Median speed? C) Mode speed?

Problem 3 - If the transit time to Earth is 42 hours for a CME traveling at a speed of 1,000 km/s, to the nearest hour, what is A) the range of speeds spanned by this complete sample of 100 CMES? B) The average transit time of a CME?

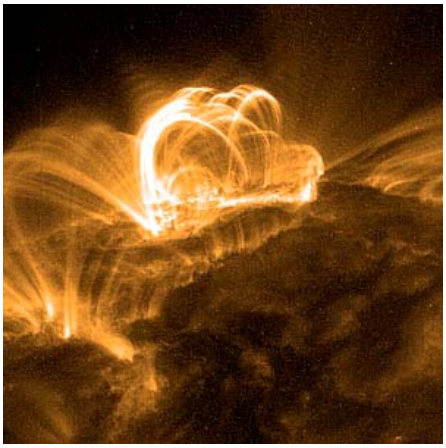


Problem 1 - Create a histogram of the CME speeds using bins that are 100 km/sec wide over the domain of speeds [1300, 2600]. Answer: See above.

Problem 2 - What is the: A) Average speed? B) Median speed? C) Mode speed?
Answer: A) The average speed is found by adding all 100 speeds and dividing by 100. The answer is **<V> = 1,851 km/sec**. B) The median speed is the speed for which half of the CME speeds are below and half are above the value. The table is ranked from fastest to slowest so CME #50 will be close to the median speed for this 'even' numbered set. **Vm = 1,773 km/sec**. C) The mode speed is the most frequently occurring speed in the sample which is **V = 2,115 km/sec**.

Problem 3 - If the transit time to Earth is 42 hours for a CME traveling at a speed of 1,000 km/s, to the nearest hour, what is A) the range of speeds spanned by this complete sample of 100 CMES? B) The average transit time of a CME?

Answer: A) The range is from 1,380 to 3,387 km/sec B) so by scaling we get 42 x (1000/1380) = 30 hours for the slowest and 42 x (1000/3387) = 12 hours. the average transit time is T = 42 x (1000/1851) = **23 hours**.



The Sun is an active star, which produces solar flares (F) and explosions of gas (C). Astronomers keep watch for these events because they can harm satellites and astronauts in space. Predicting when the next storm will happen is not easy to do. The problems below are solved by writing out all of the possibilities, then calculating the probability of the particular outcome!

Solar flare photo courtesy TRACE/NASA

1 – During a week of observing the sun, astronomers detected 1 solar flare (F). What was the probability (as a fraction) that it happened on Wednesday?

2 – During the same week, two gas clouds were ejected (C), but not on the same days. What is the probability (as a fraction) that a gas cloud was ejected on Wednesday?

3 – Suppose that the flares and the gas clouds had nothing to do with each other, and that they occurred randomly. What is the probability (as a fraction) that both a flare and a gas cloud were spotted on Wednesday? (Astronomers would say that these phenomena are uncorrelated because the occurrence of one does not mean that the other is likely to happen too).

Answer Key

1 – Answer: There are only 7 possibilities:

F X X X X X X	X X X F X X X	X X X X X X F
X F X X X X X	X X X X F X X	
X X **F** X X X X	X X X X X F X	

So the probability for any one day is **1/7.**

2 – Here we have to distribute 2 storms among 7 days. For advanced students, there are 7! / (2! 5!) = 7 x 6 / 2 = 21 possibilities which the students will work out by hand:

C C X X X X X	X C **C** X X X X	X X **C** C X X X	X X X C C X X
C X **C** X X X X	X C X C X X X	X X **C** X C X X	X X X C X C X
C X X C X X X	X C X X C X X	X X **C** X X C X	X X X C X X C
C X X X C X X	X C X X X C X	X X **C** X X X C	X X X X C C X
C X X X X C X	X C X X X X C		X X X X C X C
C X X X X X C			X X X X X C C

There are 6 possibilities (in red) for a cloud appearing on Wednesday (Day 3), so the probability is **6/21.**

3 – We have already tabulated the possibilities for each flare and gas cloud to appear separately on a given day. Because these events are independent of each other, the probability that on a given day you will spot a flare and a gas cloud is just 1/7 x 6/21 or 6/147. This is because for every possibility for a flare from the answer to Problem 1, there is one possibility for the gas clouds.

There are a total of 7 x 21 = 147 outcomes for both events taken together. Because there are a total of 1 x 6 outcomes where there is a flare and a cloud on a particular day, the fraction becomes (1 x 6)/147 = **6/147.**

Are you ready for Sunspot Cycle 24?

In 2007, the best estimates and observations suggested that we had just entered sunspot minimum, and that the next solar activity cycle might begin during the first few months of 2008. In May, 2007, solar physicist Dr. William Pesnell at the NASA, Goddard Spaceflight Center tabulated all of the current predictions for when the next sunspot cycle (2008-2019) will reach its peak. These predictions, reported by many other solar scientists are shown in the table below.

Current Predictions for the Next Sunspot Maximum

Author	Prediction Year	Spots	Year	Method Used
Horstman	2005	185	2010.5	Last 5 cycles
Thompson	2006	180		Precurser
Tsirulnik	1997	180	2014	Global Max
Podladchikova	2006	174		Integral SSN
Dikpati	2006	167		Dynamo Model
Hathaway	2006	160		AA Index
Pesnell	2006	160	2010.6	Cycle 24 = Cycle22
Maris & Onicia	2006	145	2009.9	Neural Network Forecast
Hathaway	2004	145	2010	Meridional Circulation
Gholipour	2005	145	2011.5	Spectral Analysis
Chopra & Davis	2006	140	2012.5	Disturbed Day Analysis
Kennewell	2006	130		H-alpha synoptic charts
Tritakis	2006	133	2009.5	Statistics of Rz
Tlatov	2006	130		H-alpha Charts
Nevanlinna	2007	124		AA at solar minimum
Kim	2004	122	2010.9	Cycle parameter study
Pesnell	2006	120	2010	Cycle 24 = Cycle 23
Tlatov	2006	115		Unipolar region size
Tlatov	2006	115		Large Scale magnetic field
Prochasta	2006	119		Average of Cycles 1 to 23
De Meyer	2003	110		Transfer function model
Euler & Smith	2006	122	2011.2	McNish-Lincoln Model
Hiremath	2007	110	2012	Autoregressive Model
Tlatov	2006	110		Magnetic Moments
Lantos	2006	108	2011	Even/Odd cycle pattern
Kane	1999	105	2010.5	Spectral Components
Pesnell	2006	101	2012.6	Linear Prediction
Wang	2002	101	2012.3	Solar Cycle Statistics
Roth	2006	89	2011.1	Moving averages
Duhau	2003	87		Sunspot Maxima and AA
Baranovski	2006	80	2012	Non-Linear Dynamo model
Schatten	2005	80	2012	Polar Field Precurser
Choudhuri	2007	80		Flux Transport Dynamo
Javariah	2007	74		Low-Lat. Spot Groups
Svalgaard	2005	70		Polar magnetic field
Kontor	2006	70	2012.9	Statistical extrapolation
Badalyan	2001	50	2010.5	Coronal Line
Cliverd	2006	38		Atmospheric Radiocarbon
Maris	2004	50		Flare energy during Cycle 23

Problem 1: What is the average year for the predicted sunspot maximum?
Problem 2: What is the average prediction for the total number of sunspots during the next sunspot maximum?
Problem 3: Which scientist has offered the most predictions? Do they show any trends?
Problem 4: What is the average prediction for the total sunspots during each prediction year from 2003 to 2006?
Problem 5: As we get closer to sunspot minimum in 2008, have the predictions for the peak sunspots become larger, smaller, or remain unchanged on average?
Problem 6: Which methods give the most different prediction for the peak sunspot number, compared to the average of the predictions made during 2006?

Answer Key:

Problem 1: What is the average year for the predicted sunspot maximum?
Answer: There are 21 predictions, with an average year of 2011.3.
This corresponds to about March, 2011.

Problem 2: What is the average prediction for the total number of sunspots during the next sunspot maximum?
Answer: The average of the 39 estimates in column 3 is 116 sunspots at sunspot maximum.

Problem 3: Which scientist has offered the most predictions? Do they show any trends?
Answer: Tlatov has offered 4 predictions, all made in the year 2006. The predicted numbers were 130, 115, 115 and 110. There does not seem to be a significant trend towards larger or smaller predictions by this scientist. The median value is 115 and the mode is also 115.

Problem 4: What is the average prediction for the total sunspots during each prediction year from 2003 to 2006?
Answer: Group the predictions according to the prediction year and then find the average for that year.

2003: 110, 87 average = 98

2004: 145, 122, 50 average= 106

2005: 185,145,80,70 average= 120

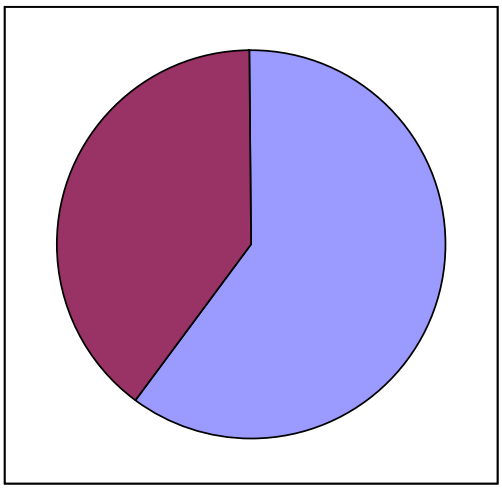
2006: 180,174,167,160,160,145,140,130,133,130,120,
115,115,119,122,110,108,101,89,80,70,38 **average= 123**

Problem 5: As we get closer to sunspot minimum in 2008, have the predictions for the peak sunspots become larger, smaller, or remain unchanged on average?
Answer: Based on the answer to problem 4, it appears that the predictions have tended to get larger, increasing from about 98 to 123 between 2003 and 2006.

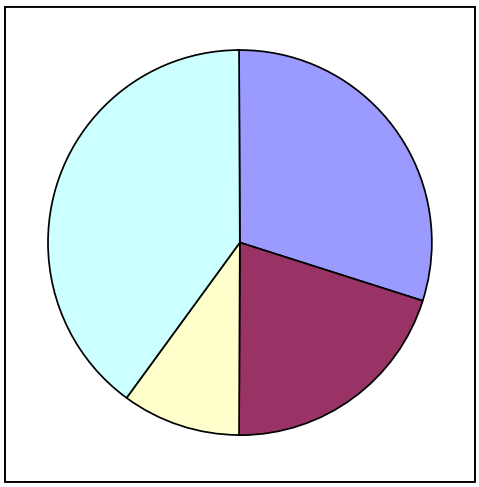
Problem 6: Which methods give the most different prediction for the peak sunspot number, compared to the average of the predictions made during 2006?
Answer: Cliverd's Atmospheric Radiocarbon Method (38 spots), Badalyan's Coronal Line Method (50 spots), and Maris's Flare Energy during Cycle 23 (50 spots) seem to be the farthest from the average predictions that have been made by other forecasting methods.

Solar Storm Energy and Pie Graphs

The pie charts below show approximately how various forms of energy are involved in a solar flare. Flares occur when stored magnetic energy is suddenly released. The chart on the left shows how much of this magnetic energy is available for creating a flare (purple) and how much is lost (blue). The chart on the right shows how much of the available magnetic flare energy goes into four different phenomena: Light green represents forms of radiation such as visible light and x-rays. Blue represents (kinetic) energy in ejected clouds of gas called Coronal Mass Ejections. Purple represents flare energy that goes into heating local gases to millions of degrees Celsius, and white is the portion of the flare energy that is lost to working against gravity.



Graph of stored magnetic energy



Graph of solar flare energy forms

Problem 1 - About what percentages of each of the four forms of energy are represented in the right-hand chart?

Problem 2 - About what percentage of the original, stored magnetic energy is available for flares?

Problem 3 - About what fraction of the original magnetic energy ends up as solar flare radiation, assuming all forms of energy can be interchanged with each other?

Problem 4 - About what fraction of the original magnetic energy ends up in CME ejection?

Problem 5 - A typical large flare has enough total energy to meet the world-wide power demands of human civilization for 10,000 years. How many years would be equivalent to A) causing the flare to shine and B) ejecting a CME?

Problem 1 – What percentages of each of the four forms of energy are represented in the right-hand chart?

Answer: **Radiation = 40%, CME = 30%, Gas heating = 20 % and Gravity = 10%**

Problem 2 - What percentage of the original, stored magnetic energy is available for flares?

Answer: The size of the purple sector is **40%**

Problem 3 – What fraction of the original magnetic energy ends up as solar flare radiation?

Answer: 40% of the original magnetic energy is available for a flare, and 40% of the flare energy ends up as radiation, so the fraction of the original magnetic energy involved is 0.40 x 0.40 = **0.16**

Problem 4 – What fraction of the original magnetic energy ends up in CME ejection?

Answer: 40% of the original magnetic energy is available for a flare, and 30% of the flare energy ends up as CME (kinetic) energy, so the fraction of the original magnetic energy involved is 0.40 x 0.30 **= 0.12**

Problem 5 – A typical large flare has enough total energy to meet the power demands of human civilization for 10,000 years. How many years would be equivalent to A) causing the flare to shine and B) ejecting a CME?

Answer: A) 40% of the flare energy ends up as radiation so this is equivalent to 0.40 x 10,000 years = *4,000 years* of human energy consumption.

B) 30% of the flare energy is available for CME kinetic energy, so this equals an equivalent of 0.30 x 10,000 years **= 3,000 years** of human energy consumption.

Probability of Compound Events

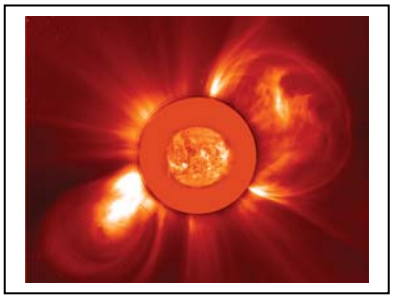
One of the most basic activities that scientists perform with their data is to look for correlations between different kinds of events or measurements in order to see if a pattern exists that could suggest that some new 'law' of nature might be operating. Many different observations of the Sun and Earth provide information on some basic phenomena that are frequently observed. The question is whether these phenomena are related to each other in some way. Can we use the sighting of one phenomenon as a prediction of whether another kind of phenomenon will happen?

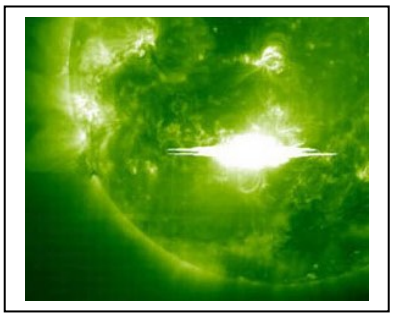
During most of the previous sunspot cycle (January-1996 to June-2006), astronomers detected 11,031 coronal mass ejections, (CME: Top image) of these 1186 were 'halo' events. Half of these were directed toward Earth.

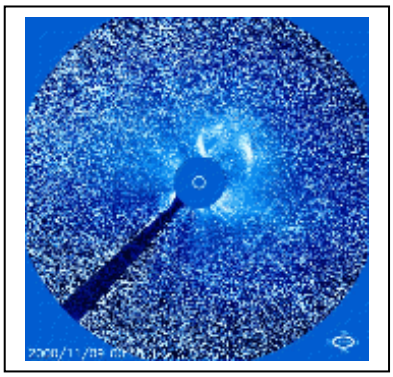
During the same period of time, 95 solar proton events (streaks in the bottom image were caused by a single event) were recorded by the GOES satellite network orbiting Earth. Of these SPEs, 61 coincided with Halo CME events.

Solar flares (middle image) were also recorded by the GOES satellites. During this time period, 21,886 flares were detected, of which 122 were X-class flares. Of the X-class flares, 96 coincided with Halo CMEs, and 22 X-class flares also coincided with 22 combined SPE+Halo CME events. There were 6 X-flares associated with SPEs but not associated with Halo CMEs. A total of 28 SPEs were not associated with either Halo CMEs or with X-class solar flares.

From this statistical information, construct a Venn Diagram to interrelate the numbers in the above findings based on resent NASA satellite observations, then answer the questions below.







1 - What are the odds that a CME is directed toward Earth?

2 - What fraction of the time does the sun produce X-class flares?

3 - How many X-class flares are not involved with CMEs or SPEs?

4 - If a satellite spotted both a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur?

5 - What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both?

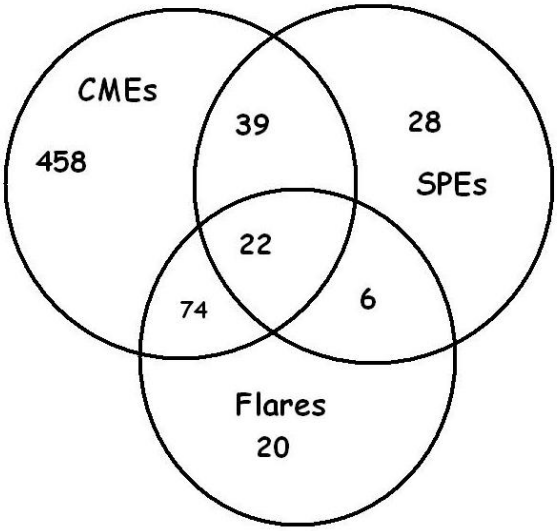
6 - If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed?

7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed?

8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?

9 - If you observed 100 CMEs, X-class flares and SPEs, how many times might you expect to see all three phenomena?

Answer Key:



Venn Diagram Construction.

1. There are 593 Halo CMEs directed to Earth so 593 = 74 with flares + 39 with SPEs + 22 both SPEs and Flares + 458 with no SPEs or Flares..

2. There are 95 SPEs. 95 = 39 with CMEs + 6 with flares + 22 with both flares and CMEs + 28 with no flares or CMEs

3. There are 122 X-class flares. 122 = 74 With CMEs only + 6 with SPEs only + 22 both CMEs and SPEs + 20 with no CMEs or SPEs.

1 - What are the odds that a CME is directed toward Earth? 593/11031 = 0.054 **odds = 1 in 19**

2 - What fraction of the time does the sun produce X-class flares? 122/21886 = **0.006**

3 - How many X-class flares are not involved with CMEs or SPEs? 122 - 74 - 22 - 6 = **20.**

4 - If a satellite spotted BOTH a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur? 22/(74+22) = **0.23**

5 - What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both?
100% x (39+22+6 / 95) = **70.1 %**

6 - If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed?
39+22+74 / 593 = 0.227 so the odds are 1/0.227 or about **1 in 4.**

7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed?
(6+22)/95 = 0.295 or 1 out of 3 times for X-flares
(39+22)/95 = 0.642 or 2 out of 3 for Halo CMEs
It is more likely to detect an SPE if a Halo CME occurs by 2 to 1.

8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?
39+6 / 95 = 0.50 so the odds are 1/0.50 or **2 to 1 .**

9 - If you observed 100 CMEs, X-class flares and SPEs, how many times might you expect to see all three phenomena?

100 x 22/(95+122+593) = **3 times**

Table of Event Frequencies for Cycle 23

	Flares (Class)			SPEs (pfu)				Halo
	C	M	X	10	100	1000	10000	CMEs
1996	76	4	1	1	1	0	0	1
1997	288	20	3	1	1	0	0	5
1998	1198	96	14	5	2	1	0	29
1999	1860	170	4	8	2	2	1	42
2000	2265	215	17	6	3	0	1	48
2001	3107	311	20	8	5	2	1	41
2002	2319	219	12	7	3	2	0	30
2003	1315	160	20	2	1	1	1	21
2004	912	122	12	3	1	1	1	87
2005	578	103	18	2	1	3	0	110
2006	150	10	4	0	1	1	0	33
2007	73	10	0	0	0	0	0	11
2008	8	1	0	0	0	0	0	12

The term 'solar storm' can refer to many different kinds of energetic phenomena including solar flares, Solar Proton Events (SPEs) and coronal mass ejections (CMEs). Solar flares are the most well-known, and are classified according to their x-ray power as C, M or X, with X being the most powerful. SPEs are classed according to the number of protons that pass through a surface area per second at Earth's orbit in units of particle Flux Units (pFUs). SPEs with 10,000 pFUs or higher can be deadly to astronauts not properly shielded. The table above gives the number of flares detected during each year of the previous solar activity cycle

Problem 1 - During sunspot maximum in 2001, what was the average number of hours you would have to wait between A) each of the three classes of X-ray flares? Each of the four classes of SPEs? And C) Each of the Halo CMEs? (Hint: There are 8,760 hours in 1 year)

Problem 2 - Assuming that Halo CMEs and X-class flares occur randomly over the year, what is the probability that during 2001 you will see both a Halo CME and an X-class flare on the same day?

Problem 1 - During sunspot maximum in 2001, what was the average number of hours you would have to wait between A) each of the three classes of X-ray flares? Each of the four classes of SPEs? and C) Each of the Halo CMEs? (Hint: There are 8,760 hours in 1 year)

Answer: From the table we have

	Flares (Class)			**SPEs (pfu)**				Halo
	C	M	X	10	100	1000	10000	CMEs
2001	3107	311	20	8	5	2	1	41

A) C-class flares: 8760 hours /3107 flares = **2.8 hours.**
M-class flares: 8760 / 311 = **28 hours = 1 day**
X-class flares: 8760 / 20 = **438 hours = 18 days**

B) 10 pFU: 8760/8 = **1,095 hours = 1.5 months**
100 pFU: 8760/5 = **1,752 hours = 2.5 months**
1000 pFU: 8760/2 = **4,380 hours = 6 months**
10000 pFU: 8760/1 = **8,760 hours = 1 year**

C) Halo CME: 8760 / 41 = **214 hours or 9 days**.

Problem 2 - Assuming that Halo CMEs and X-class flares occur randomly over the year, what is the probability that during 2001 you will see both a Halo CME and an X-class flare on the same day?

Answer: During this year there were 20 X-class flares and 41 Halo CMEs. Each day the probability is 20/365 = 1/18 that an X-class flare will occur, and 41/365 = 1/9 that a Halo CME will occur. The probability is then P(Halo) x P(Xflare) = 1/18 x 1/9 = **P(both) = 1/162** = 0.006 that both will happen on the same day.

Cosmic Rays and the Sunspot Cycle

Year	SSN	Flux	Year	SSN	Flux	Year	SSN	Flux
1950	84	93	1970	105	86	1990	142	76
1951	69	94	1971	67	88	1991	146	80
1952	31	95	1972	69	98	1992	94	86
1953	14	96	1973	38	96	1993	55	94
1954	4	98	1974	34	100	1994	30	96
1955	38	100	1975	15	97	1995	18	98
1956	142	98	1976	13	99	1996	9	99
1957	190	94	1977	27	100	1997	21	99
1958	185	80	1978	93	96	1998	64	100
1959	159	84	1979	155	90	1999	93	96
1960	112	82	1980	155	89	2000	120	88
1961	54	84	1981	140	84	2001	111	86
1962	38	88	1982	116	88	2002	104	88
1963	28	92	1983	67	80	2003	64	86
1964	10	96	1984	46	90	2004	40	86
1965	15	98	1985	18	92	2005	30	94
1966	47	98	1986	13	96	2006	15	96
1967	94	93	1987	29	98	2007	8	98
1968	106	90	1988	100	94	2008	3	100
1969	106	88	1989	158	84			

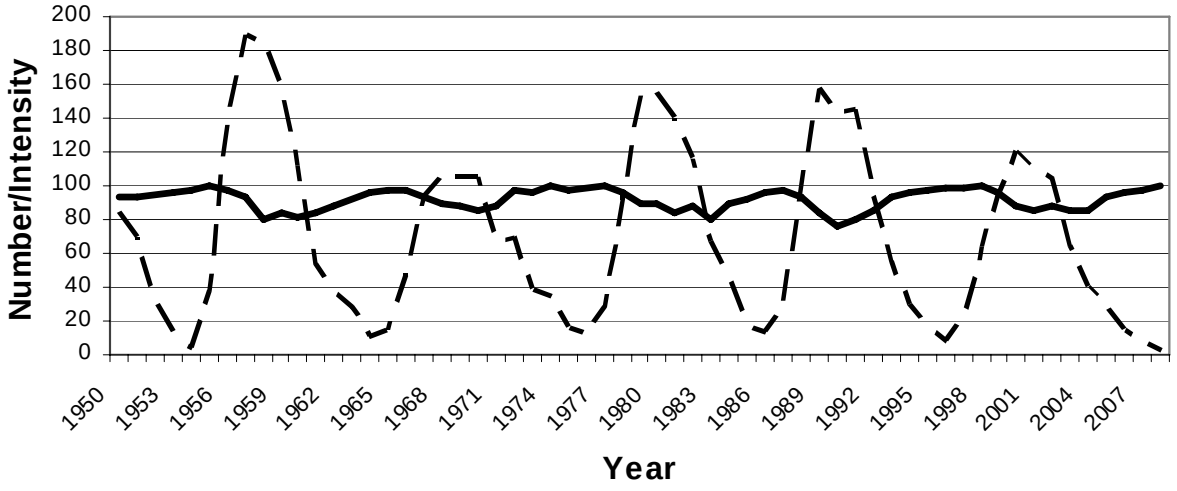
Cosmic rays are high-energy particles that enter our solar system from distant sources in the Milky Way galaxy and beyond. These particles are a radiation hazard for satellite electronics and for astronauts exposed to them for prolonged periods of time. The table above presents the intensity of the cosmic rays measured by instruments located in Climax, Colorado during the period from 1950 to 2008, along with the annual sunspot counts.

Problem 1 - On the same graph, plot the sunspot numbers and the cosmic ray intensities.

Problem 2 - What do you notice about the relationship between cosmic rays and sunspots?

Problem 3 - If you were planning a 1-year trip to Mars, what time would you make your journey relative to the sunspot cycle?

Problem 1 - On the same graph, plot the sunspot numbers and the cosmic ray intensities. Answer: **See below.**



Problem 2 - What do you notice about the relationship between cosmic rays and sunspots?
Answer: **When the sunspot number is highest, the cosmic ray intensity is lowest. The sunspot number peaks (maxima) match up with the cosmic ray valleys (minima).**

Problem 3 - If you were planning a 1-year trip to Mars, what time would you make your journey relative to the sunspot cycle?
Answer: **You would probably want to make the journey during sunspot maximum conditions when the cosmic ray intensity is lowest. This will reduce your total accumulated radiation dosage during the 1-year trip by 20%.**

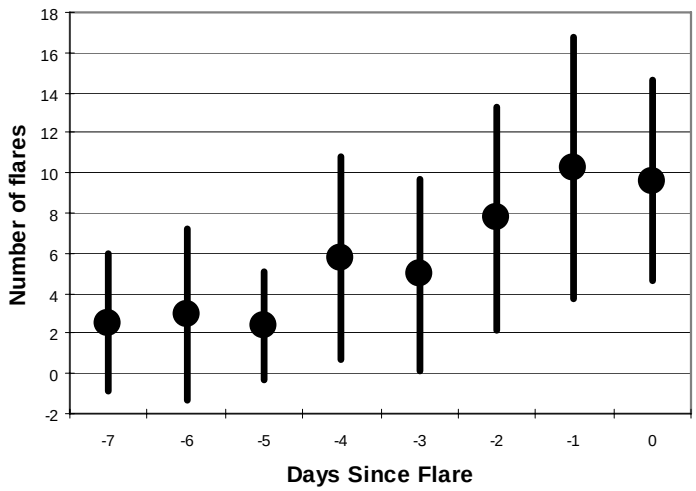
Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
-7	**0**	8	**0**	1	0	5	7	0	0	**0**	**0**	**0**	7	**0**	**0**	9	**0**	**0**	4	11	2	0	3	4	2
-6	**0**	15	2	2	0	8	3	0	0	**0**	**0**	**0**	5	**0**	**0**	13	**0**	**0**	6	7	0	4	0	2	7
-5	**0**	8	3	1	0	8	7	2	0	0	0	**0**	1	0	2	5	6	2	4	5	0	2	0	2	1
-4	0	8	3	2	11	6	15	14	10	6	0	0	3	1	9	16	8	4	11	8	0	2	0	6	1
-3	1	13	4	0	7	3	8	4	12	0	0	0	11	1	13	15	6	3	7	6	0	3	1	5	0
-2	1	12	14	9	8	8	12	18	15	0	0	1	10	3	15	13	15	7	6	8	2	9	0	6	2
-1	3	9	11	30	5	9	10	12	14	8	9	6	15	3	14	22	14	8	6	10	18	6	0	3	12
0	5	15	8	20	2	19	13	11	6	7	6	6	16	7	9	14	11	12	10	5	16	7	1	6	8

The table above gives the number of C and M-class flares recorded in the 7 days just before the eruption of an X-class flare from the solar surface. On Day-0 the X-class flare appeared. We would like to know if there is some way to predict whether an X-class flare will occur based on the amount of solar activity during previous days leading up to the X-class flare.

Problem 1 - For each day, what is the average number of C and M-class flares recorded?

Problem 2 - Graph the average number of flares versus the number of days prior to the X-class flare event. What do you notice about the data?

Problem 3 - A statistical measure of the possible range in a measured value is the 'sigma' or 'standard deviation'. For a given day, subtract the average value from each of the 25 events. Next compute the square of this difference and sum these squared quantities. Finally, divide the sum by (25-1) = 24 and take the square root of the answer. The most likely range of the measurement, A, (1-sigma) is then A +s and A-s. Compute the sigma for each day, and add 'error bars' to your graph to show the possible range of values due to statistical sampling.



Problem 1 - For each day, what is the average number of C and M-class flares recorded? Answer: **See the dots plotted on the graph above, and column 2 of the table below.**

Problem 2 - Graph the average number of flares versus the number of days prior to the X-class flare event. What do you notice about the data? Answer: **See the table below, column 2, for the averages. These are found by summing the total number of flares each day, N, and dividing by 25 'events'. The trend is that there is an increasing number of flares as you get closer to the day of the X-class flare.**

Day	Average	N	Sigma
-7	3	63	3
-6	3	74	4
-5	2	59	3
-4	6	144	5
-3	5	123	5
-2	8	194	6
-1	10	257	7
0	10	240	5

Problem 3 - A statistical measure of the possible range in a measured value is the 'sigma' or 'standard deviation'. For a given day ,subtract the average value from each of the 25 events. Next compute the square of this difference and sum these squared quantities. Finally, divide the sum by (25-1) = 24 and take the square root of the answer. The most likely range of the measurement, A, (1-sigma) is then A +s and A-s. Compute the sigma for each day, and add 'error bars' to your graph to show the possible range of values due to statistical sampling. Answer: See graph above. Example, Day -7, Average = 3. Sum = $(0-3)^2 + (8-3)^2 + (0-3)^2 + (1-3)^2 + \ldots = 280.3$, sigma = (280.3/24) = 3.4, so the error bar for day -7 should extend from 3-3.4 = -0.4 to 3+3.4 = 6.4.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
-7		2		0	0	0	2	0		0					4			2			2	2	0	0	0	0		0
-6		5	0	1	0	1	1	0		0					0			2			1	2	0	0	0	0		0
-5		3	1	0	0	0	0	0		0	0	0			0	0	0	4	2	0	0	1	0	0	0	0		0
-4	0	3	0	0	0	0	4	0	7	0	0	0		0	0	0	0	2	1	0	2	1	0	0	0	0	0	0
-3	0	4	0	0	2	0	2	0	5	1	0	0	1	0	1	0	1	3	0	0	1	0	0	1	1	1	1	0
-2	0	1	3	0	2	0	2	0	2	5	0	0	2	0	0	0	1	2	3	0	0	0	0	0	0	0	0	0
-1	1	1	5	4	1	0	4	1	4	2	0	0	7	2	1	0	0	4	3	1	0	0	3	1	0	1	1	0
0	0	2	4	6	0	4	0	4	0	0	1	2	3	2	2	0	1	1	0	2	1	0	2	0	0	0	0	2

The table above lists the number of M-class flares that occurred on the sun in the 7 days preceding the appearance of an X-class flare for 28 separate solar events.

Problem 1 - For each day, what is the average number of M-class flares that occurred?

Problem 2 - What is the variance (sigma) for the average value during each day?

Problem 3 - Graph the average number of M-class flares counted for each day including the range determined by 1-sigma error bars.

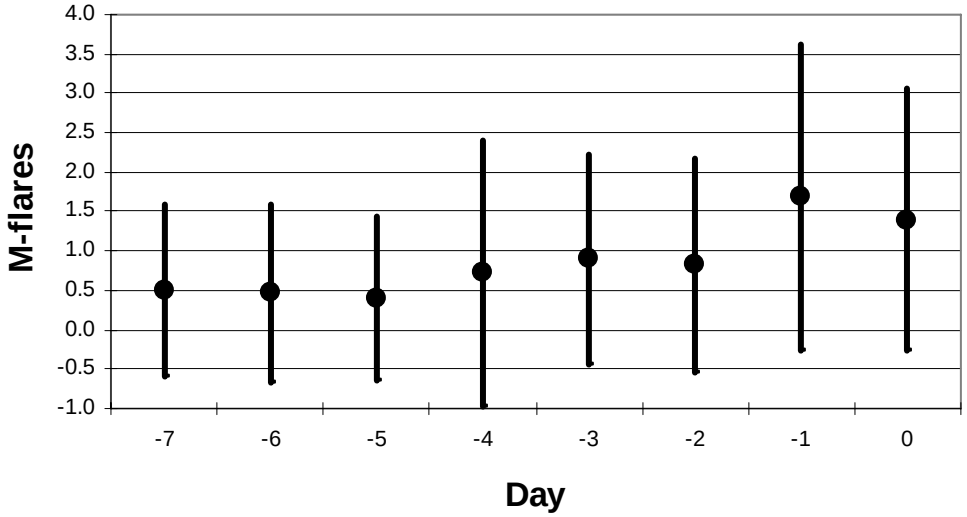
Problem 4 - Is the M-flare activity a good predictor of whether an X-class flare will occur or not?

Problem 1 - For each day, what is the average number of M-class flares that occurred? Answer: **See table below.**

Problem 2 - What is the variance (sigma) for the average value during each day? Answer: **See table below.**

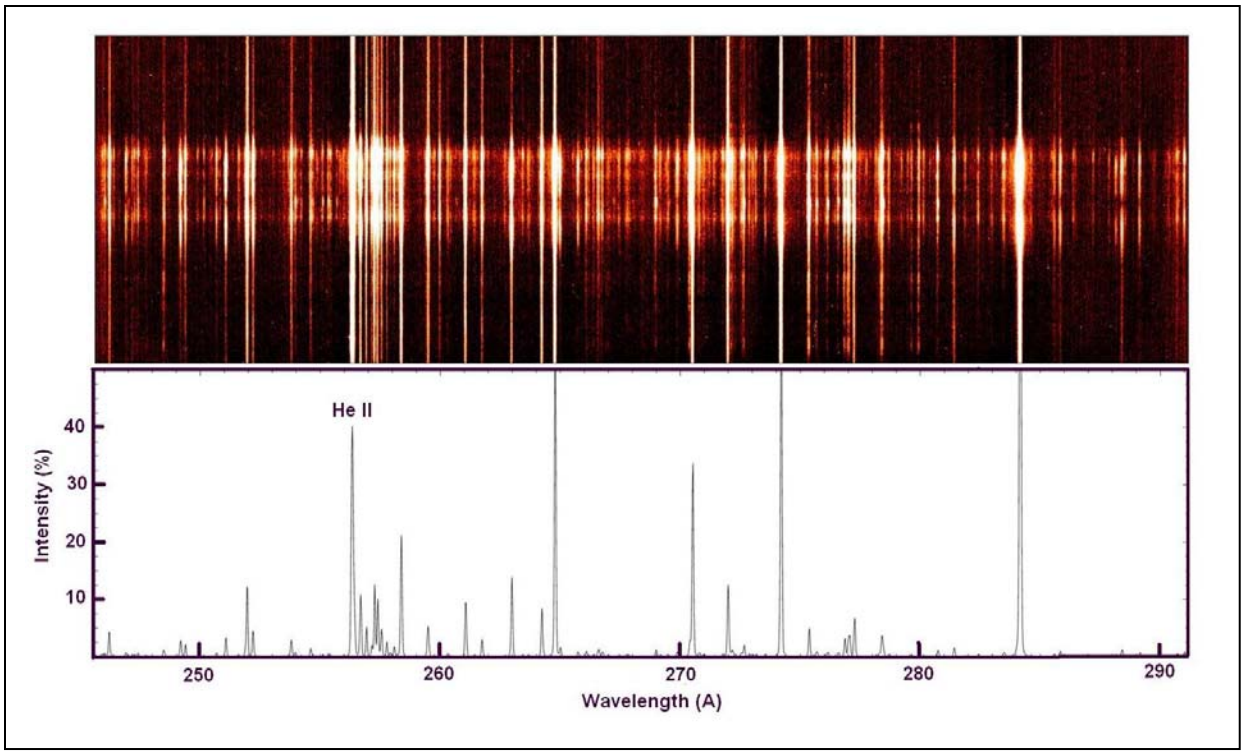
Day	Average	Sigma
-7	0.5	1.0
-6	0.5	1.1
-5	0.4	1
-4	0.7	1.6
-3	0.9	1.3
-2	0.8	1.3
-1	1.7	1.8
0	1.4	1.6

Problem 3 - Graph the average number of M-class flares counted for each day including the range determined by 1-sigma error bars. Answer: **See below.**



Problem 4 - Is the M-flare activity a good predictor of whether an X-class flare will occur or not? **Answer: The averages show a weak trend upwards, but the 1-sigma error bars are so large that the apparent trend is not statistically significant. All of the daily averages are within 1-sigma of each other.**

Find the mystery lines...if you can!



This is an image from the Hinode satellite's Extreme Ultraviolet Spectrometer (EIS). The graph below the spectrum provides the wavelength scale and line locations for the Hinode image. Each line in the bottom graph has an intensity that is indicated by its length along the vertical axis of the figure. The most intense lines have a value of 30 % of larger. The helium II (He II) line is about 40% on this intensity scale. The table to the right gives the wavelengths of some spectral lines that fall within the wavelength range of the figure. Let's use this information to identify a 'mystery line' in the above spectrum.

The scale of the horizontal axis is 0.31 Angstroms/millimeter For example, the difference in wavelength between He II and the next strong line to the right of He II is 27 mm or 27 x 0.31 = 8.4 Angstroms. Since the wavelength of He II is 256.32 Angstroms and the scale increases in wavelength to the right, the wavelength of the mystery line is 256.32 + 8.4 = 264.7 A. the table to the right suggests that this is the ion iron-14 (Fe XIV).

Problem 1 - From the wavelengths of the tabulated lines, calculate from the graph scale where these lines should be in the Hinode spectrum. Match up the tabulated lines with the lines shown in the above figure. Which lines can you match up?

Problem 2 - A careful count will show that there are about 127 lines in the above spectrum. What percentage of lines in the Hinode solar data are not identified in the table?

Table 1: List of ionic lines

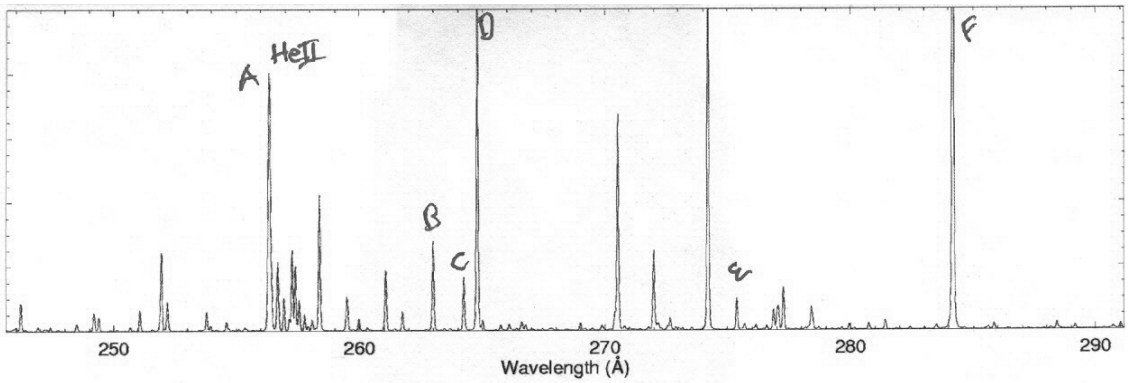
Ion	Wavelength
He II	256.32 A
Fe XVI	262.98
S X	264.23
Fe XIV	264.79
Si VII	275.35
Fe XV	284.16

Note: Wavelengths are given in the traditional Angstrom (A) units used by spectroscopists in which 10 Angstroms = 1 nanometer

Ionic Notation: The naming convention for atoms that have lost some of their electrons (to be come ions) is as follows for the case of iron (Fe) which has 26 electrons:

Fe I - neutral atom
Fe II - 1 electron lost
Fe III - 2 electrons lost
Fe IV - 3 electrons lost
…
Fe XIV - 13 electrons lost
Fe XV - 14 electrons lost
…
Fe XXVII - All 26 electrons lost

Answer Key:



Problem 1 - From the wavelengths of the tabulated lines, calculate from the graph scale where these lines should be in the Hinode spectrum. Match up the tabulated lines with the lines shown in the above figure. Which lines can you match up?

Answer: For example, Select the He Ii line at 256.32 A and subtract its wavelength from 250.0 .Convert this to millimeters using the scale 0.31 A/mm to get its location at 20.4 mm to the right of '250 Angstroms'. Do the same for the other tabulated lines, and locate them based on their distance in mm from the '250' mark.

A	He II	(256.32 - 250) / 0.31 = 20.4 mm to the right of '250'
B	Fe XII	(262.98 - 250)/0.31 = 41.9 mm to the right of '250'
C	S X	(264.23-250)/0.31 = 45.9 mm to the right of '250'
D	Fe XIV	(264.79 - 250) / 0.31 = 47.7 mm to the right of '250'
E	Si VII	(275.35 - 250) / 0.31 = 81.8 mm to the right of '250'
F	Fe XV	(284.16 - 250) / 0.31 = 110.2 mm to the right of '250'

Problem 2 - What percentage of lines in the Hinode solar data are not identified?

Answer: There are about 127 lines in the top Hinode spectrum, but only 6 lines are known from the table, so (121/127) x 100 = 95% of the lines in the Hinode spectrum are unknown.

Artist rendering courtesy NASA/G. Bacon (STScI)

Our sun is an active star that produces a variety of storms, such as solar flares and coronal mass ejections. Typically, these explosions of matter and radiation are harmless to Earth and its living systems, thanks to our great distance from the sun, a thick atmosphere, and a strong magnetic field. The most intense solar flares rarely exceed about 10^{21} Watts and last for an hour, which is small compared to the sun's luminosity of 3.8×10^{26} Watts.

A long-term survey with the Hubble Space Telescope of 215,000 red dwarf stars for 7 days each revealed 100 'solar' flares during this time. Red dwarf stars are about 1/20000 times as luminous as our sun. Average flare durations were about 15 minutes, and occasionally exceeded 2.0×10^{21} Watts.

Problem 1 - By what percentage does a solar flare on our sun increase the brightness of our sun?

Problem 2 - By what percentage does a stellar flare on an average red dwarf increase the brightness of the red dwarf star?

Problem 3 - Suppose that searches for planets orbiting red dwarf stars have studied 1000 stars for a total of 480 hours each. How many flares should we expect to see in this survey?

Problem 4 - Suppose that during the course of the survey in Problem 3, 5 exoplanets were discovered orbiting 5 of the surveyed red dwarf stars. To two significant figures, about how many years would inhabitants on each planet have to wait between solar flares?

Problem 1 - By what percentage does a solar flare on our sun increase the brightness of our sun?

Answer: P = 100% x (1.0 x 10^{21} Watts)/(3.8 x 10^{26} Watts)

P = 0.00026 %

Problem 2 - By what percentage does a stellar flare on an average red dwarf increase the brightness of the red dwarf star?

Answer: The average luminosity of a red dwarf star is stated as 1/20000 times our sun's luminosity, which is 3.8 x 10^{26} Watts, so the red dwarf star luminosity is about 1.9 x 10^{22} Watts.

P = 100% x (2.0 x 10^{21} Watts)/(1.9 x 10^{22} Watts)

P = 10 %

Problem 3 - Suppose that searches for planets orbiting red dwarf stars have studied 1000 stars for a total of 480 hours each. How many flares should we expect to see in this survey?

Answer: We have two samples: N1 = 215,000 stars for 7 days each producing 100 flares. N2 = 1000 stars for 480 hours each, producing x flares.

From the first survey, we calculate a rate of flaring per star per day:

Rate = 100 flares x (1/215,000 stars) x (1 / 7days)

= 0.000066 flares/star/day

Now we multiply this rate by the size of our current sample to get the number of flares to be seen in the 480-hour (20-day) period.

N = 0.000066 flares/star/day) x (1000 stars) x (20 days)

= 1.32 or **1 flare event**.

Problem 4 - Suppose that during the course of the survey in Problem 3, 5 exoplanets were discovered orbiting 5 of the surveyed red dwarf stars. To two significant figures, about how long would inhabitants on each planet have to wait between solar flares?

Answer: The Flare rate was 0.000066 flares/star/day. For 1 star, we have T = 0.000066 flares/star/day x (1 star) = 0.000066 flares/day so the time between flares (days/flare) is just T = 1/0.000066 = 15,151 days. Since there are 365 days/year, this is about **42 years between flares on the average**.

This spectacular night-time image taken by photographed Dominic Agostini shows the launch of the STEREO mission from Pad 17B at the Kennedy Space Center. The duration of the time-lapse image was 2.5 minutes. The distance to the horizon was 10 km, and the width of the image was 40 degrees. Assume that the trajectory is at the distance of the horizon, and in the plane of the photograph (the camera was exactly perpendicular to the plane of the launch trajectory).

Problem 1 – By using trigonometry formula, what is the horizontal scale of this image in meters/mm at the distance of the horizon?

Problem 2 - What was the altitude, in kilometers, of the rocket when it crossed the point directly in front of the camera's field of view at a position 2.4 km from the launchpad?

Problem 3 – From three measured points along the trajectory, what is the formula for the simplest parabola that can be fitted to this portion of the rocket's trajectory with the form $H(x) = ax^2 + bx + c$? (use kilometers for all measurements)

Problem 4 – Because of projection effects, although the rocket is continuing to gain altitude beyond the vertex of the parabola shown in the photograph, it appears as though the rocket reaches a maximum altitude and then starts to lose altitude as it travels further from the launch pad. About what is the apparent maximum altitude, in kilometers, that the rocket attains from the vantage point of the photographer?

More of Dominic's excellent night launch images can be found at www.dominicphoto.com

Problem 1 – By using trigonometry formula, what is the horizontal scale of this image in meters/mm at the distance of the horizon?
Answer: Solve the right-triangle to determine the hypotenuse h as h = 10 km/cos(20) = 11 km, then the width of the field of view at the horizon is just 2(11sin(20)) = 2(3.8 km) = 7.6 km. This is the width of the photograph along the distant horizon. With a millimeter ruler, the width of the mage is 125 mm, so the scale of this image at the horizon is about 7600m/125mm = **61 meters/mm**.

Problem 2 - What was the altitude, in kilometers, of the rocket when it crossed the point directly in front of the camera's field of view at a position 2.4 km from the launchpad? Answer: Draw a vertical line through the center of the image. Measure the length of this line between the distant horizon and the point on the trajectory. Typical values should be about 40 mm. Then from the scale of the image of 61 meters/mm, we get an altitude of H = 40 mm x 61 m/mm = **2440 meters or 2.4 kilometers**.

Problem 3 – From three measured points along the trajectory, what is the formula for the simplest parabola that can be fitted to this trajectory with the form $H(x) = ax^2 + bx + c$? (use kilometers for all measurements with the launch pad as the origin of the coordinates)

Answer: Because the launch pad is the origin of the coordinates, one of the two x-intercepts must be at (0,0) so the value for c = 0. We already know from Problem 2 that a second point is (2.4,2.4) so that $2.4 = a(2.4)^2 + b(2.4)$ and so the first equation for the solution is just
1 = 2.4a + b. We only need one additional point to solve for a and b. If we select a point at (1.2,1.5) we have a second equation $1.5 = a(1.2)^2 + 1.2b$ so that 1.5 = 1.4a + 1.2b. Our two equations to solve are now just

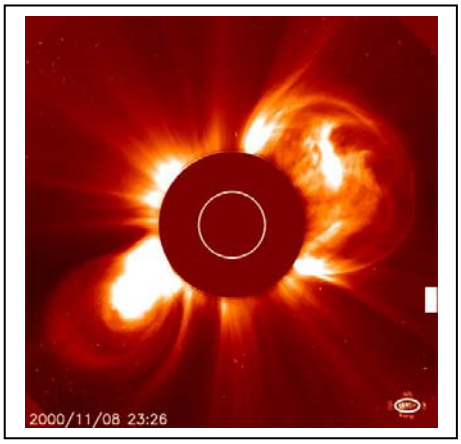
$$2.4\,a + 1.0\,b = 1.0$$
$$1.4\,a + 1.2\,b = 1.5$$

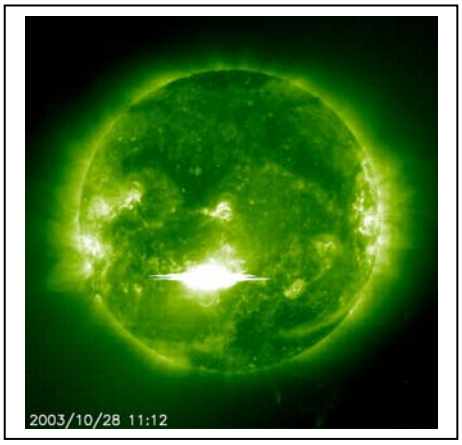
which we can solve by elimination of b to get a = -0.2 and then b = 1.48
so **$H(x) = -0.2\,x^2 + 1.48\,x$**

Problem 4 – Because of projection effects, although the rocket is continuing to gain altitude beyond the vertex of the parabola shown in the photograph, it appears as though the rocket reaches a maximum altitude and then starts to lose altitude as it travels further from the launch pad. About what is the apparent maximum altitude, in kilometers, that the rocket attains from the vantage point of the photographer?

Answer: Students can use a ruler to determine this as about **2.7 kilometers**. Alternatively they can use the formula they derived in Problem 3 to get the vertex coordinates at
x = -b/2a
= +3.7 km and so h = **2.7 km**.

Do fast CMEs produce intense SPEs?





The sun produces two basic kinds of storms; coronal mass ejections (SOHO satellite: top left) and solar flares (SOHO satellite: bottom left). These are spectacular events in which billions of tons of matter are launched into space (CMEs) and vast amounts of electromagnetic energy are emitted (Flares). A third type of 'space weather storm' can also occur.

Solar Proton Events (SPEs) are invisible, but intense, showers of high-energy particles near Earth that can invade satellite electronics and cause serious problems, even malfunctions and failures. Some of the most powerful solar flares can emit these particles, which streak to Earth within an hour of the flare event. Other SPE events, however, do not seem to arrive at Earth until several days latter.

Here is a complete list of Solar Proton Events between 1976-2005: http://umbra.nascom.nasa.gov/SEP/

Here is a complete list of coronal mass ejections 1996 - 2006: http://cdaw.gsfc.nasa.gov/CME_list/

Between January 1, 1996 and June 30, 2006 there were 11,031 CMEs reported by the SOHO satellite. Of these, 1186 were halo events. Only half of the halo events are actually directed towards Earth. The other half are produced on the far side of the sun and directed away from Earth. During this same period of time, 90 SPE events were recorded by GOES satellite sensors orbiting Earth. On the next page, is a list of all the SPE events and Halo CMEs that corresponded to the SPE events. There were 65 SPEs that coincided with Halo CMEs. Also included is the calculated speed of the CME event.

From the information above, and the accompanying table, draw a Venn Diagram to represent the data, then answer the questions below.

Problem 1: A) What percentage of CMEs detected by the SOHO satellite were identified as Halo Events? B) What are the odds of seeing a halo Event? C) How many of these Halo events are directed towards Earth?

Problem 2: A) What fraction of SPEs were identified as coinciding with Halo Events? B) What are the odds that an SPE occurred with a Halo CME? C) What fraction of all halo events directed towards earth coincided with SPEs?

Problem 3: A) What percentage of SPEs coinciding with Halo CMEs are more intense than 900 pFUs? B) What are the odds that, if you detect a 'Halo- SPE', it will be more intense than 900 pFUs?

Problem 4: A) What percentage of Halo-SPEs have speeds greater than 1000 km/sec? B) What are the odds that a Halo-SPE in this sample has a speed of > 1000 km/sec?

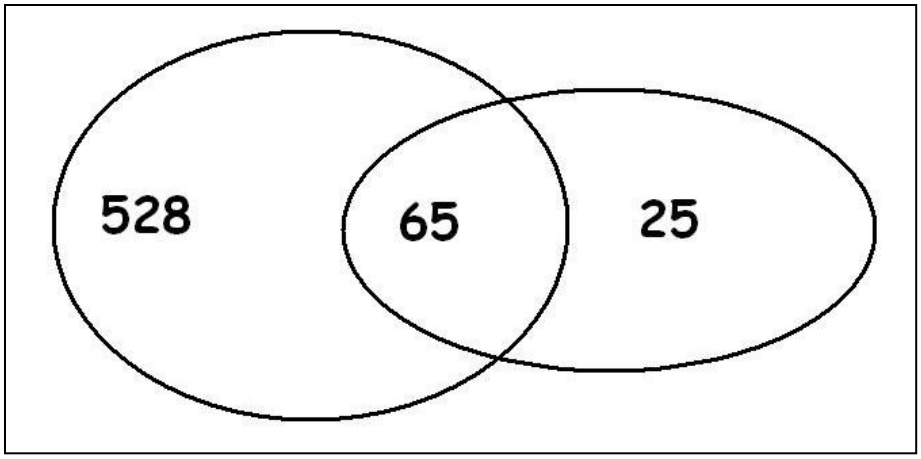
Problem 5: From what you have calculated as your answers above, what might you conclude about SPEs and CMEs? How would you use this information as a satellite owner and operator?

Data Tables showing dates and properties of Halo CMEs and Solar Proton Events.

Date	CME Speed (km/s)	SPE (pfu)
November 4, 1997	785	72
November 6, 1997	1556	490
April 20, 1998	1863	1700
May 2, 1998	938	150
May 6, 1998	1099	210
May 3, 1999	1584	14
June 1, 1999	1772	48
June 4, 1999	2230	64
February 18, 2000	890	13
April 4, 2000	1188	55
June 6, 2000	1119	84
June 10, 2000	1108	46
July 14, 2000	1674	24000
July 22, 2000	1230	17
September 12, 2000	1550	320
October 16, 2000	1336	15
October 25, 2000	770	15
November 8, 2000	1738	14800
November 24, 2000	1289	940
January 28, 2001	916	49
March 29, 2001	942	35
April 2, 2001	2505	1100
April 10, 2001	2411	355
April 15, 2001	1199	951
April 18, 2001	2465	321
April 26, 2001	1006	57
August 9, 2001	479	17
September 15, 2001	478	11
September 24, 2001	2402	12900
October 1, 2001	1405	2360
October 19, 2001	901	11
October 22, 2001	618	24
November 4, 2001	1810	31700
November 17, 2001	1379	34
November 22, 2001	1437	18900
December 26, 2001	1446	779

Date	CME Speed (km/s)	SPE (pfu)
January 8, 2002	1794	91
January 14, 2002	1492	15
February 20, 2002	952	13
March 15, 2002	957	13
March 18, 2002	989	19
March 22, 2002	1750	16
April 17, 2002	1240	24
April 21, 2002	2393	2520
May 22, 2002	1557	820
July 15, 2002	1151	234
August 14, 2002	1309	24
August 22, 2002	998	36
August 24, 2002	1913	317
September 5, 2002	1748	208
November 9, 2002	1838	404
May 28, 2003	1366	121
May 31, 2003	1835	27
June 17, 2003	1813	24
October 26, 2003	1537	466
November 4, 2003	2657	353
November 21, 2003	494	13
April 11, 2004	1645	35
July 25, 2004	1333	2086
September 12, 2004	1328	273
November 7, 2004	1759	495
January 15, 2005	2861	5040
July 13, 2005	1423	134
July 27, 2005	1787	41
August 22, 2005	2378	330

Note: Solar Proton Event strengths are measured in the number of particles that pass through a square centimeter every second, and is given in units called Particle Flux Units or PFUs.



Problem 1: Answers: A) 1186/11031 = **11%** B) 1 / 0.11 = **1 chance in 9** C) From the text, only half are directed to Earth so 1186/2 = **593 Halos**.

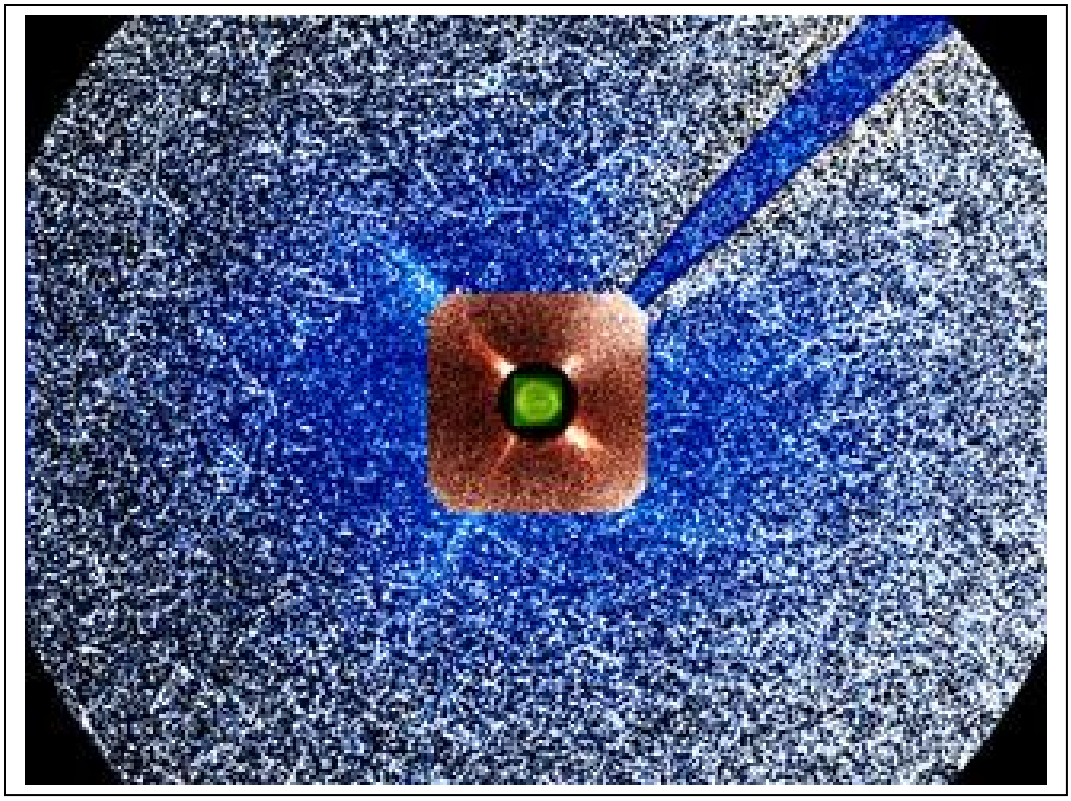
Problem 2: Answers: A) 65 table entries / 90 SPEs = **72%** B) 1 / 0.72 = 1 chance in 1.38 or about **2 chances in 3** C) 65 in Table / (528+65) Halos = **11%**

Problem 3: Answers: A) From the table, there are 12 SPEs out of 65 in this list or 12/65 = **18 %** B) 1 / 0.18 = **1 chance in 5**.

Problem 4: Answers: A) There are 50 out of 65 or 50/65 = **77%** B) 1 / 0.77 = 1 chance in 1.3 or **2 chances in 3**.

Problem 5: From what you have calculated as your answers above, what might you conclude about Solar Proton Events and CMEs? How would you use this information as a satellite owner and operator?

A reasonable student response is that Halo CMEs occur only 11% of the time, and of the ones directed towards Earth only 1 out of 9 coincide with SPEs. However, in terms of SPEs, virtually all of the SPEs coincide with Halo events (2 out of 3) and SPEs are especially common when the CME speed is above 1000 km/sec. As a satellite owner, I would be particularly concerned if scientists told me there was a halo CME headed towards Earth AND that it had a speed of over 1000 km/sec. Because the odds are now 2 chances out of 3 that an SPE might occur that could seriously affect my satellite. I would try to put my satellite in a safe condition to protect it from showers of high-energy particles that might damage it.

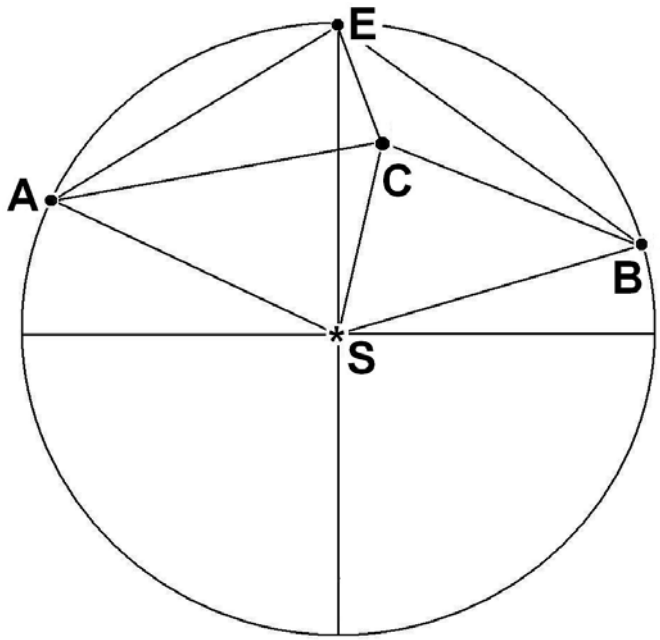


The January 20, 2005 solar proton event (SPE) was by some measures the biggest since 1989. It was particularly rich in high-speed protons packing more than 100 million electron volts (100 MeV) of energy. Such protons can burrow through 11 centimeters of water. A thin-skinned spacesuit would have offered little resistance, and the astronaut would have been radiation poisoned, and perhaps even killed.

The above image was taken by the SOHO satellite during this proton storm. The instrument, called LASCO, was taking an image of the sun in order for scientists to study the coronal mass ejection (CME) taking place. Each of the individual white spots in the image is a track left by a high-speed proton as it struck the imaging CCD (similar to the 'chip' in your digital camera). As you see, the proton tracks corrupted the data being taken.

The high-speed particles from these proton storms also penetrate satellites and can cause data to be lost, or even false commands to be given by on-board computers, causing many problems for satellite operators.

Seeing Solar Storms in STEREO - II



The two STEREO spacecraft are located along Earth's orbit and can view gas clouds ejected by the sun as they travel to Earth. From the geometry, astronomers can accurately determine their speeds, distances, shapes and other properties.

By studying the separate 'stereo' images, astronomers can determine the speed and direction of the cloud before it reaches Earth.

Use the diagram, (angles and distances not drawn to the same scale of the 'givens' below) to answer the following question.

The two STEREO satellites are located at points A and B, with Earth located at Point E and the sun located at Point S, which is the center of a circle with a radius ES of 1.0 Astronomical unit (150 million kilometers). Suppose that the two satellites spot a Coronal Mass Ejection (CME) cloud at Point C. Satellite A measures its angle from the sun mSAC as 45 degrees while Satellite B measures the corresponding angle to be mSBC=50 degrees. In the previous math problem the astronomers knew the ejection angle of the CME, mESC, but in fact they didn't need to know this in order to solve the problem below!

Problem 1 - The astronomers want to know the distance that the CME is from Earth, which is the length of the segment EC. The also want to know the approach angle, mSEC. Use either a scaled construction (easy: using compass, protractor and millimeter ruler) or geometric calculation (difficult: using trigonometric identities) to determine EC from the available data.

Givens from satellite orbits:

SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km
mASE = 54 degrees mBSE = 48 degrees
mEAS = 63 degrees mEBS = 66 degrees mAEB = 129 degrees

Find the measures of all of the angles and segment lengths in thee above diagram rounded to the nearest integer.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?

Givens from satellite orbits:
SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km
mASE = 54 degrees mBSE = 48 degrees
mEAS = 63 degrees mEBS = 66 degrees mAEB = 129 degrees
use units of megakilometers i.e. 150 million km = 150 Mkm.

Method 1: Students construct a scaled model of the diagram based on the angles and measures, then use a protractor to measure the missing angles, and from the scale of the figure (in millions of kilometers per millimeter) they can measure the required segments. **Segment EC is about 49 Mkm at an angle, mSEB of 28 degrees.**

Method 2 use the Law of Cosines and the Law of Sines to solve for the angles and segment lengths exactly.

mASB = mASE+mBSE = 102 degrees
mASC = theta
mACS = 360 - mCAS - mASC = 315 - theta
mBSC = mASB - theta = 102 - theta
mBCS = 360 - mCBS - mBSC = 208 + theta

Use the Law of Sines to get
sin(mCAS)/L = sin(mACS)/150 Mkm and sin(mBCS)/L = sin(mBCS)/150 Mkm.

Eliminate L : 150sin(45)/sin(315-Theta) = 150 sin(50)/sin(208+theta)

Re-write using angle-addition and angle-subtraction:
sin50 [sin(315)cos(theta)-cos(315)sin(theta)] = sin(45) [sin(208)cos(theta) + cos(208)sin(theta)]

Compute numerical factors by taking indicated sines and cosines:
-0.541cos(theta) - 0.541sin(theta) = -0.332cos(theta) - 0.624sin(theta)

Simplify: cos(theta) = 0.397sin(theta)

Use definition of sine: $\cos(\text{theta})^2 = 0.158\,(1 - \cos(\text{theta})^2)$

Solve for cosine: $\cos(\text{theta}) = (0.158/1.158)^{1/2}$ so theta = 68 degrees. And so **mASC=68**

Now compute segment CS = 150 sin(45)/sin(315-68) = **115 Mkm.**
BC = 115 sin(102-68)/sin(50) = **84 Mkm**.
Then $EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(\text{mEBS-mCBS})$
$EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(66-50)$ So **EC = 47 Mkm**.

mCEB from Law of Cosines: $84^2 = 122^2 + 47^2 - 2(122)(47)\cos(\text{mCEB})$ so **mCEB = 29 degrees**

And since mAES = 180 - mASE - mEAS = 180 - 54 - 63 = 63 degrees
so mSEC = mAEB - mAES - mCEB = 129 - 63 - 29 = **37 degrees**

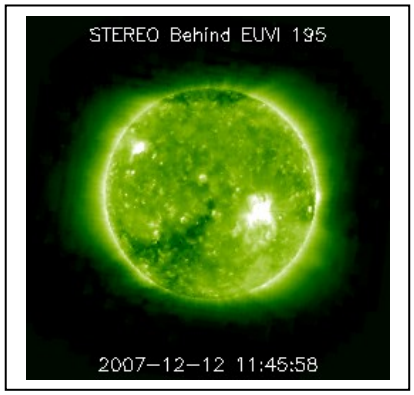
So, the two satellites are able to determine that the CME is 49 million kilometers from Earth and approaching at an angle of 37 degrees from the sun.

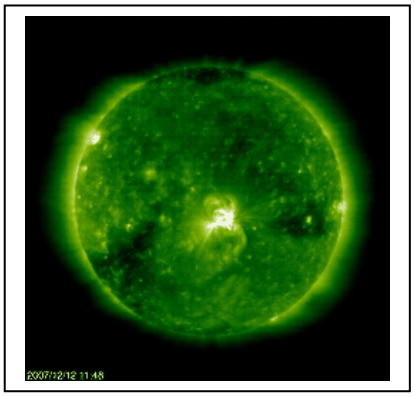
Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?
Answer: 115 million kilometers / 2 million km/hr = **58 hours or 2.4 days**.

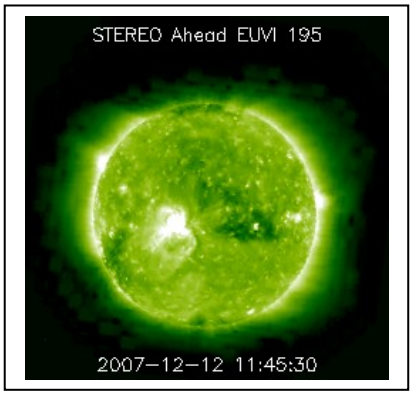
Seeing Solar Storms in STEREO

Two NASA, STEREO satellites take images of the sun and its surroundings from two separate vantage points along Earth's orbit. From these two locations, one located ahead of the Earth, and the other located behind the Earth along its orbit, they can create stereo images of the 3-dimensional locations of solar storms on or near the solar surface.

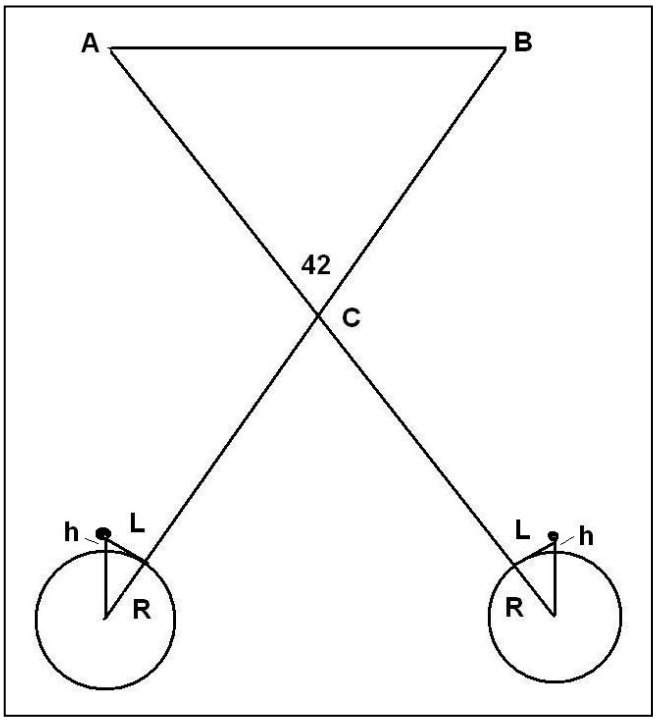
The three images below, taken on December 12, 2007, combine the data from the two STEREO satellites (left and right) taken from these two locations, with the single image taken by the SOHO satellite located half-way between the two STEREO satellites (middle). Notice that there is a large storm event, called Active Region 978, located on the sun. The changing location of AR978 with respect to the SOHO image shows the perspective change seen from the STEREO satellites. You can experience the same *Parallax Effect* by holding your thumb at arms length, and looking at it, first with the left eye, then with the right eye. The location of your thumb will shift in relation to background objects in the room.







The diagram to the right shows the relevant parallax geometry for the two satellites A and B, separated by an angle of 42 degrees as seen from the sun. The diagram lengths are not drawn to scale. The radius of the sun is 696,000 km.



Problem 1: With a millimeter ruler, determine the scale of each image in km/mm. How many kilometers did AR978 shift from the center position (SOHO location for AR) between the two STEREO images? This is the average measure of 'L' in the diagram.

Problem 2: Using the Pythagorean Theorem, determine the equation for the height, h, in terms of R and L. Assume the relevant triangle is a right triangle.

Problem 3: How high (h) above the sun's surface, called the photosphere, was the AR978 viewed by STEREO and SOHO on December 12, 2007?

Answer Key

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Answer:

STEREO-Left image, sun diameter = 28 mm, actual = 1,392,000 km, so the scale is
1392000 km / 28mm = **49,700 km/mm**
SOHO-center sun diameter = 36 mm, so the scale is
1392000 km/36mm = **38,700 km/mm**
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Taking the location of the SOHO image for AR978 as the reference, the left-hand image shows that AR978 is about 5 mm to the right of the SOHO location which equals 5 mm x 49,700 km/mm = 248,000 km. From the right-hand STEREO image, we see that AR978 is about 5 mm to the left of the SOHO position or 5 mm x 48,000 km/mm = 240,000 km.
The average is 244,000 kilometers.

Problem 2: Using the Pythagorean Theorem, determine the equation for the height, h, in terms of R and L.

Answer:

$$(R + h)^2 = R^2 + L^2$$

$$h = (R^2 + L^2)^{1/2} - R$$

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Answer:

$$h = ((244{,}000)^2 + (696{,}000)^2)^{1/2} - 696{,}000$$

$$h = 737{,}500 - 696{,}000$$

$$h = 41{,}500 \text{ kilometers}$$

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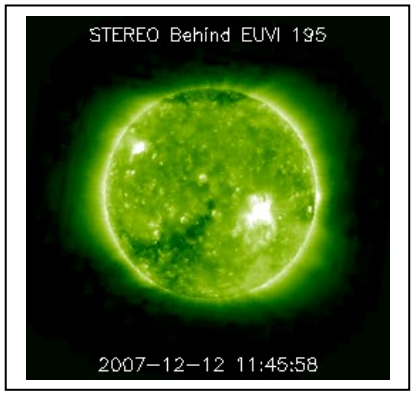

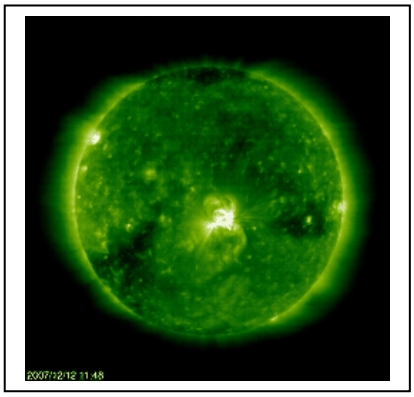

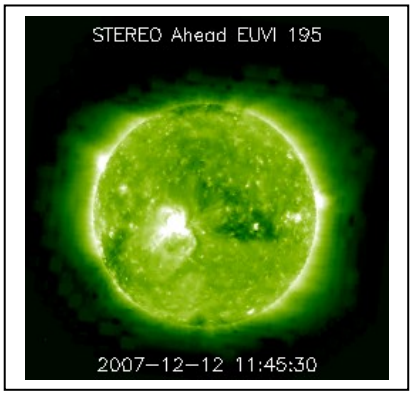

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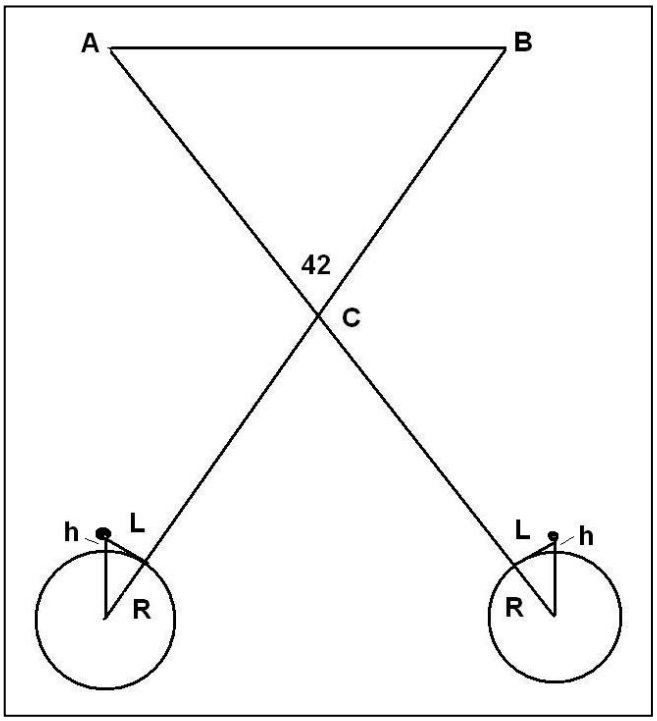

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$$h = 737{,}500 - 696{,}000$$

$$h = 41{,}500 \text{ kilometers}$$

Hinode Studies Loopy Sunspots!

The solar surface is not only a hot, convecting ocean of gas, but is laced with magnetism. The sun's magnetic field can be concentrated into sunspots, and when solar gases interact with these magnetic fields, their light lets scientists study the complex 'loopy' patterns that the magnetic fields make as they expand into space. The above image was taken by NASA's TRACE satellite and shows one of these magnetic loops rising above the surface near two sunspots. The horseshoe shape of the magnetic field is anchored at its two 'feet' in the dark sunspot regions. The heated gases become trapped by the magnetic forces in sunspot loops, which act like magnetic bottles. The gases are free to flow along the lines of magnetic force, but not across them. The above image only tells scientists where the gases are, and the shape of the magnetic field, which isn't enough information for scientists to fully understand the physical conditions within these magnetic loops. Satellites such as Hinode carry instruments like the EUV Imaging Spectrometer, which lets scientists measure the temperatures of the gases and their densities as well.

Problem 1: The Hinode satellite studied a coronal loop on January 20, 2007 associated with Active Region AR 10938, which was shaped like a semi-circle with a radius of 20,000 kilometers, forming a cylindrical tube with a base radius of 1000 kilometers. What was the total volume of this magnetic loop in cubic centimeters assuming that it is shaped like a cylinder?

Problem 2: The Hinode EUV Imaging Spectrometer was able to determine that the density of the gas within this magnetic loop was about 2 billion hydrogen atoms per cubic centimeter. If a hydrogen atom has a mass of 1.6×10^{-24} grams, what was the total mass of the gas trapped within this cylindrical loop in metric tons?

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Answer: The length (h) of the cylinder is 1/2 the circumference of the circle with a radius of 20,000 km or $h = 1/2\ (2\pi R) = 3.14 \times 20{,}000$ km = 62,800 km

The volume of a cylinder is $V = \pi R^2 h$ so that the volume of the loop is

$V = \pi\ (1000\ \text{km})^2 \times 62{,}800$ km

$= 2.0 \times 10^{11}$ cubic kilometers.

1 cubic kilometer = 10^{15} cubic centimeters so

= **2.0×10^{26} cubic centimeters**

Problem 2: The Hinode EUV Imaging Spectrometer was able to determine that the density of the gas within this magnetic loop was about 2 billion hydrogen atoms per cubic centimeter. If a hydrogen atom has a mass of 1.6×10^{-24} grams, what was the total mass of the gas trapped within this cylindrical loop in metric tons?

Answer: The total mass is the product of the density times the volume, so

Density = 2×10^9 particles/cc $\times$ (1.6×10^{-24} grams/particle) = 3.2×10^{-15} grams/cm^3

The approximate volume of the magnetic loop in cubic centimeters is

$V = (2.0 \times 10^{11}\ \text{km}^3) \times (1.0 \times 10^{15}\ \text{cm}^3/\text{km}^3$

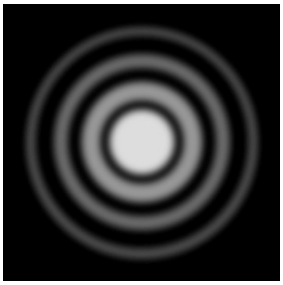
$= 2.0 \times 10^{26}\ \text{cm}^3$

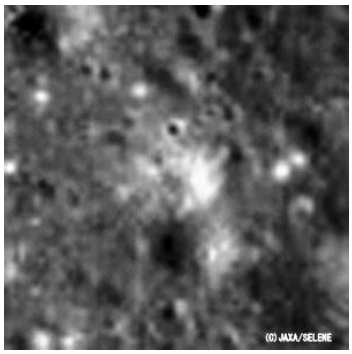
Mass = Density x Volume = $(3.2 \times 10^{-15}\ \text{grams/cm}^3) \times (2.0 \times 10^{26}\ \text{cm}^3) = 6.4 \times 10^{26-15}$

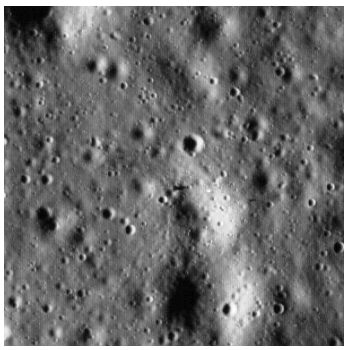
$= 6.4 \times 10^{11}$ grams or 6.4×10^8 kilograms or **640,000 metric tons.**

The Most Important Equation in Astronomy!

$$R = 1.22 \frac{L}{D}$$







There are many equations that astronomers use to describe the physical world, but none is more important and fundamental to the research that we conduct than the one to the left! You cannot design a telescope, or a satellite sensor, without paying attention to the relationship that it describes.

In optics, the best focused spot of light that a perfect lens with a circular aperture can make, limited by the diffraction of light. The diffraction pattern has a bright region in the center called the Airy Disk. The diameter of the Airy Disk is related to the wavelength of the illuminating light, L, and the size of the circular aperture (mirror, lens), given by D. When L and D are expressed in the same units (e.g. centimeters, meters), R will be in units of angular measure called radians (1 radian = 57.3 degrees).

You cannot see details with your eye, with a camera, or with a telescope, that are smaller than the Airy Disk size for your particular optical system. The formula also says that larger telescopes (making D bigger) allow you to see much finer details. For example, compare the top image of the Apollo-15 landing area taken by the Japanese Kaguya Satellite (10 meters/pixel at 100 km orbit elevation: aperture = about 15cm) with the lower image taken by the LRO satellite (0.5 meters/pixel at a 50km orbit elevation: aperture =). The Apollo-15 Lunar Module (LM) can be seen by its 'horizontal shadow' near the center of the image.

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

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Answer: First convert all numbers to centimeters, then use the formula to calculate the resolution in radian units: L = 21 centimeters, D = 100 meters = 10,000 centimeters, then R = 1.22 x 21 cm /10000 cm so R = 0.0026 radians. There are 57.3 degrees to 1 radian, so A) 0.0026 radians x (57.3 degrees/ 1 radian) = **0.14 degrees.** And B) There are 60 arc minutes to 1 degrees, so 0.14 degrees x (60 minutes/1 degrees) = **8.4 arcminutes.**

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Answer: R = 1.22 x (0.00006 cm/10400 cm) = 0.000000069 radians. A) Since 1 microradian = 0.000001 radians, the resolution of this telescope is **0.069 microradians.** B) Since 1 radian = 57.3 degrees, and 1 degree = 3600 arcseconds, the resolution is 0.000000069 radians x (57.3 degrees/radian) x (3600 arcseconds/1 degree) = 0.014 arcseconds. One thousand milliarcsecond = 1 arcseconds, so the resolution is 0.014 arcsecond x (1000 milliarcsecond / arcsecond) = **14 milliarcseconds.**

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

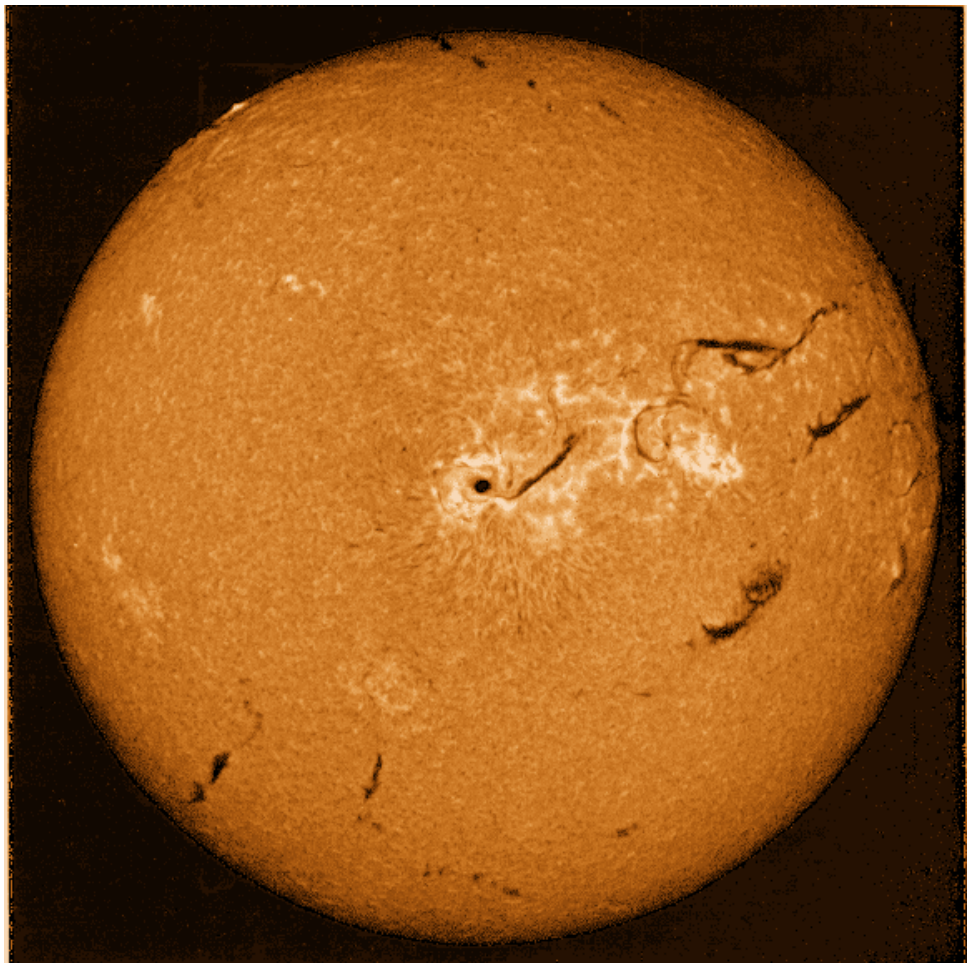
Answer: From R = 1.22 L/D we have R = 1 arcsecond and L = 20 micrometers and need to calculate D, so with algebra we re-write the equation as D = 1.22 L/R.
Convert R to radians:

R = 1 arcsecond x (1 degree/3600 arcsecond) x (1 radian / 57.3 degrees) = 0.0000048 radians.

L = 20 micrometers x (1 meter/1,000,000 micrometers) = 0.00002 meters.

Then D = 1.22 (0.00003 meters)/(0.0000048 radians) = **5.1 meters.**

The Sun in Hydrogen Light



This is an image taken by the National Solar Observatory on Sacramento Peak using light from hydrogen atoms (H-alpha). It was obtained at a wavelength of 635 nm, or a frequency of 460 teraHertz (4.6×10^{14} Hz), and reveals hydrogen gas at temperatures between 10,000 and 30,000 K,. These gases are found in prominences and near sunspots. The image shows considerable detail in the photosphere and chromosphere of the sun that cannot be as well seen in broad-band optical light. Once reserved only for the most advanced observatories, these H-alpha images can routinely be produced by amateur astronomers using equipment costing less than $1,000.

Problem - A cloud of gas in front of a light source will absorb light at the specific wavelengths of its atoms, and then re-emit this light in different directions at exactly the same wavelengths. From one vantage point, the cloud will appear dark, while from other vantage point the cloud can appear to be emitting light. This is an example of Kirchoff's Law in spectroscopy. Can you explain how this principle accounts for the details seen in the H-alpha image of the sun above?

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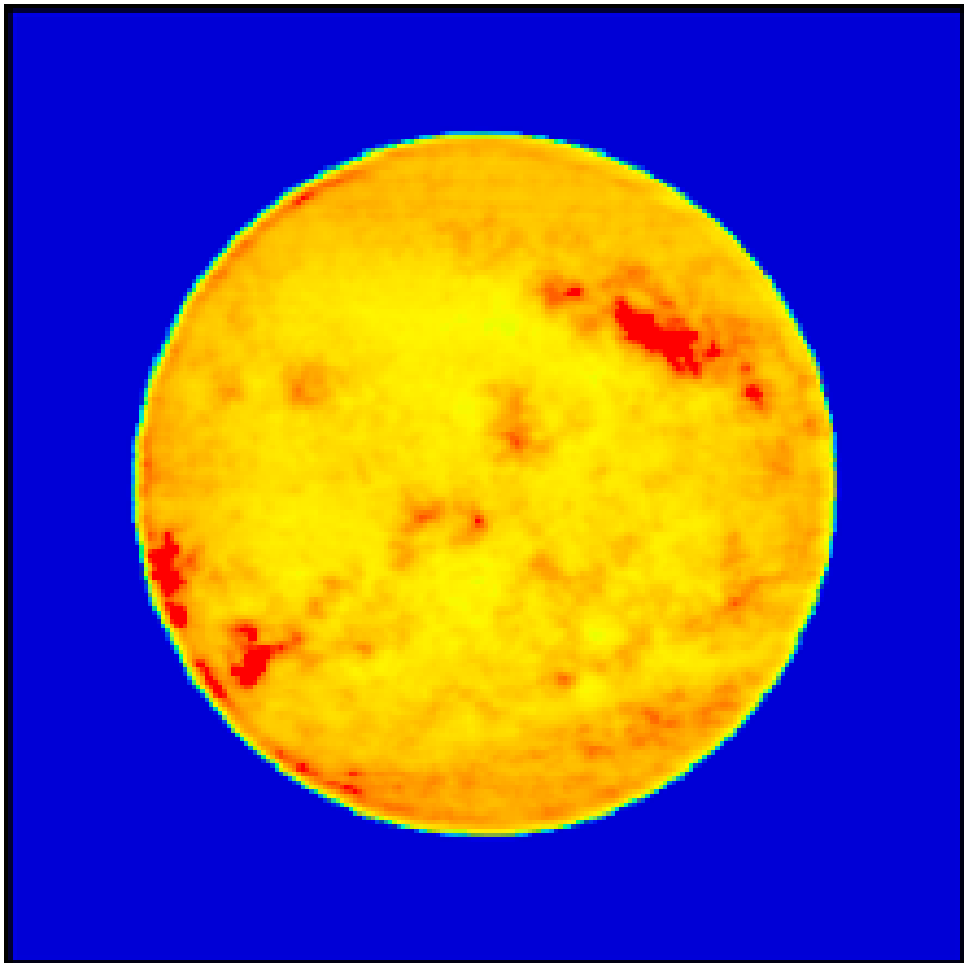
Answer: A dense hydrogen gas cloud (prominence) seen against the hot solar surface absorbs and re-emits light at the specific wavelengths of hydrogen such as the H-alpha spectral line. When we see a cloud on the face of the sun, most of the scattered light does not travel towards the observer, so the cloud appears to dim the light from the sun at this wavelength. When seen against the dark sky at the limb of the sun, the same hydrogen cloud appears to glow with a reddish color.

At this wavelength, the sun has a brightness of about

2.6×10^{-21} watts/meter^2

Radio Image of the Sun



This solar image at a wavelength of 1 millimeter (0.001 meters) or a frequency of 300 gigaHertz (3.0×10^{11} Hz) was obtained by the Atacama Large Millimeter Array (ALMA). The telescope array consists of 64 antennas each 12 meters in diameter. The antennas may all be grouped together in an area 150 meters in diameter to provide about 1.0 arcsecond angular resolution, or they may be distributed over an area 14 kilometers in extent to provide an angular resolution as high as 0.01 arcseconds (10 milliarcsec). Millimeter radio emission is extremely sensitive to dynamic processes in the chromosphere; a layer of the sun located just above the photosphere but below the corona. The image above shows features that coincide with large sunspot groups in the photosphere.

Problem - At these frequencies, electrons are emitting radio-wave energy by spiraling in magnetic fields. This causes them to lose energy very rapidly. The lifetime of an electron before it can no longer emit much energy is given by the formula $T = 422 / (B^2E)$ where T is in seconds, B is the magnetic field strength in Gauss, and E is the electron energy in electron-Volts (eV). If B = 100 Gauss and E = 0.001 eV, how long do these electrons continue to radiate radio waves?

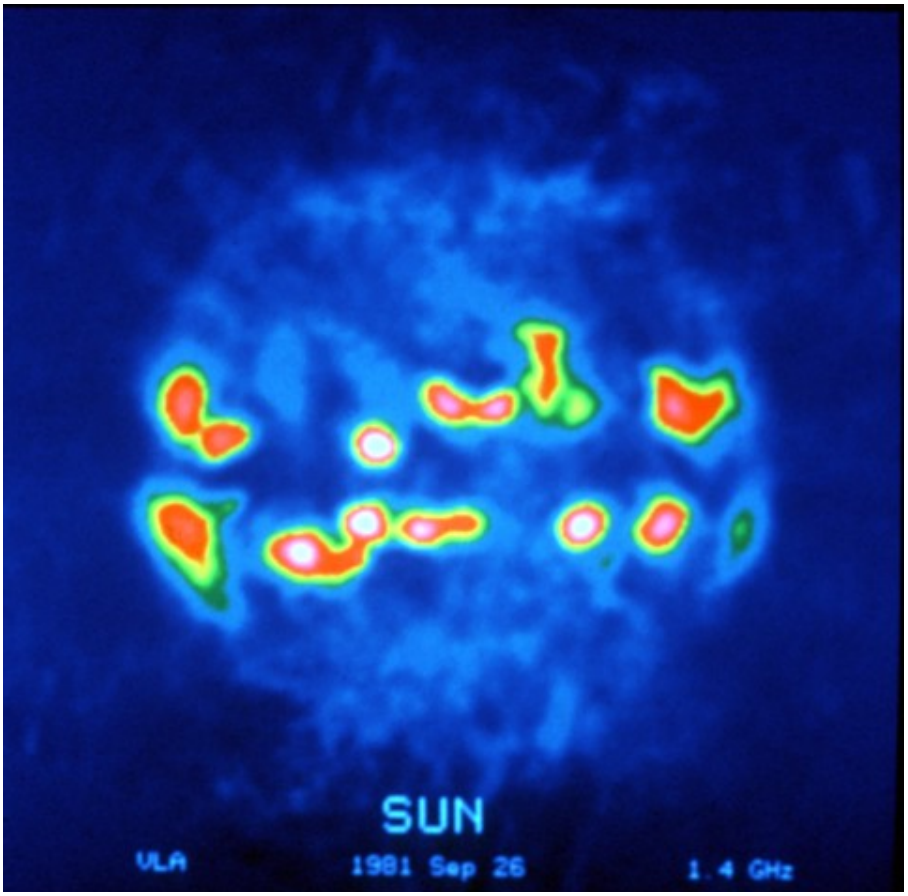
Answer Key

Problem - At these frequencies, electrons are emitting radio-wave energy by spiraling in magnetic fields. This causes them to lose energy very rapidly. The lifetime of an electron before it can no longer emit much energy is given by the formula T = 422 / (B2E) where T is in seconds, B is the magnetic field strength in Gauss, and E is the electron energy in eV. If B = 100 gauss and E = 0.001 eV, how long doe these electrons continue to radiate radio waves?

Answer: $T = \frac{422}{(100)^2 (0.001)}$ = **42 seconds.**

Note: At the typical speed of the electrons of about 500 km/sec, they can travel a distance of about 42 sec x 500 km/sec = 21,000 kilometers. At the scale of the radio image (1,300,000 km/90mm) of 14,500 km/mm, this distance is about 1 - 2 millimeters.

Radio Image of the Sun



False-color image of the sun obtained at a wavelength of 20 centimeters (0.2 meters) or a frequency of 1.5 gigaHertz (1.5×10^9 Hz) with the Very Large Array (VLA) radio telescope in New Mexico on September 26, 1981.

The image shows a dozen intense sources of radio emission forming two belts of activity parallel to the solar equator. The radio emission comes from electrons at temperatures of over 2 million degrees K. The fainter 'blue' regions are produced by plasma at lower temperatures near 50,000 K. The corona appears as a patchwork of cloudy regions and 'holes' depending on whether magnetic fields are confining the plasma (bright) or whether the plasma is free to escape into interplanetary space (dark).

Problem - If you approximated each active region as a sphere, about what is the total volume occupied by the radio-emitting hot plasma compared to the volume of Earth (1 trillion km^3).

Problem - If you approximated each active region as a sphere, about what is the total volume occupied by the radio-emitting hot plasma compared to the volume of Earth (1 trillion km^3).

Answer: The diameter of the solar disk is about 90 millimeters, so the scale is 1,300,000 km/90 mm = 14,500 km/mm. The active regions are about 5 mm in diameter or 5 mm x 14,500 km/mm = 73,000 km. Assuming a spherical volume, each active region is about $V = 4/3\ (3.14)\ (73{,}000/2)^3 = 2.0 \times 10^{14}\ km^3$ so for 12 regions we have a total volume of $2.4 \times 10^{15}\ km^3$. The number of equivalent Earth volumes is just

$$N = \frac{2.4 \times 10^{15}\ km^3}{1.0 \times 10^{12}\ km^3} = \textbf{2,400 Earths}$$

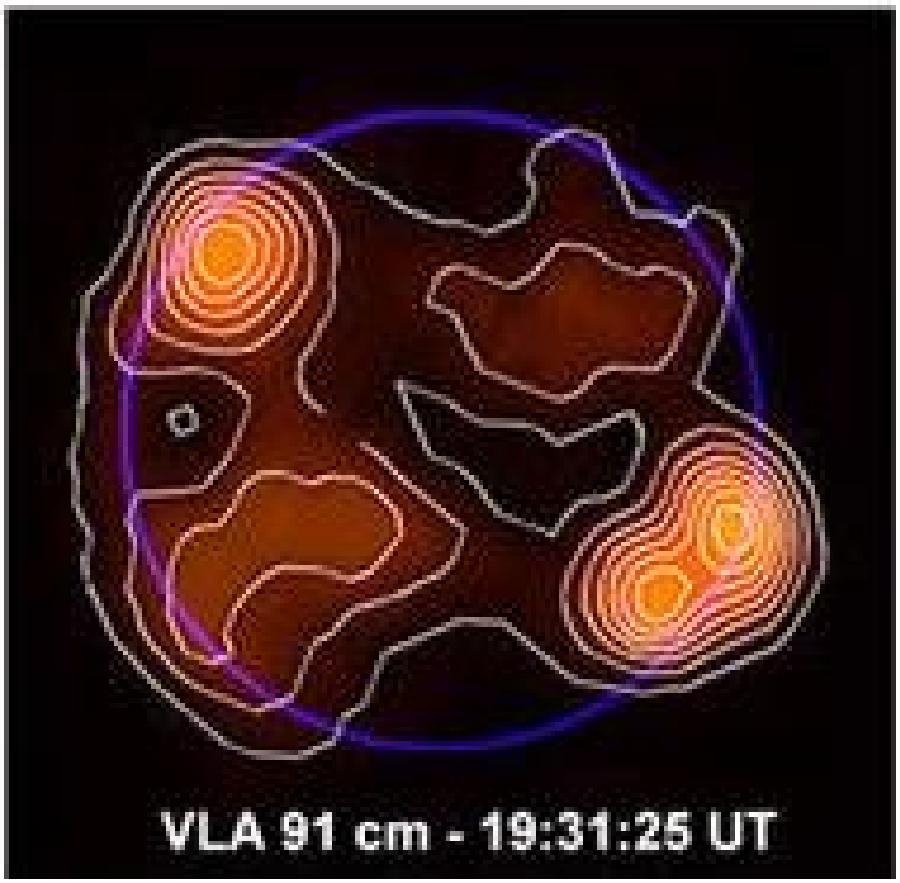
The brightness of the Sun at these wavelengths is about

$$F = 6.0 \times 10^{-21}\ watts/meter^2$$

But during disturbed conditions near sunspot maximum, the brightness is about

$$F = 8.0 \times 10^{-19}\ watts/meter^2$$

Radio Image of the Sun



This radio map of the sun at a wavelength of 91 centimeters (0.92 meters), or a frequency of 327 megaHertz (3.27×10^6 Hz), was obtained with the Very Large Array (VLA) during a solar flare episode on February 22, 2004. This is the longest wavelength at which this high-resolution array can operate, revealing features larger than about 56 arcseconds. High resolution VLA observations of the Sun at 20 and 91 cm have made it possible to locate and resolve sites of impulsive energy release at two different heights in the solar corona. Investigations of microwave burst emission have also yielded insights to the pre-flare heating of coronal loops, the successive interaction of adjacent loops . The VLA map shows intense Type I noise storm emission above the active region at the limb. This time-variable radio emission followed, by a few minutes, a solar flare and coronal mass ejection detected by SOHO and TRACE, This suggests that these evolving EUV sources may have played a role in accelerating electrons that were eventually seen in the noise storm at greater heights. (Courtesy; Tufts Solar Physics Group and NRAO/VLA)

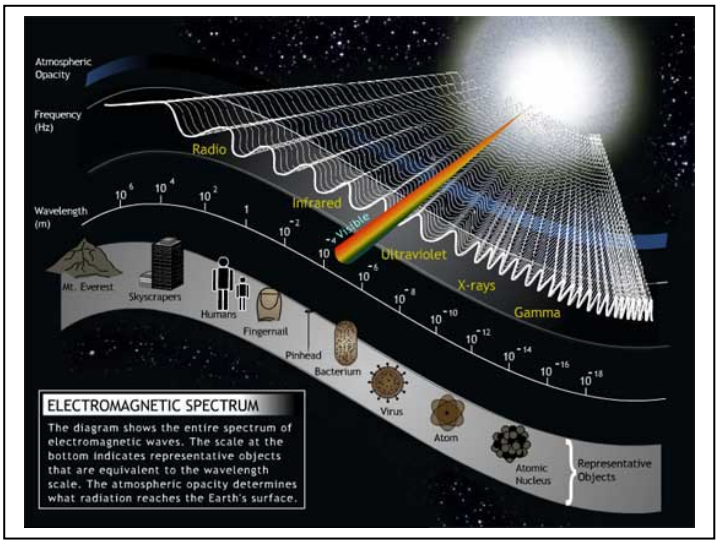
Problem - The volume of Earth is 1 trillion km^3. From the scale of this image and the size of the radio burst in the upper left corner of the solar image, about how many Earth's could fit inside the plasma volume of the radio burst? (Sun

Problem - The volume of Earth is 1 trillion km^3. From the scale of this image and the size of the radio burst in the upper left corner of the solar image, about how many Earth's could fit inside the plasma volume of the radio burst? (Sun diameter = 1,300,000 km)

Answer: The disk of the sun has a diameter of about 85 millimeters, so the scale of the image is 1,300,000 km / 85 mm = 15,000 km/mm .The approximate diameter of the radio burst region is 30 mm, so its physical size is about 30 mm x 15,000 km/mm = 450,000 km. The volume of a sphere with this diameter is just
$V = 4/3\ (3.141)\ (450000/2)^3 = 4.8 \times 10^{16}\ km^3$.

The number of equivalent Earth volumes is just

$$N = \frac{4.8 \times 10^{16}\ km^3}{1.0 \times 10^{12}\ km^3} = \textbf{48,000 Earths.}$$



The sun emits electromagnetic energy at all wavelengths from gamma-ray to radio. Astronomers study the sun at each wavelength band in order to examine different aspects of how solar phenomena work.

To see all of this information at one time, astronomers use a Log-Log plot to graph the emission at each wavelength (or frequency).

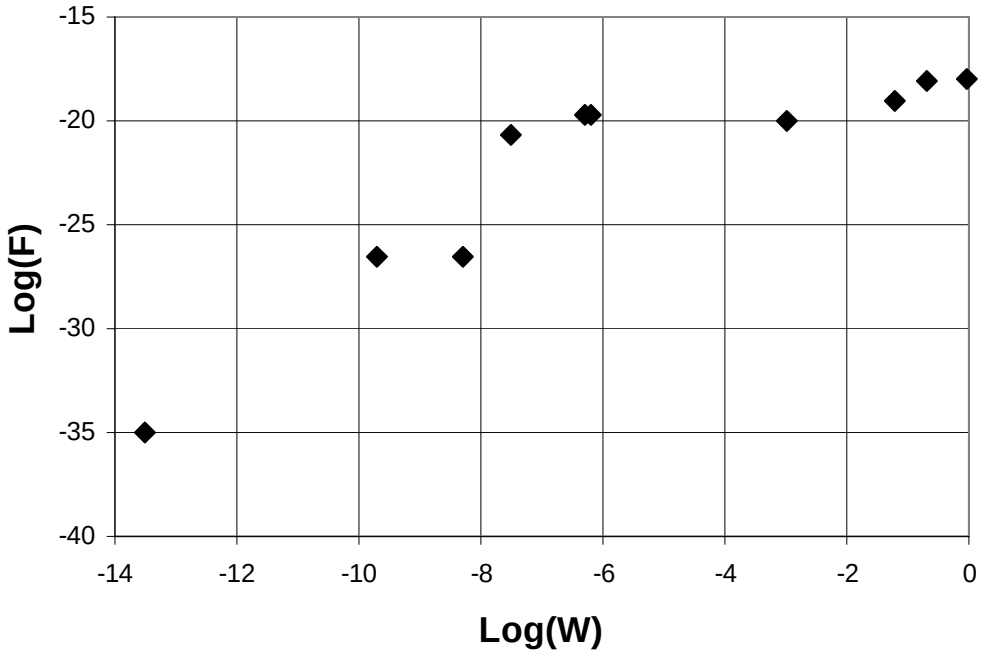
In a Log-Log graph, instead of using the units of 'x' and 'y', we use the units $Log_{10}(x)$ and $Log_{10}(y)$. For example, the point (100, 100000) would be graphed at (2.0, 5.0). The x-axis scale would be marked in uniform intervals of '1.0, 2.0, 3.0…' as before, but these would represent magnitudes of '10, 100 and 1000…' An x-value of '2.5' represents $10^{2.5}$ which is 316 on the usual 'linear' scale.

Problem - In this book, the problems showing the face of the sun at a variety of wavelengths also indicate the brightness of the sun at these wavelengths. The numbers are indicated in the table below. From the brightness data, create a Log(w) vs Log(F) grid, and plot the wavelength and brightness of the sun, F in watts/meter^2, in each electromagnetic wavelength, w, in meters.

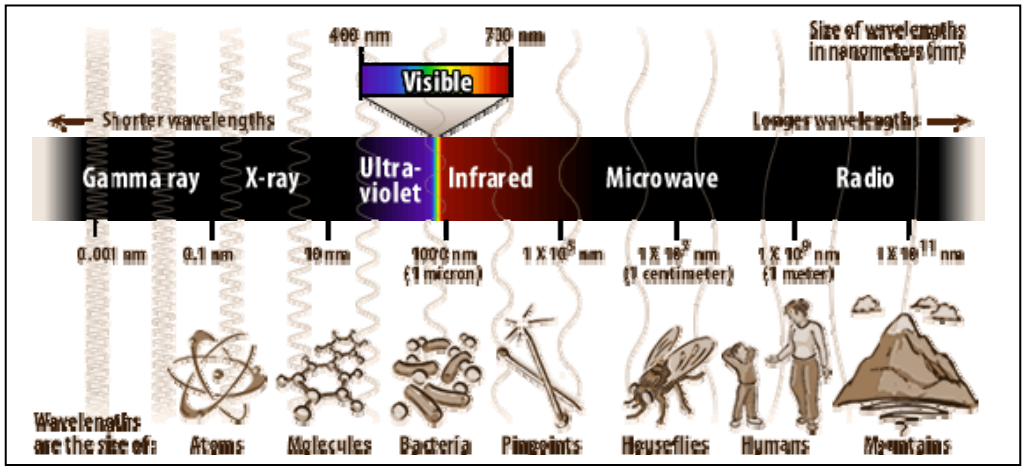
Example: For the 'Gamma ray' point, $F = \log_{10}(1.0 \times 10^{-35}) = -35.0$; and $W = \log_{10}(1.0 \times 10^{22}) = +22.0$, so plot this point at (+22,-35) on your graph.

Band	Frequency (Hertz)	Brightness (watts/meter^2)
Gamma ray	1.0×10^{22}	1.0×10^{-35}
Hard X-ray	1.5×10^{18}	3.0×10^{-27}
Soft X-ray	6.0×10^{16}	5.0×10^{-27}
Extreme Ultraviolet	1.0×10^{16}	3.0×10^{-26}
Visible	6.0×10^{14}	2.0×10^{-21}
Millimeter	3.0×10^{11}	5.0×10^{-20}
Microwave	5.0×10^{9}	1.0×10^{-19}
Microwave	1.5×10^{9}	8.0×10^{-19}
Radio	3.3×10^{6}	1.0×10^{-19}

Problem 1 - Answer: In each of the 10 solar images, a notation has been made as to the brightness of the Sun at the indicated wavelength. Students should calculate Log(W) and Log(F) for each image, and plot this point on a Log-Log graph. The result should resemble the graph below:



The Energy of Light



Light is defined as electromagnetic radiation, but 'EM radiation' comes in many other forms too. The X-rays that doctors use to probe your body carry enough energy to penetrate skin and bones, while infrared radiation does nothing more than warm you on a summer's day. EM radiation can be conveniently classified according to its frequency (or wavelength). But there is another way to classify it that helps physicists study its interaction with matter: the energy that it carries.

Light consists of particles called photons that travel at the speed of light - 300,000 km/s. Each of these particles carry a different amount of energy depending on their frequency. The formula that relates the energy in Joules (E) of a photon to its frequency in Hertz (F) is just

$$E = 6.6 \times 10^{-34} F \quad \text{Joules.}$$

Another form of this formula takes advantage of the fact that, for an electron, 1 'electron Volt' (eV) equals 1.6×10^{-19} Joules so

$$E = 4.1 \times 10^{-15} F \quad \text{electron-Volts}$$

Problem 1 - A photon of visible light has a frequency of 6×10^{14} Hertz, so how much energy does it carry in A)Joules? B) Electron volts.

Problem 2 - A radio station broadcasts at 100 megaHertz, how many eVs of energy do these radio-wave photons carry in the FM band?

Problem 3 - A dental X-ray machine uses X-rays that carry 150 keV of energy. A) What is the frequency of these X-rays? B) How many times more energy do they carry than visible light?

Problem 4 - If Frequency(Hertz) x Wavelength(meters) = 3×10^{8} meters/sec, re-write the second equation in terms of the wavelength of light given in units of nanometers (10^{-9} meters). Check your equation with your answer for the visible light photons in Problem 1.

Problem 1 - A photon of visible light has a frequency of 6×10^{14} Hertz, so how much energy does it carry in

A) Joules: $E = 6.6 \times 10^{-34} \times 6 \times 10^{14}$ = **4.0×10^{-19} Joules.**

B) Electron volts: $E = 4.1 \times 10^{-15} \times 6 \times 10^{14}$ = **2.5 electron Volts**

Problem 2 - A radio station broadcasts at 100 megaHertz, how many eVs of energy do these radio-wave photons carry in the FM band?

Answer: $E = 4.1 \times 10^{-15} \times 1 \times 10^{8}$ = **4.1×10^{-7} electron Volts**

Problem 3 - A dental X-ray machine uses X-rays that carry 150 keV of energy. A) What is the frequency of these X-rays? B) How many times more energy do they carry than visible light?

Answer: A) Use the energy formula in electron volts and solve for F to get $F = 2.4 \times 10^{14}$ E(volts) and so $F = 2.4 \times 10^{14}$ (150,000) = **3.6×10^{19} Hertz**.

B) From Problem 1, E = 2.6 eV, so the X-rays carry 150,000 eV/2.5 eV = **60,000 times more energy per photon.**

Problem 4 - If Frequency(Hertz) x wavelength(meters) = 3×10^{8} meters/sec. Re-write the second equation in terms of the wavelength of light given in nanometers (1.0×10^{-9} meters).

Answer: Let W be the wavelength in meters, then

$F = 3 \times 10^{8} / W$

But 1 meter = 10^{9} nanometers so, $F = 3 \times 10^{17} / W$

Then upon substitution for F,

$E = 4.41 \times 10^{-15} \times (3 \times 10^{17} / W)$ and we get

$$E = \frac{1230}{W(nm)} \text{ eV}$$

Visible light $F = 6 \times 10^{14}$ Hertz ; $W = 3 \times 10^{17} / F$ = 500 nm, then E = 2.5 eV - check!

$$E = 6.63 \times 10^{-34} \; f \quad \text{Joules}$$

$$E = 4.1 \times 10^{-15} \; f \quad \text{eV}$$

where F = frequency in Hertz

The electric and magnetic properties of electromagnetic waves allow them to carry energy from place to place. The quantum character of this radiation allows this energy to be carried in packets called photons. A simple relationship connects the frequency of an EM photon with a specific amount of energy being delivered by each photon in the EM wave.

The two equations above make it convenient to compute the energy in two different energy units in common usage in the study of EM radiation, once the frequency of the EM wave is given in Hertz. The 'Joule' is a common measure of energy used in the so-called MKS or 'SI' system of units, however it is very cumbersome when measuring the minute energies carried by atomic systems. The 'electron Volt' is widely used by physicists, chemists and astronomers when referring to atomic systems measured under laboratory conditions or in the universe at-large.

For example, the frequency of visible 'yellow' light is about 520 trillion Hertz, so the energy carried by one 'yellow' photon is $E = 6.63 \times 10^{-34} (5.2 \times 10^{14}) = 3.4 \times 10^{-19}$ Joules, or from the second equation we would get 2.1 eV.

Problem 1 - Although these formula refer to the frequency of EM radiation, we can also re-write them to refer to the wavelength of EM radiation using the familiar rule that $f = c/L$ where c is the speed of light in meters/sec and L is the wavelength in meters. If c = 300 million meters/sec, what are the two new energy laws expressed in terms of wavelength in meters?

Problem 2 - How much energy, in eV, is carried by each photon in the FM radio band if the frequency of your station is 101.5 MHz?

Problem 3 - How much energy, in eV, is carried by a single photon in the infrared band if the wavelength of the photon is 25 microns (1 micron = 1.0×10^{-6} meters)?

Problem 4 - How much energy is carried by a single gamma-ray photon if its wavelength is the same as the diameter of an atomic nucleus (1×10^{-14} meters) computed in A) Joules? B) eV?

Problem 1 - Although these formula refer to the frequency of EM radiation, we can also re-write them to refer to the wavelength of EM radiation using the familiar rule that f = c/L where c is the speed of light in meters/sec and L is the wavelength in meters. If c = 300 million meters/sec, what are the two new energy laws expressed in terms of wavelength in meters?

Answer: From $E = 6.63 \times 10^{-34}$ f we have, with substitution,

$E = 6.63 \times 10^{-34} (3 \times 10^{8} / L)$ or

$\mathbf{E = 2.0 \times 10^{-25} / L}$ **Joules.**

From $E = 4.1 \times 10^{-15}$ f we have, with substitution

$E = 4.1 \times 10^{-15} (3 \times 10^{8} / L)$ or

$\mathbf{E = 1.23 \times 10^{-6} / L}$ **eV**.

Problem 2 - How much energy, in eV, is carried by each photon in the FM radio band if the frequency of your station is 101.5 MHz?
Answer: The information is stated in terms of frequency, so use the fact that 1MHz = 1 million Hertz and so $E = 4.1 \times 10^{-15} (101.5 \times 1 \times 10^{6})$ = $\mathbf{4.2 \times 10^{-7}}$ **eV**.

Problem 3 - How much energy, in eV, is carried by a single photon in the infrared band if the wavelength of the photon is 25 microns (1 micron = 1.0×10^{-6} meters)?
Answer: Use the relationship for E in terms of wavelength to get
$E = 1.32 \times 10^{-6} /(25.0 \times 1.0 \times 10^{-6})$ = **0.053 eV**.

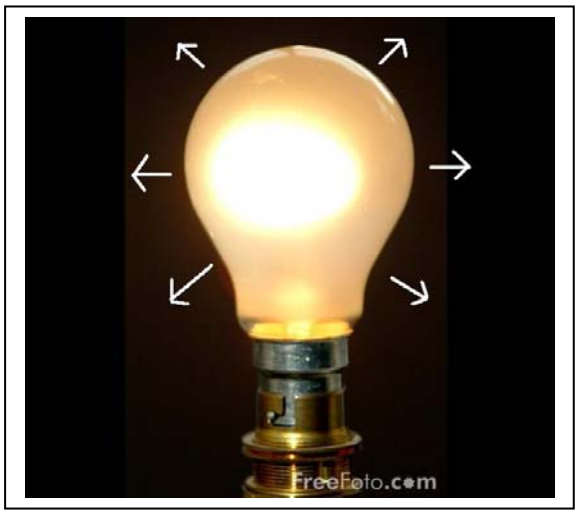
Problem 4 - How much energy is carried by a single gamma-ray photon if its wavelength is the same as the diameter of an atomic nucleus (1×10^{-15} meters) computed in A) Joules? B) eV?

Answer: A) $E = 2.0 \times 10^{-25} / 1.0 \times 10^{-15}$ = $\mathbf{2 \times 10^{-10}}$ **Joules**.

B) $E = 1.32 \times 10^{-6} / (1.0 \times 10^{-15})$ = $\mathbf{1.32 \times 10^{9}}$ **eV**

or 1.32 giga eV (also abbreviated as 1.32 GeV)

The Flow of Light Energy: Power



Because electromagnetic waves carry energy from place to place, there are a number of different ways to express how rapidly, and how intensely, this energy is being moved for a given source. The first of these has to do with the transmitted power, measured in watts.

One watt is equal to one Joule of energy being transmitted, or received, each second. The watt is the same unit that you use when you select light bulbs at the store, or measure the amount of work that something is doing. (A 350 horse-power car engine delivers 260,000 watts!).

Problem 1 - At a frequency of 100 million Hertz (100 MHz) a single photon of electromagnetic energy carries about 6×10^{-26} Joules of energy. If 10 trillion of these '100 MHz' photons are transmitted every 3 seconds, how many watts of electromagnetic energy are being moved?

Problem 2 - On your FM dial, the radio signal for station WXYZ is 101.5 MHz. The weakest signal that your radio can detect at this frequency is 10^{-12} watts. How many photons per second can you detect at this sensitivity level if the photon energy, in Joules, is given by $E = 6.63 \times 10^{-34}$ f, where f is the frequency in Joules?

Problem 3 - The faintest star you can see in the sky has an apparent magnitude of +6.0 in the visual band. The light from such a star carries just enough power for your retinal 'rods' to detect the star. If the frequency of visible light is 520 trillion Hertz, and the minimum power you can detect is about 3×10^{-17} watts, how many photons are being detected by your retina each second at this light threshold?

Problem 4 - Staring at the sun can have harmful consequences, including blindness! If the sun delivers 0.01 watts of energy to your retina, and the frequency of a visible light photon is 520 trillion Hertz, how many photons per second reach your eye?

Problem 1 - At a frequency of 100 million Hertz (100 MHz) a single photon of electromagnetic energy carries about 6×10^{-26} Joules of energy. If 10 trillion of these '100 MHz' photons are transmitted every 3 seconds, how many watts of electromagnetic energy are being moved?
Answer: We know how much energy is carried by a single photon, so multiplying this by the number of photons gives the total energy: $E = 6 \times 10^{-26}$ Joules/photon x (10×10^{12} photons) = 6×10^{-13} Joules <u>every 3 seconds</u>. Since the definition of a watt is 1 Joule/1 second, we have the power transmitted as $P = 6 \times 10^{-13}$ Joules / 3 seconds = **2×10^{-13} Watts**.

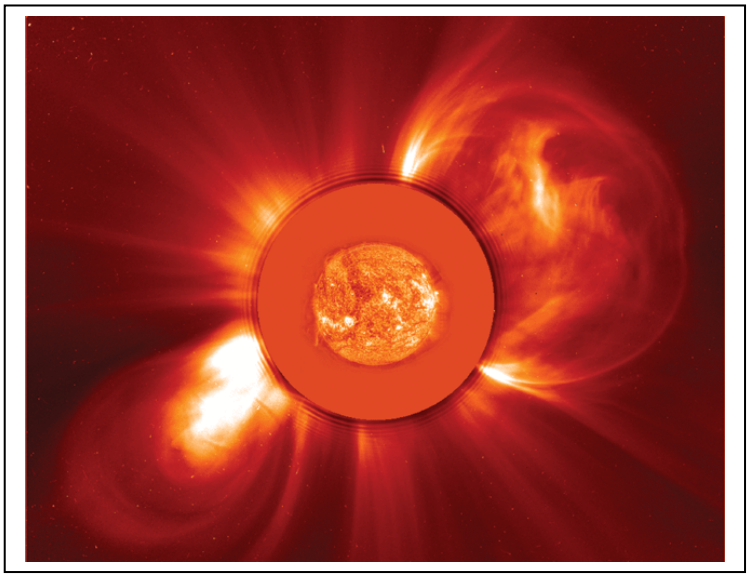
Problem 2 - On your FM dial, the radio signal for station WXYZ is 101.5 MHz. The weakest signal that your radio can detect at this frequency is 10^{-12} watts. How many photons per second can you detect at this sensitivity level if the photon energy, in Joules, is given by $E = 6.63 \times 10^{-34}$ f, where f is the frequency in Joules?
Answer: Each photon carries $E = 6.63 \times 10^{-34}$ (101.5×10^{6}) = 6.7×10^{-26} Joules/photon. Since the minimum detected power is 10^{-12} watts, we re-write this as 10^{-12} Joules/second. Then we do the conversion: $N = 10^{-12}$ Joules/second x (1 photon/6.7×10^{-26} Joules) = 15 trillion photons/second.

Problem 3 - The faintest star you can see in the sky has an apparent magnitude of +6.0 in the visual band. The light from such a star carries just enough power for your retinal 'rods' to detect the star. If the frequency of visible light is 520 trillion Hertz, and the minimum power you can detect is about 3×10^{-17} watts, how many photons are being detected by your retina each second at this light threshold?
Answer: A single photon carries $E = 6.63 \times 10^{-34}$ (520 trillion) = 3.4×10^{-19} Joules.
The threshold power is 3×10^{-17} watts, or 3×10^{-17} Joules/sec, so
$N = 3 \times 10^{-17}$ Joules/sec x (1 photon/3.4×10^{-19} Joules) = **88 photons/sec**.

Problem 4 - Staring at the sun can have harmful consequences, including blindness! If the sun delivers 0.01 watts of energy to your retina, and the frequency of a visible light photon is 520 trillion Hertz, what is the threshold level for retinal damage in photons/sec?
Answer: A single photon carries $E = 6.63 \times 10^{-34}$ (520 trillion) = 3.4×10^{-19} Joules.
The threshold power is 0.01 watts, or 0.01 Joules/sec, so
N = 0.01 Joules/sec x (1 photon/3.4×10^{-19} Joules) = **2.9×10^{16} photons/sec**.

Note: The range of the human eye is then a factor of $2.9 \times 10^{16}/88$ = 329 trillion!

CME Kinetic Energy and Mass



Kinetic energy is the energy that a body has by virtue of its mass and speed. Mathematically, it is expressed as one-half of the product of the mass of the object (in kilograms), times the square of the objects speed (in meters/sec).

$$\textbf{K.E.} = 0.5\ m\ V^2$$

Between October 1996 and May 2006, the SOHO satellite detected and cataloged 11,031 coronal mass ejections (CMEs) like the one seen in the figure to the left. There was enough data available to determine the properties for 2,131 events. The table below gives values for ten of these CMEs.

Date	Speed (km/s)	K.E. (Joules)	Mass (kilograms)
4/8/1996		1.1×10^{20}	2.2×10^{9}
8/22/2000	388	1.3×10^{22}	
6/10/2001	731	8.2×10^{23}	
1/18/2002	64		2.6×10^{10}
5/16/2002	1,310		7.8×10^{10}
10/7/2002		7.8×10^{21}	3.0×10^{10}
1/24/2003	387	9.1×10^{18}	
10/31/2003	2,198	1.6×10^{24}	
11/2/2003		9.3×10^{25}	4.5×10^{13}
11/10/2004	3,387		9.6×10^{12}

Problem 1 - Complete the table by determining the value of the missing entries using the formula for Kinetic Energy.

Problem 2 - What is the minimum and maximum range for the observed kinetic energies for the 10 CMEs? The largest hydrogen bomb ever tested was the Tsar Bomba in 1961 and was equivalent to 50 megatons of TNT. It had a yield of 5×10^{23} Joules. What is the equivalent yield for the largest CME in megatons, and 'Tsar Bombas'?

Problem 3 - What are the equivalent masses of the smallest and largest CMEs in metric tons?

Problem 4 - Compare the mass of the largest CME to the mass of a small mountain. Assume that the mountain can be represented as a cone with a volume given by $1/3\ \pi\ R^2\ H$ where R is the base radius and H is the height in meters, and assume the density of rock is 3 grams/cm^3.

Date	Speed (km/s)	K.E. (Joules)	Mass (kilograms)
4/8/1996	**316**	1.1×10^{20}	2.2×10^{9}
8/22/2000	388	1.3×10^{22}	$\mathbf{1.7 \times 10^{11}}$
6/10/2001	731	8.2×10^{23}	$\mathbf{3.1 \times 10^{12}}$
1/18/2002	64	$\mathbf{5.3 \times 10^{19}}$	2.6×10^{10}
5/16/2002	1,310	$\mathbf{6.7 \times 10^{22}}$	7.8×10^{10}
10/7/2002	**721**	7.8×10^{21}	3.0×10^{10}
1/24/2003	387	9.1×10^{18}	$\mathbf{1.2 \times 10^{8}}$
10/31/2003	2,198	1.6×10^{24}	$\mathbf{6.6 \times 10^{11}}$
11/2/2003	**2,033**	9.3×10^{25}	4.5×10^{13}
11/10/2004	3,387	$\mathbf{5.5 \times 10^{25}}$	9.6×10^{12}

Problem 1: Complete the table by determining the value of the missing entries using the formula for Kinetic Energy.

Answer: See above answers.

Problem 2: What is the minimum and maximum range for the observed kinetic energies for the 10 CMEs?

Answer: Maximum = 9.3×10^{25} Joules. Minimum = 5.3×10^{19} Joules

The largest hydrogen bomb ever tested was the *Tsar Bomba* in 1961 and was equivalent to 50 megatons of TNT. It had a yield of 5×10^{23} Joules. What is the equivalent yield for the largest CME in megatons, and 'Tsar Bombas'?

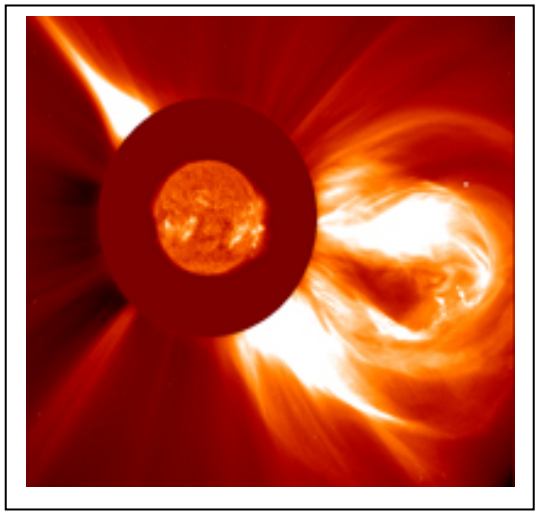
Answer: The CME on November 2, 2003 was equal to
$(9.3 \times 10^{25}/5 \times 10^{23}) =$ 186 Tsar Bombas,
and an equivalent TNT yield of 186 x 50 megatons = **9,300 megatons!**

Problem 3: What are the equivalent masses of the smallest and largest CMEs in metric tons?

Answer: One metric ton is 1,000 kilograms. The smallest mass was for the January 24, 2003 CME with about 120,000 tons. The largest mass was for the November 2, 2003 'Halloween Storm' with about **45 billion metric tons.**

Problem 4: Compare the mass of the largest CME to the mass of a small mountain. Assume that the mountain can be represented as a cone with a volume given by $V = 1/3\ \pi R^2 H$ where R is the base radius and H is the height in meters, and assume the density of rock is 3 grams/cm^3.

Answer: One possibility is for a mountain with a base radius of R = 1 kilometers, and a height of 50 meters. The cone volume is $0.33 \times 3.14 \times (1000)^2 \times 50 = 5.2 \times 10^7$ cubic meters. The rock density of 3 gm/cm^3 converted into kg per cubic meters is 0.003 kg/(.01)^3 = 3000 kg/m^3. This yields a mountain with a mass of **156 billion tons,** which is close to the largest CME mass in the table. So CMEs, though impressive in size, carry no more mass than a small hill on Earth!



Solar storms can affect our satellite and electrical technologies, and can also produce health risks. For over 100 years, scientists have kept track of these harsh 'space weather' events, which come and go with the 11-year sunspot cycle.

During times when many sunspots are present on the solar surface, daily storms are not uncommon. These storms come in two distinct types: Solar flares, which cause radio interference and health risks, and coronal mass ejections, which affect satellites and cause the Northern Lights.

Problem 1 – During a particular week in 2001, on a given day of the week, the Sun produced either a coronal mass ejection, S, or an X-ray solar flare, X. Use the Binomial Theorem to compute all of the possible terms for $(S + X)^7$ that represents the number of possible outcomes for S and X on each of the seven days.

Problem 2 – What does the term represented by ${}_7C_3\, S^4X^3 = 21\, S^4X^3$ represent?

Problem 3 – In counting up all of the possible ways that the two kinds of storms can occur during a 7-day week, what are the most likely number of S and X-type storms you might expect to experience during this week?

Problem 4 – How much more common are weeks with 3 S-type storms and 4 X-type storms than weeks with 1 S-type storm and 6 X-type storms?

Problem 1 – During a particular week in 2001, on a given day of the week, the Sun produced either a coronal mass ejection, S, or an X-ray solar flare, X. Use the Binomial Theorem to compute all of the possible terms for $(S + X)^7$.

Answer: The Binomial expansion is

$${}_7C_7\, S^0X^7 + {}_7C_6\, S^1X^6 + {}_7C_5\, S^2X^5 + {}_7C_4\, S^3X^4 + {}_7C_3\, S^4X^3 + {}_7C_2\, S^5X^2 + {}_7C_1\, S^6X^1 + {}_7C_0\, S^7X^0$$

which can be evaluated using the definition of $\mathbf{{}_nC_r}$ to get:

$$\mathbf{= X^7 + 7\, S^1X^6 + 21\, S^2X^5 + 35\, S^3X^4 + 35\, S^4X^3 + 21\, S^5X^2 + 7\, S^6X^1 + S^7}$$

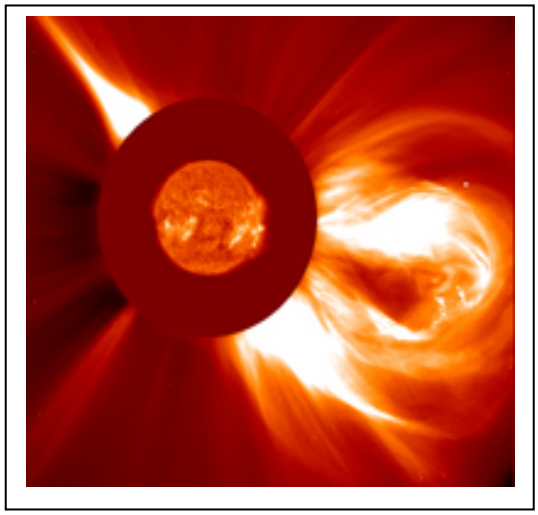
Problem 2 – What does the term represented by ${}_7C_3\, S^4X^3 = 21\, S^4X^3$ represent?
Answer:
If you were to tally up the number of ways that 4 S-type storms and 3 X-type storms could be distributed among the 7 days in a week, you would get 21 different 'line ups' for the sequence of these storms. For instance during the 7 consecutive days, one of these would be S,X,X,S,S,X,S

Problem 3 – In counting up all of the possible ways that the two kinds of storms can occur during a 7-day week, what are the most likely number of S and X-type storms you might expect to experience during this week?

Answer: **The two possibilities consisting of 3 days of S-type and 4 days of X-type storms, and 4 days of S-type and 3 days of X-type storms each have the largest number of possible combinations: 35.**

Problem 4 – How much more common are weeks with 3 S-type storms and 4 X-type storms than weeks with 1 S-type storm and 6 X-type storms?

Answer: From the Binomial Expansion, the two relevant terms are $\mathbf{7\, S^1X^6}$ and $\mathbf{35\, S^3X^4}$**.** The ratio of the leading coefficients gives the ratio of the relative frequency from which we see that 35/7 = **5 times more likely to get 3 S-type and 4 X-type storms.**



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During times when many sunspots are present on the solar surface, daily storms are not uncommon. These storms come in two distinct types: Solar flares, which cause radio interference and health risks, and coronal mass ejections, which affect satellites and cause the Northern Lights.

Problem 1 – During a particular week in 2001, on a given day of the week, the Sun produced either a coronal mass ejection, S, or an X-ray solar flare, X. From an additional study, astronomers determined that it is twice as likely for a coronal mass ejection to occur than an X-ray solar flare. Use the Binomial Theorem to compute all of the possible terms for $(2S + X)^7$.

Problem 2 – How much more common are weeks with 5 S-type storms and 2 X-type storms than weeks with 1 S-type storm and 6 X-type storms?

Problem 1 – During a particular week in 2001, on a given day of the week, the Sun produced either a coronal mass ejection, S, or an X-ray solar flare, X. From an additional study, astronomers determined that it is twice as likely for a coronal mass ejection to occur than an X-ray solar flare. Use the Binomial Theorem to compute all of the possible terms for $(2S + X)^7$.

Answer: First let a = 2S and b = X, then use the binomial expansion to determine $(a + b)^7$:

$${}_7C_7\, a^0 b^7 + {}_7C_6\, a^1 b^6 + {}_7C_5\, a^2 b^5 + {}_7C_4\, a^3 b^4 + {}_7C_3\, a^4 b^3 + {}_7C_2\, a^5 b^2 + {}_7C_1\, a^6 b^1 + {}_7C_0\, a^7 b^0$$

which can be evaluated using the definition of ${}_nC_r$ to get:

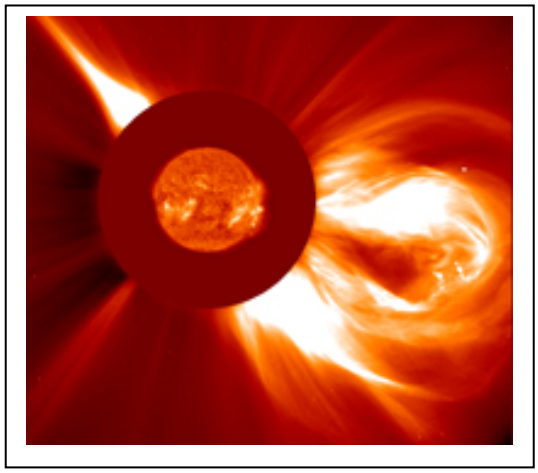
$$= b^7 + 7\, a^1 b^6 + 21\, a^2 b^5 + 35\, a^3 b^4 + 35\, a^4 b^3 + 21\, a^5 b^2 + 7\, a^6 b^1 + a^7$$

now substitute a = 2S and b = X to get

$$= X^7 + 14\, S^1 X^6 + 84\, S^2 X^5 + 280\, S^3 X^4 + 560\, S^4 X^3 + 672\, S^5 X^2 + 448\, S^6 X^1 + 128 S^7$$

Problem 2 – How much more common are weeks with 5 S-type storms and 2 X-type storms than weeks with 1 S-type storm and 6 X-type storms?

Answer: The two relevant terms are $\mathbf{14\ S^1X^6}$ and $\mathbf{672\ S^5X^2}$**.** The ratio of the leading coefficients gives the ratio of the relative frequency from which we see that 672/14 = **48 times more likely to get 5 S-type and 2 X-type storms.**



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Problem 2 – What does the term represented by ${}_7C_3\, S^4X^3 = 21\, S^4X^3$ represent?

Problem 3 – During an average month in 2001 there were 25 coronal mass ejections and 5 X-flares. What is the probability of A) observing a coronal mass ejection during a given day? B) Observing an X-flare during a given day?

Problem 4 – Using the probabilities derived in Problem 3, what is the most likely number of X-flares and coronal mass ejections spotted during a given week?

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Answer: The Binomial expansion is

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If you were to tally up the number of ways that 4 S-type storms and 3 X-type storms could be distributed among the 7 days in a week, you would get 21 different 'line ups' for the sequence of these storms. For instance during the 7 consecutive days, one of these would be S,X,X,S,S,X,S

Problem 3 – During an average month in 2001 there were 25 coronal mass ejections and 5 X-flares. What is the probability of A) observing a coronal mass ejection during a given day? B) Observing an X-flare during a given day?

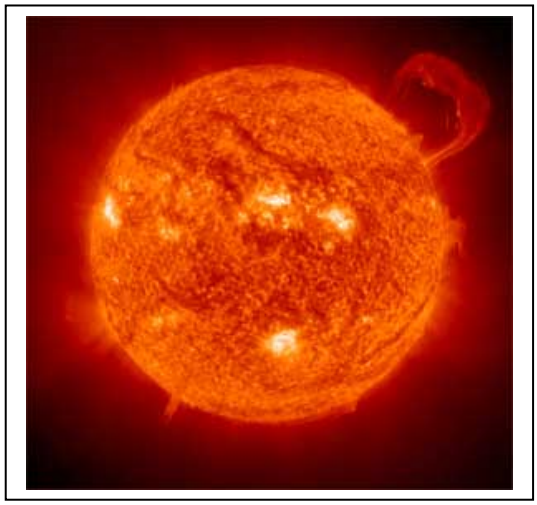
Answer: **P(X) = 5/30 = 0.17 and P(S) = 25/30 = 0.83**

Problem 4 – Using the probabilities derived in Problem 3, what is the most likely number of X-flares and coronal mass ejections spotted during a given week?

Answer: Substitute X = P(X) = 0.17 and S = P(S) = 0.83 into the binomial expansion and evaluate each term.

$$\mathbf{X^7 + 7\, S^1X^6 + 21\, S^2X^5 + 35\, S^3X^4 + 35\, S^4X^3 + 21\, S^5X^2 + 7\, S^6X^1 + S^7}$$

The probabilities are:
0.000004 + 0.00014 + 0.002 + 0.017 + 0.082 + 0.24 + 0.39 + 0.27
so the most likely case would be 6 coronal mass ejections and one x-flare during an average week.



The sun is an active star. When it is 'stormy' it can release giant clouds of plasma called coronal mass ejections, or intense bursts of X-rays in events called solar flares. Solar flares can cause radio interference or black outs, or be harmful to astronauts. Meanwhile, coronal mass ejections can cause satellite and electric power grid problems.

The sun can produce coronal mass ejections and flares as separate events, or sometimes both happen at the same time.

The Binomial Theorem is helpful when two outcomes are possible, such as the flip of a two-sided coin. But suppose you had three, four or more possible outcomes? The case of N=3 is a natural extension of the Binomial Theorem and can be explored by determining the coefficients and terms that result from expanding $P = (a + b + c)^{N}$.

Problem 1 – Let's investigate the case where a = probability that only corona mass ejection events occur, b = probability that only X-flares occur; and c = probability that both coronal mass ejections and X-flares occur. Expand the factors for the case of:
A) A 2-day study (N=2); B) A 3-day study (N=3); C) A 4-day study (N=4)

Problem 2 – A careful study of solar activity determined that for 619 storm events, 497 were only coronal mass ejections, 26 were only solar flares, and 96 were both solar flares and coronal mass ejections. What are the probabilities P(C), P(X) and P(both)?

Problem 3 – Three days were selected at random from a typical month. What two combinations of C, X and 'Both' were the most likely to occur given the probabilities determined in Problem 2?

Problem 1 – Let's investigate the case where a = probability that only corona mass ejection events occur, b = probability that only X-flares occur; and c = probability that both coronal mass ejections and X-flares occur. Expand the factors for the case of:
A) A 2-day study (N=2); B) A 3-day study (N=3); C) A 4-day study (N=4)
Answer:

(N=2): $P(2) = \mathbf{C^2 + X^2 + B^2 + 2XC + 2CB + 2XB}$

(N=3) $P(3) = C^3 + X^3 + B^3 + 3CX^2 + 3CB^2 + 3XC^2 + 3BC^2 + 3XB^2 + 3X^2B + 6XBC$

(N=4) $P(4) = C^4 + X^4 + B^4 + 4CX^3 + 4CB^3 + 6C^2X^2 + 6C^2B^2 + 4XC^3 + 4BC^3 + 12 CXB^2 + 12 CBX^2 + 12XBC^2 + 4XB^3 + 3X^2B^2 + 3BX^3 + BX^3$

Problem 2 – A careful study of solar activity determined that for 619 storm events, 497 were only coronal mass ejections, 26 were only solar flares, and 96 were both solar flares and coronal mass ejections. What are the probabilities P(C), P(X) and P(both)?

Answer: P(C) = 497/619 so **P(C) = 0.8**
P(X) = 26/619 so **P(X) = 0.04**
P(Both) = 96 / 619 so **P(Both) = 0.16.**

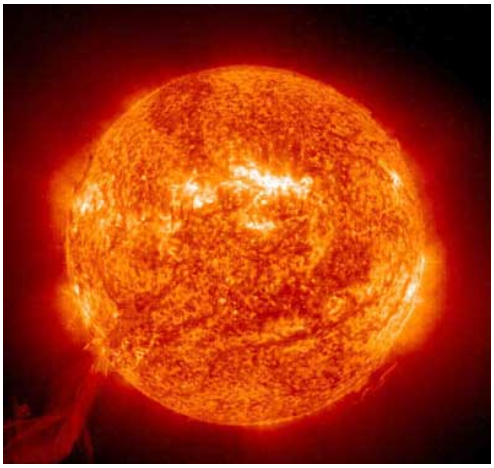
Problem 3 – Three days were selected at random from a typical month. What two combinations of C, X and 'Both' were the most likely to occur given the probabilities determined in Problem 2?

Answer: For N=3:
$P(3) = C^3 + X^3 + B^3 + 3CX^2 + 3CB^2 + 3XC^2 + 3BC^2 + 3XB^2 + 3X^2B + 6XBC$

Since X = 0.04, C=0.8 and B=0.16 we have

P(3) = **0.51** + 0.000064 + 0.0041 + 0.0038 + 0.061 + 0.077 + **0.31** + 0.0031 + 0.00077 + 0.031

The two most common combinations are for:
1) C^3 : All three days to only have coronal mass ejections (C); and
2) $3BC^2$: two coronal mass ejections (C) and one day where both a flare and a coronal mass ejection occur (B).



What does it really mean to determine the average and standard deviation of a set of measurements?

Astronomers have to deal with this basic question all the time because they are dealing with objects that cannot be visited to make normal measurements as we would under laboratory conditions.

Here is a step-by-step example!

An astronomer wants to determine the temperature of 10 stars like our own sun that are identical in mass, diameter and luminosity. From the detailed knowledge of our Sun, suppose the Sun's exact surface temperature is 5,700 K. If the other stars are exact duplicates of our Sun, they should also have the same temperature. The particular ten stars that the astronomer selected are, in fact, slightly different than the Sun in ways impossible for the astronomer to measure with the best available instruments. The table below gives the measured temperatures along with their reasons for not matching our Sun exactly; reasons that the astronomer is not aware of:

Star	Temp. (K)	Reason for Difference
A	5,750	There is 1% more hydrogen in the star
B	5,685	The star is 100 million years younger than the sun
C	5,710	The radius of the star is 1% smaller than the sun
D	5,695	The radius of the star is 2% larger than the sun
E	5,740	The luminosity of the star is 1 percent larger than the sun
F	5,690	The luminosity is 1% smaller than the sun
G	5,725	The star has 3% more helium than the sun
H	5,701	The star has a bright sunspot with a high temperature
I	5,735	The abundance of calcium is 5% lower than the sun
J	5,681	The amount of sodium is 1% greater than for the sun

Problem 1 – What is the average temperature, and standard deviation, σ, of this sample of 'twins' to our sun?

Problem 2 – How does this sampling compare with a classroom experiment to determine the distribution of student heights?

Problem 1 – What is the average temperature of this sample of 'twins' to our sun?
Answer: Average
Ta = (5750+5685+5710+5695+5740+5690+5725+5701+5735+5681)/10 = **5712 K**.

To calculate the S.D first subtract the average temperature, 5712 from each temperature, square this number, sum all the ten 'squared' numbers; divide by 10-1 = 9, and take the square-root:

Star	Temp. (K)	T-Tave	Squared	Sigma range
A	5,750	+38	1444	1-2
B	5,685	-27	729	1-2
C	5,710	-2	4	0-1
D	5,695	-17	289	0-1
E	5,740	+28	784	1-2
F	5,690	-22	484	0-1
G	5,725	+13	169	0-1
H	5,701	-11	121	0-1
I	5,735	+23	529	0-1
J	5,681	-31	961	1-2

Sum (squares)= 5514
$\sigma = (5514/9)^{1/2}$ **σ = 25 K**

Problem 2 – How does this sampling compare with a classroom experiment to determine the distribution of student heights?

Answer: The table shows the range of sigmas in which the various points fall. For example, Star B: T=5,685 so 5685-5712 = -27, but 1-sigma = 25K so this point is between -1 and -2 sigma from the mean value. Counting the number of stars whose temperatures fall within each rank in the temperature distribution we see that 60% fall between +/- 1 sigma, and 100% fall between 0 and 2 sigma. This is similar to the Normal Distribution for which 68% should fall within 1-sigma and 95.4% within 2-sigma in a random sample (Note: 10 x 0.954 = 9.54 stars or 10 stars). There are no exceptional points in this limited sample that represent larger deviations from the mean value than 2-sigma.

Each student has their own physical reason for being the height that they are in a specific classroom. Most of these reasons cannot be measured such as genetic pre-disposition, and nutrition. Just as the astronomer selected 10 stars because they are 'sun-like' we would select a classroom of 30 students in a specific school with students of a common age, give or take a few months. We measure the height of each student of the same age range, and calculate an average height for that age, and a standard deviation that is influenced by the factors that we cannot measure (genetics, nutrition). If we could, we would further divide our groups and calculate a new average and S.D..

The Solar Probe Plus mission will be launched in 2018 for a rendezvous with the sun in 2024. To lose enough energy to reach the sun, the spacecraft will make seven fly-bys of Venus.

The 480 kg spacecraft, costing $740 million, will approach the sun to a distance of 5.8 million km. Protected by the heat shield will be five instruments that will peak over the edge of the heat shield and measure the particles and radiation fields in the outer solar corona.

At the closest approach distance, the sun will occupy a much larger area of the sky than what it does at the distance of Earth.

The angular size of an object as it appears to a viewer depends on the actual physical size of the object, and its distance from the viewer. Although physical size and diameter are usually measured in terms of meters or kilometers, the angular diameter of an object is measured in terms of degrees or radians. The angular size, t, depends on its actual size d and its distance r according to the simple formula

$$\theta = 2arcTan\left(\frac{d}{2R}\right)$$

For example, the angular size of your thumb with your arm fully extended for d = 2 centimeters and r = 60 cm is just θ = 2 arcTan(0.016) so θ = 1.8 degrees.

Problem 1 – At the distance of Earth, 147 million kilometers, what is the angular diameter of the sun whose physical diameter is 1.4 million kilometers?

Problem 2 – At the distance of closest approach to the solar surface Solar Probe Plus will be at a distance of 5.8 million kilometers from the solar surface. What will be the angular diameter of the sun if its physical diameter is 1.4 million kilometers?

Problem 3 – A DVD disk measures 12 cm in diameter. How far from your eyes will you have to hold the DVD disk in order for it to subtend the same angular size as the disk of the sun viewed by Solar Probe Plus at its closest distance?

Problem 1 – At the distance of Earth, 147 million kilometers, what is the angular diameter of the sun whose physical diameter is 1.4 million kilometers?

Answer: $\theta = 2arcTan\left(\frac{1.4million}{2x147million}\right)$

θ = 2 arctan(0.0048)

so θ **= 0.55 degrees**.

Problem 2 – At the distance of closest approach to the solar surface Solar Probe Plus will be at a distance of 5.8 million kilometers from the solar surface. What will be the angular diameter of the sun if its physical diameter is 1.4 million kilometers?

Answer: θ = 2 arctan (1.4/(2 x 5.8)) so θ **= 14 degrees**.

Problem 3 – A DVD disk measures 12 cm in diameter. How far from your eyes will you have to hold the DVD disk in order for it to subtend the same angular size as the disk of the sun viewed by Solar Probe Plus at its closest distance?

Answer: The desired angular diameter of the sun is θ = 14 degrees, and for the DVD disk we have d = 12 cm, so we need to solve the formula for r.

Tan (θ/2) = d/2r so

2r = d / tan(θ/2) then

r = 6 cm / tan(7) yields

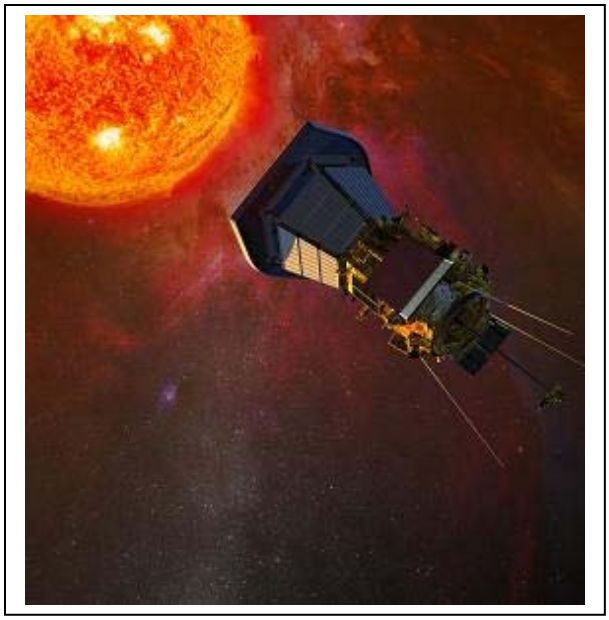
r = **49 cm. This is about 18 inches**.

Note: From the vantage point of the Solar Probe Plus spacecraft, the sun's disk at a temperature of 5570 K is much larger (14 degrees) than it appears in Earth's sky (0.5 degrees), and this results in the spacecraft heat shield being heated to over 1600 K.

For more mission details, visit:

http://solarprobe.jhuapl.edu/

http://www.nasa.gov/topics/solarsystem/sunearthsystem/main/solarprobeplus.html



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As the spacecraft approaches the sun, its heat shield must withstand temperatures exceeding 2500° F and blasts of intense radiation.

The 480 kg spacecraft, costing $740 million, will approach the sun to a distance of 6 million km. Protected by the heat shield will be five instruments that will peak over the edge of the heat shield and measure the particles and radiation fields in the outer solar corona.

A simple formula that predicts the temperature, in Kelvins, of a surface exposed to solar radiation is given by

$$T = 396\frac{(1-A)^{\frac{1}{4}}}{\sqrt{R}}$$

where R is the distance to the solar surface in Astronomical Units, and A is the fraction of the incoming radiation that is reflected by the surface. Because only the amount of absorbed sunlight determines how hot a body becomes, the final temperature depends on the quantity (1 – A) rather than A alone. (Note 1 Astronomical Unit is the distance of Earth from the center of the sun; 147 million kilometers).

Problem 1 – An astronaut's white spacesuit reflects 80% of the incoming radiation at Earth's orbit (1 Astronomical Unit). From the formula, about what is the temperature of the surface of the spacesuit?

Problem 2 - The Solar Probe Plus spacecraft will use a heat shield facing the sun with a reflectivity of about A = 0.60. What will be the temperature of the heat shield, called the Thermal Protection System or TPS, at the distance of 5.9 million km (0.040 AU), A) In Kelvins? B) In degrees Celsius? C) In degrees Fahrenheit?

Problem 3 – Suppose that the TPS consisted of a highly-reflective mirrored coating with a reflectivity of 99%. What would be the temperature in Kelvins, of the back of the heat shield when the Solar Probe is closest to the sun at a distance of 0.04 AU?

Problem 1 – An astronaut's white spacesuit reflects 80% of the incoming radiation at Earth's orbit (1 Astronomical Unit). From the formula, about what is the temperature of the surface of the spacesuit?

Answer: $T = 396\frac{(1-0.8)^{\frac{1}{4}}}{\sqrt{1AU}}$ so **T = 265 Kelvin**.

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Answer: A) **T = 1600 K**.
B) Tc = Tk – 273 so **Tc = 1,300^{o} C**
C) Tf = 9/5 (Tc) + 32 so **Tf = 2,400^{o} F**.

Problem 3 – Suppose that the TPS consisted of a highly-reflective mirrored coating with a reflectivity of 99%. What would be the temperature in Kelvins, of the back of the heat shield when the Solar Probe is closest to the sun at a distance of 0.04 AU?

Answer: A = 0.99 then

$T = 396\frac{(1-0.99)^{\frac{1}{4}}}{\sqrt{0.04AU}}$ so Tk = **630 Kelvins**.

For more mission details, visit:

http://solarprobe.jhuapl.edu/

http://www.nasa.gov/topics/solarsystem/sunearthsystem/main/solarprobeplus.html

$$Fg = \frac{G M m}{R^2}$$

$$Fc = \frac{m V^2}{R}$$

$$V = \frac{2 \pi R}{T}$$

One of the neatest things in astronomy is being able to figure out the mass of a distant object, without having to 'go there'. Astronomers do this by employing a very simple technique. It depends only on measuring the separation and period of a pair of bodies orbiting each other. In fact, Sir Issac Newton showed us how to do this over 300 years ago!

Imagine a massive body such as a star, and around it there is a small planet in orbit. We know that the force of gravity, **Fg**, of the star will be pulling the planet inwards, but there will also be a centrifugal force, **Fc**, pushing the planet outwards.

This is because the planet is traveling at a particular speed, **V**, in its orbit. When the force of gravity and the centrifugal force on the planet are exactly equal so that **Fg = Fc**, the planet will travel in a circular path around the star with the star exactly at the center of the orbit.

Problem 1) Use the three equations above to derive the mass of the primary body, M, given the period, T, and radius, R, of the companion's circular orbit.

Problem 2) Use the formula $\mathbf{M = 4 \pi^2 R^3 / (G T^2)}$ where $G = 6.6726 \times 10^{-11} N\text{-}m^2/kg^2$ and M is the mass of the primary in kilograms, R is the orbit radius in meters and T is the orbit period in seconds, to find the masses of the primary bodies in the table below. (Note: Make sure all units are in meters and seconds first! 1 light years = 9.5 trillion kilometers)

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	
Earth	Moon	27.3 days	385,000 km	
Jupiter	Callisto	16.7 days	1.9 million km	
Pluto	Charon	6.38 days	17,530 km	
Mars	Phobos	7.6 hrs	9,400 km	
Sun	Earth	365 days	149 million km	
Sun	Neptune	163.7 yrs	4.5 million km	
Sirius A	Sirius B	50.1 yrs	20 AU	
Polaris A	Polaris B	30.5 yrs	290 million miles	
Milky Way	Sun	225 million yrs	26,000 light years	

Answer Key

Problem 1: Answer

$$\frac{G M m}{R^2} = \frac{m V^2}{R}$$

Cancil the companion mass, m, on both sides, and isolate the primary mass, M, on the left side:

$$M = \frac{R V^2}{G}$$

Now use the definition of V to eliminate it from the equation,

$$M = \frac{R}{G}\left(\frac{2 \pi R}{T}\right)^2$$

and simplify

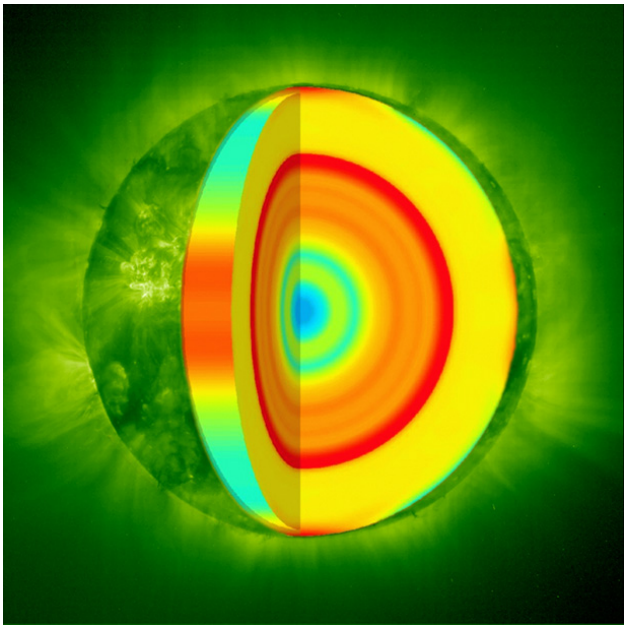
$$M = \frac{4 \pi^2 R^3}{G T^2}$$

Problem 2:

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	**6.1×10^{24} kg**
Earth	Moon	27.3 days	385,000 km	**6.1×10^{24} kg**
Jupiter	Callisto	16.7 days	1.9 million km	**1.9×10^{27} kg**
Pluto	Charon	6.38 days	17,530 km	**1.3×10^{22} kg**
Mars	Phobos	7.6 hrs	9,400 km	**6.4×10^{23} kg**
Sun	Earth	365 days	149 million km	**1.9×10^{30} kg**
Sun	Neptune	163.7 yrs	4.5 million km	**2.1×10^{30} kg**
Sirius A	Sirius B	50.1 yrs	298 million km	**6.6×10^{30} kg**
Polaris A	Polaris B	30.5 yrs	453 million km	**6.2×10^{28} kg**
Milky Way	Sun	225 million yrs	26,000 light years	**1.7×10^{41} kg**

Note: The masses for Sirius A and Polaris A are estimates because the companion star has a mass nearly equal to the primary so that our mass formula becomes less reliable.

A Mathematical Model of the Sun



Once an astronomer knows the radius and mass of a planet or star, one of the first things they might do is to try to figure out what the inside looks like.

Gravity would tend to compress a large body so that its deep interior was at a higher density than its surface layers. Imagine that the sun consisted of two regions, one of high density (the core shown in red) and a second of low density (the outer layers shown in yellow). All we know about the sun is its radius (696,000 km) and its mass (2.0×10^{30} kilograms). What we would like to estimate is how dense these two regions would be, in order for the final object to have the volume of the sun and its total observed mass. We can use the formula for the volume of a sphere, and the relationship between density, volume and mass to create such a model.

Suppose the inner zone has a radius of 417,000 km. The volume of this 'core' is

$$Vc = \frac{4}{3}\pi\,(417{,}000{,}000\ \mathrm{m})^3 = 3.0 \times 10^{26}\ \mathrm{m}^3$$

The volume of the entire sun is

$$V^* = \frac{4}{3}\pi\,(696{,}000{,}000\ \mathrm{m})^3 = 1.4 \times 10^{27}\ \mathrm{m}^3$$

So the outer shell zone has a volume of

$$Vs = V^* - Vc$$

So, $Vs = 1.1 \times 10^{27}\ \mathrm{km}^3$

Mass = Density x Volume,
so if we estimate the density of the sun's core as $Dc = 100{,}000\ \mathrm{kg/m}^3$, and the outer shell as $Ds = 10{,}000\ \mathrm{kg/m}^3$, the masses of the two parts are:

$Mc = 100{,}000 \times (3.0 \times 10^{26}\ \mathrm{m}^3) = 3.0 \times 10^{31}$ **kg**

$Ms = 10{,}000 \times (1.1 \times 10^{27}\ \mathrm{m}^3) = 1.1 \times 10^{31}$ **kg**

So the total solar mass in our model would be:

$$M^* = Mc + Ms = 4.1 \times 10^{31}\ \mathrm{kg}$$

This is about 20 times more massive than the sun actually is!

Inquiry Problem.

With the help of an Excel spreadsheet, program the spreadsheet so that you can adjust the radius of the core and calculate the volume of the core and shell zones. Then create a formula so that you can enter various choices for the densities of the core and shell zones and then sum-up the total mass of your mathematical model for the sun.

What are some possible ranges for the core radius, and zone densities (in grams per cubic centimeter) that give about the right mass for the modeled sun? How do these compare with what you find from searching the web literature? How could you improve your mathematical model to make it more accurate? Show how you would apply this method to modeling the interior of the Earth, or the planet Jupiter?

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With the help of an Excel spreadsheet, program the spreadsheet so that you can adjust the radius of the core and calculate the volume of the core and shell zones. Then create a formula so that you can enter various choices for the densities of the core and shell zones and then sum-up the total mass of your mathematical model for the sun.

Notes: The big challenge for students who haven't used the volume formula is how to work with it for large objects…requiring the use of scientific notation as well as learning how to use the volume formula. Students will program a spreadsheet in whatever way works best for them, although it is very helpful to lay out the page in an orderly manner with appropriate column labels. Students, for example, can select ranges for the quantities, and then generate several hundred models by just copying the formula into several hundred rows. Those familiar with spreadsheets will know how to do this ,and it saves entering each number by hand…which would just as easily have been done with a calculator and does not take advantage of spreadsheet techniques.

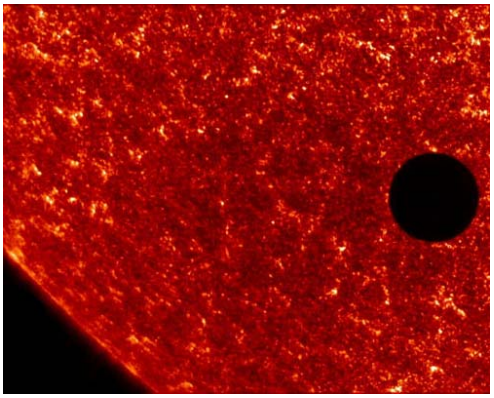
What are some possible ranges for the core radius, and zone densities (in grams per cubic centimeter) that give about the right mass for the modeled sun? How do these compare with what you find from searching the web literature? How could you improve your mathematical model to make it more accurate? Show how you would apply this method to modeling the interior of the Earth, or the planet Jupiter?

Notes: Most solar models suggest a core density near 160 grams/cc, and an outer 'convection zone' density of about 0.1 grams/cc. The average value for the shell radius is about 0.7 x the solar radius. Students will encounter, using GOOGLE, many pages on the solar interior, but the best key words are things like 'solar core density' and 'convection zone density'. These kinds of interior models are best improver by 1) using more shells to divide the interior of the body, and 2) developing a mathematical model of how the density should change from zone to zone. Does the density increase linearly as you go from the surface to the core, or does it follow some other mathematical function? Students may elect to also test various models for the interior density change such as

$$D = \frac{100 \text{ gm/cc}}{R} \quad \text{or even} \quad 100 \text{ gm/cc} \times e^{-R/R^*}$$

This method can be applied to any spherical body (planet, asteroid, etc) for which you know a radius and mass ahead of time.

Alternately, if you know a planet's mass and its composition (this gives an average density), then you can estimate its radius!



On June 5, 2012 the planet Venus will pass across the face of the sun as viewed from Earth. The last time this happened was on June 6, 2004. A similar event, called the Transit of Venus' will not happen again until December 11, 2117.

This image, taken by the TRACE satellite, shows the black disk of Venus passing across the solar disk photographed with a filter that highlights details on the solar surface.

Problem 1 – The table below gives the location of the other 7 planets at the time of the 2012 Transit of Venus, with the sun at the origin of the Cartesian coordinate system. What are the polar coordinates for each planet at the time of the next transit? Note: All distances are in terms of the Astronomical Unit (AU) for which the distance between the Sun and Earth equals exactly 1.0 AU.

Planet	X	Y	R	θ
Sun	0.000	0.000		
Mercury	-0.157	+0.279		
Venus	-0.200	-0.698		
Earth	-0.268	-0.979		
Mars	-1.450	-0.694		
Jupiter	+2.871	+4.098		
Saturn	-8.543	-4.635		
Uranus	+19.922	+1.988		
Neptune	+26.272	-14.355		

Problem 2 – In the sky <u>as viewed from Earth</u>, what is the angular distance between the Sun and Mars if the angle between the Sun and Earth as viewed from Mars is 39°?

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Planet	X	Y	R	θ
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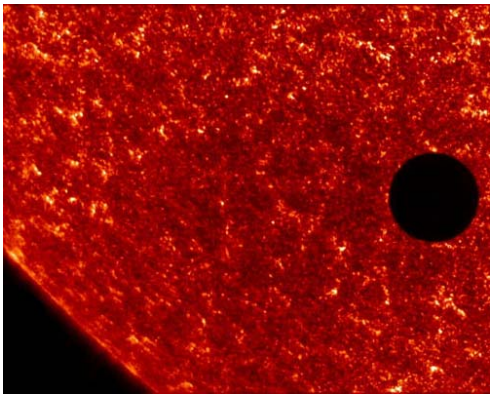
Earth	-0.268	-0.979	1.015	255
Mars	-1.450	-0.694	1.608	206
Jupiter	+2.871	+4.098	5.003	55
Saturn	-8.543	-4.635	9.719	208
Uranus	+19.922	+1.988	20.021	6
Neptune	+26.272	-14.355	29.938	331

Answer: Example for Jupiter: $R^2 = (2.871)^2 + (4.098)^2$ so R = 5.003 AU.
The angle can be determined from $\cos(\theta) = X/R$, for Earth, located in the Third Quadrant: $\cos^{-1}(-0.268/1.015) = 105^o$ so $360^o - 105^o = 255^o$.

Problem 2 – In the sky <u>as viewed from Earth</u>, what is the angular distance between the Sun and Mars if the angle between the Sun and Earth as viewed from Mars is 39°?

Answer: Sun-Earth-Mars form a triangle with the Earth at one vertex. You need to calculate this vertex angle. Use the Law of Sines:

$$\frac{\sin(39)}{1.015} = \frac{\sin(x)}{1.608}$$ so $\sin(x) = 0.9970$ and so **x = 86 degrees**.



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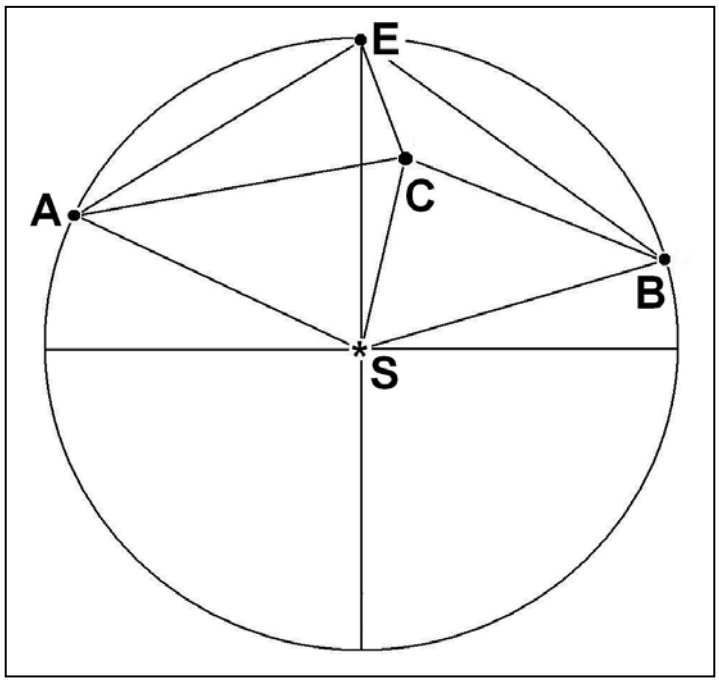
Answer: Sun-Earth-Mars form a triangle with the Earth at one vertex. You need to calculate this vertex angle. Use the Law of Cosines:

$$L^2 = a^2 + b^2 - 2ab\cos A$$

Where L = Mars-Sun distance = 1.608 AU
A = Earth-Sun distance = 1.015 AU
B = Earth-Mars distance: $B^2 = (-0.269 + 1.450)^2 + (-0.979 + 0.694)^2$ so B = 1.215 AU

Then CosA = 0.032 so **A = 88 degrees**.

Law of Cosines



The two STEREO spacecraft are located along Earth's orbit and can view gas clouds ejected by the sun as they travel to Earth. From the geometry, astronomers can accurately determine their speeds, distances, shapes and other properties.

By studying the separate 'stereo' images, astronomers can determine the speed and direction of the cloud before it reaches Earth.

Use the diagram, (angles and distances not drawn to the same scale of the 'givens' below) to answer the following question.

The two STEREO satellites are located at points A and B, with Earth located at Point E and the sun located at Point S, which is the center of a circle with a radius ES of 1.0 Astronomical unit (150 million kilometers). Suppose that the two satellites spot a Coronal Mass Ejection (CME) cloud at Point C. Satellite A measures its angle from the sun mSAC as 45 degrees while Satellite B measures the corresponding angle to be mSBC=50 degrees. In the previous math problem the astronomers knew the ejection angle of the CME, mESC, but in fact they didn't need to know this in order to solve the problem below!

Problem 1 - The astronomers want to know the distance that the CME is from Earth, which is the length of the segment EC. The also want to know the approach angle, mSEC. Use either a scaled construction (easy: using compass, protractor and millimeter ruler) or geometric calculation (difficult: using trigonometric identities) to determine EC from the available data.

Givens from satellite orbits:
SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km
mASE = 54 degrees mBSE = 48 degrees
mEAS = 63 degrees mEBS = 66 degrees mAEB = 129 degrees

Find the measures of all of the angles and segment lengths in thee above diagram rounded to the nearest integer.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?

Givens from satellite orbits:
SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km
mASE = 54 degrees mBSE = 48 degrees
mEAS = 63 degrees mEBS = 66 degrees mAEB = 129 degrees
use units of megakilometers i.e. 150 million km = 150 Mkm.

Method 1: Students construct a scaled model of the diagram based on the angles and measures, then use a protractor to measure the missing angles, and from the scale of the figure (in millions of kilometers per millimeter) they can measure the required segments. **Segment EC is about 49 Mkm at an angle, mSEB of 28 degrees.**

Method 2 use the Law of Cosines and the Law of Sines to solve for the angles and segment lengths exactly.

mASB = mASE+mBSE = 102 degrees
mASC = theta
mACS = 360 - mCAS - mASC = 315 - theta
mBSC = mASB - theta = 102 - theta
mBCS = 360 - mCBS - mBSC = 208 + theta

Use the Law of Sines to get
sin(mCAS)/L = sin(mACS)/150 Mkm and sin(mBCS)/L = sin(mBCS)/150 Mkm.

Eliminate L : 150sin(45)/sin(315-Theta) = 150 sin(50)/sin(208+theta)

Re-write using angle-addition and angle-subtraction:
sin50 [sin(315)cos(theta)-cos(315)sin(theta)] = sin(45) [sin(208)cos(theta) + cos(208)sin(theta)]

Compute numerical factors by taking indicated sines and cosines:
-0.541cos(theta) - 0.541sin(theta) = -0.332cos(theta) - 0.624sin(theta)

Simplify: cos(theta) = 0.397sin(theta)

Use definition of sine: $\cos(\text{theta})^2 = 0.158\,(1 - \cos(\text{theta})^2)$

Solve for cosine: $\cos(\text{theta}) = (0.158/1.158)^{1/2}$ so theta = 68 degrees. And so **mASC=68**

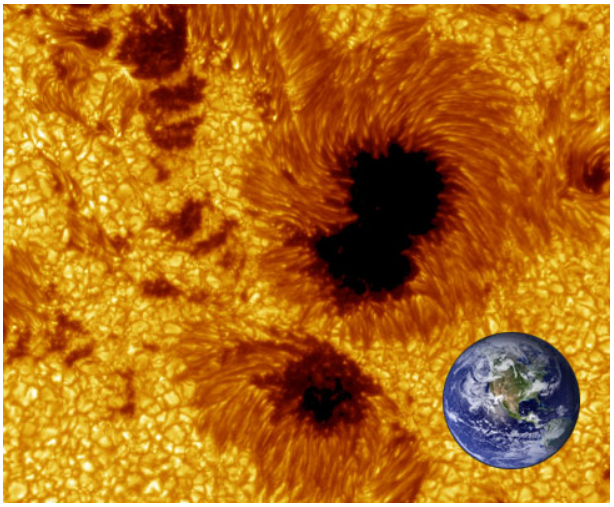
Now compute segment CS = 150 sin(45)/sin(315-68) = **115 Mkm**.
BC = 115 sin(102-68)/sin(50) = **84 Mkm**.
Then $EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(\text{mEBS-mCBS})$
$EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(66-50)$ So **EC = 47 Mkm.**

mCEB from Law of Cosines: $84^2 = 122^2 + 47^2 - 2(122)(47)\cos(\text{mCEB})$ so **mCEB = 29 degrees**

And since mAES = 180 - mASE - mEAS = 180 - 54 - 63 = 63 degrees
so mSEC = mAEB - mAES - mCEB = 129 - 63 - 29 = **37 degrees**

So, the two satellites are able to determine that the CME is 49 million kilometers from Earth and approaching at an angle of 37 degrees from the sun.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?
Answer: 115 million kilometers / 2 million km/hr = **58 hours or 2.4 days.**



The sun goes through a periodic cycle of sunspots being common on its surface, then absent. Sunspot counts during the last 200 years have uncovered many interesting phenomena in the sun that can lead to hazardous 'solar storms' here on Earth.

During the most recent sunspot cycle, Number 23, the average annual number of spots, N, discovered each year, Y, was given by the table below:

Y	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
N	9	21	64	93	120	111	104	64	40	30	15

Problem 1 - Graph this data for N(Y).

Problem 2 - How do you know that the data represents a function rather than merely a relation?

Problem 3 - What is the domain and range of the sunspot data?

Problem 4 - When did the maximum and minimum occur, and what values did N(Y) attain? Express your answers in functional notation.

Problem 5 - Over what domain was the range below 50% of the maximum?

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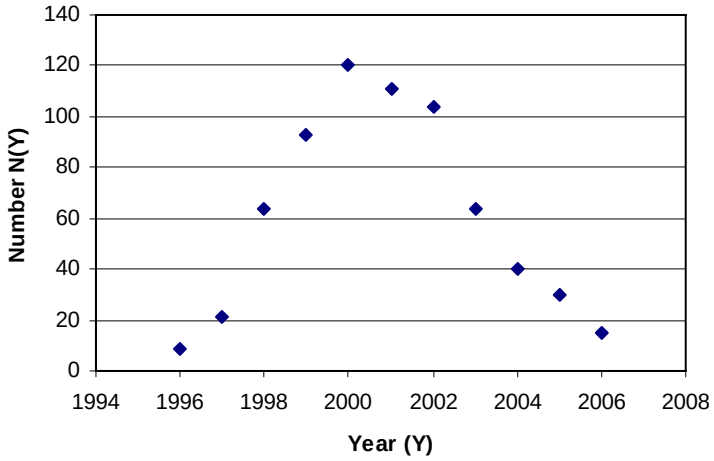
N	9	21	64	93	120	111	104	64	40	30	15

Write this information in functional notation so that it is easier to refer to this information. Answer: N(Y)

Problem 1 - Graph this data.



Problem 2 - How do you know that the data represents a function rather than merely a relation? Answer: **From the vertical line test, every Y only touches one value of N(Y).**

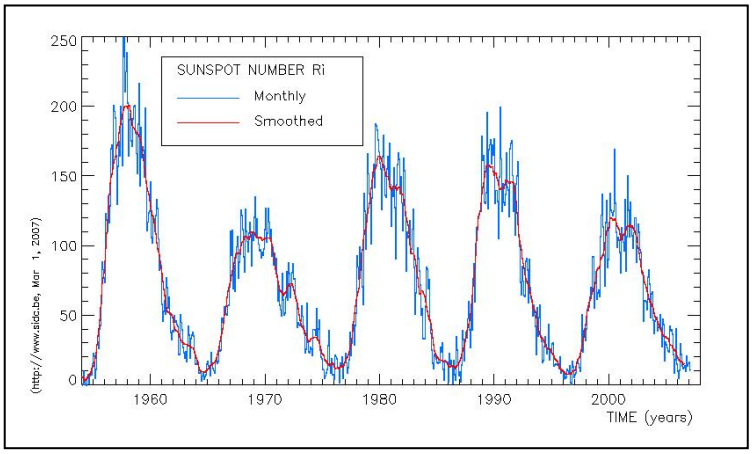
Problem 3 - What is the domain and range of the sunspot data?
Answer: **Domain [1996, 2006] Range [9,120]**

Problem 4 - When did the maximum and minimum occur, and what values did N(Y) attain? Express your answers in functional notation.

Answer: The maximum occurred for **Y=2000 with a value of N(2000)=120 sunspots**; the minimum occurred for **Y=1996 with a value N(1996) = 9 sunspots**.

Problem 5 - Over what domain was the range below 50% of the maximum?

Answer: The maximum was 120 so 1/2 the maximum is 60. N(Y) < 60 occurred for **[1996,1997] and [2004,2006]**.



Sunspots come and go in a roughly 11-year cycle. Astronomers measure the symmetry of these cycles by comparing the first 4 years with the last 4 years. If the cycles are exactly symmetric, the corresponding differences will be exactly zero.

Matrix A
Sunspot numbers at start of cycle.

	Year 1	Year 2	Year 3	Year 4
Cycle 23	**21**	**64**	**93**	**119**
Cycle 22	**13**	**29**	**100**	**157**
Cycle 21	**12**	**27**	**92**	**155**
Cycle 20	**15**	**47**	**93**	**106**

Matrix B
Sunspot numbers at end of cycle

	Year 11	Year 10	Year 9	Year 8
Cycle 23	**8**	**15**	**29**	**40**
Cycle 22	**8**	**17**	**30**	**54**
Cycle 21	**15**	**34**	**38**	**64**
Cycle 20	**10**	**28**	**38**	**54**

Problem 1 - Compute the average of the sunspot numbers for each cycle according to **C** = (**A** + **B**)/2.

Problem 2 - Compute the average difference of the sunspot numbers for the beginning and end of each cycle according to **D** = (**A** - **B**)/2.

Problem 3 – Are the cycles symmetric?

Problem 1 - Compute the average of the sunspot numbers for each cycle according to **C** = (**A** + **B**)/2. Answer:

$$C = \frac{1}{2}\begin{pmatrix} 21 & 64 & 93 & 119 \\ 13 & 29 & 100 & 157 \\ 12 & 27 & 92 & 155 \\ 15 & 47 & 93 & 106 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 8 & 15 & 29 & 40 \\ 8 & 17 & 30 & 54 \\ 15 & 34 & 38 & 64 \\ 10 & 28 & 38 & 54 \end{pmatrix}$$

$$C = \begin{pmatrix} 14.5 & 39.5 & 61.0 & 79.5 \\ 10.5 & 23.0 & 65.0 & 105.5 \\ 13.5 & 30.5 & 65.0 & 219.0 \\ 12.5 & 37.5 & 65.5 & 80.0 \end{pmatrix}$$

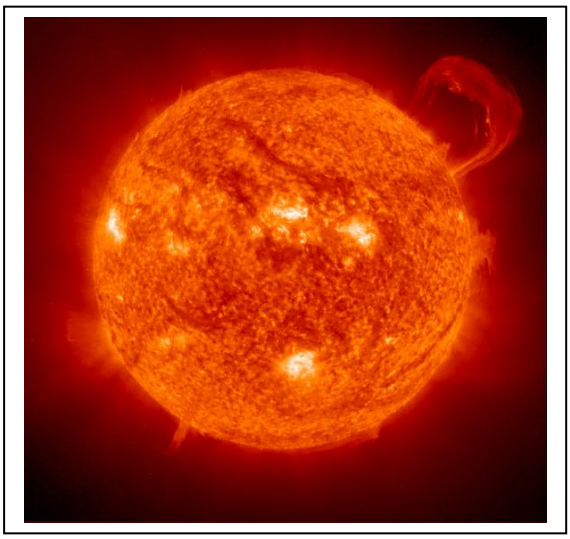
Problem 2 - Compute the average difference of the sunspot numbers for the beginning and end of each cycle according to **D** = (**A** - **B**)/2.

$$D = \frac{1}{2}\begin{pmatrix} 21 & 64 & 93 & 119 \\ 13 & 29 & 100 & 157 \\ 12 & 27 & 92 & 155 \\ 15 & 47 & 93 & 106 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 8 & 15 & 29 & 40 \\ 8 & 17 & 30 & 54 \\ 15 & 34 & 38 & 64 \\ 10 & 28 & 38 & 54 \end{pmatrix}$$

$$D = \begin{pmatrix} +6.5 & +24.5 & +32.0 & +39.5 \\ +2.5 & +6.0 & +35.0 & +66.5 \\ -1.5 & -3.5 & +27.0 & +45.5 \\ +2.5 & +9.5 & +27.5 & +26.0 \end{pmatrix}$$

Problem 3 – Are the cycles symmetric?

Answer: From the values in **D** we can conclude that the cycles are not symmetric, and from the large number of positive differences, that the start of each cycle has more spots than the corresponding end of each cycle.



Detailed mathematical models of the interior of the sun are based on astronomical observations and our knowledge of the physics of stars. These models allow us to explore many aspects of how the sun 'works' that are permanently hidden from view.

The Standard Model of the sun, created by astrophysicists during the last 50 years, allows us to investigate many separate properties. One of these is the density of the heated gas throughout the interior. The function below gives a best-fit formula, D(x) for the density (in grams/cm^3) from the core (x=0) to the surface (x=1) and points in-between.

$$D(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155$$

For example, at a radius 30% of the way to the surface, x = 0.3 and so D(x=0.3) = 14.5 grams/cm^3.

Problem 1 - What is the estimated core density of the sun?

Problem 2 - To the nearest 1% of the radius of the sun, at what radius does the density of the sun fall to 50% of its core density at x=0? (Hint: Use a graphing calculator and estimate x to 0.01)

Problem 3 - To three significant figures, what is the estimated density of the sun near its surface at x=0.9 using this polynomial approximation?

Problem 1 - Answer; At the core, x=0, do D(0) = 155 grams/cm^3.

Problem 2 - Answer: We want D(x) = 155/2 = 77.5 gm/cm^3. Use a graphing calculator, or an Excell spreadsheet, to plot D(x) and slide the cursor along the curve until D(x) = 77.5, then read out the value of x. The relevant portion of D(x) is shown in the table below:

X	D(x)
0.08	94.87
0.09	88.77
0.1	82.96
0.11	**77.43**
0.12	72.16
0.13	67.16
0.14	62.41

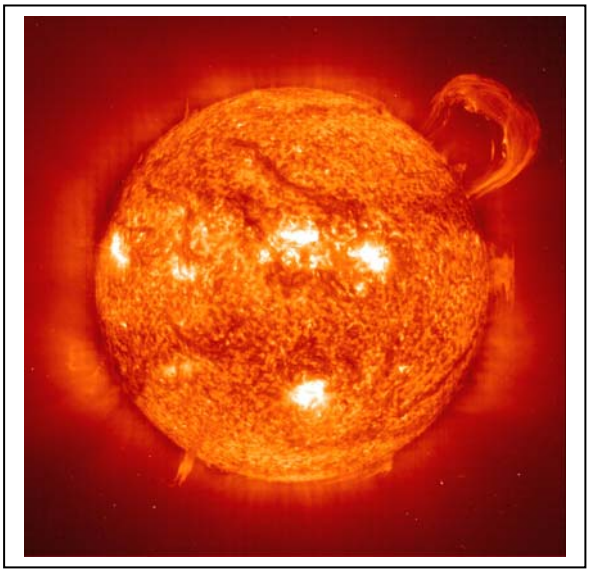
Problem 3 - Answer: At x=0.9 (i.e., a distance of 90% of the radius of the sun from the center).

$D(0.9) = 519(0.9)^4 - 1630(0.9)^3 + 1844(0.9)^2 - 889(0.9) + 155$

$D(0.9) = 340.516 - 1188.27 + 1493.64 - 800.10 + 155.00$

D(0.9) = 0.786 gm/cm^3.

Note: The density of water id 1.0 gm/cm^3 so this solar material would 'float' on water!



The sun is an active star. Matter erupts from its surface and flows into space under the tremendous magnetic forces at play on its surface.

Among the most dramatic phenomena are the eruptive prominences, which eject billions of tons of matter into space, and travel at thousands of kilometers per minute.

This image from the Solar and Heliophysics Observatory (SOHO) satellite taken on September 23, 1999 and shows a giant prominence being launched from the sun.

Problem 1 – The table below shows the height versus time data for an eruptive prominence seen on August 6, 1931. Graph the data, and find a polynomial, h(t), that fits this data.

t	**15**	**15.5**	**16**	**16.5**	**17**	**17.5**	**18**	**18.5**	**19**
h	**50**	**60**	**70**	**100**	**130**	**150**	**350**	**550**	**700**

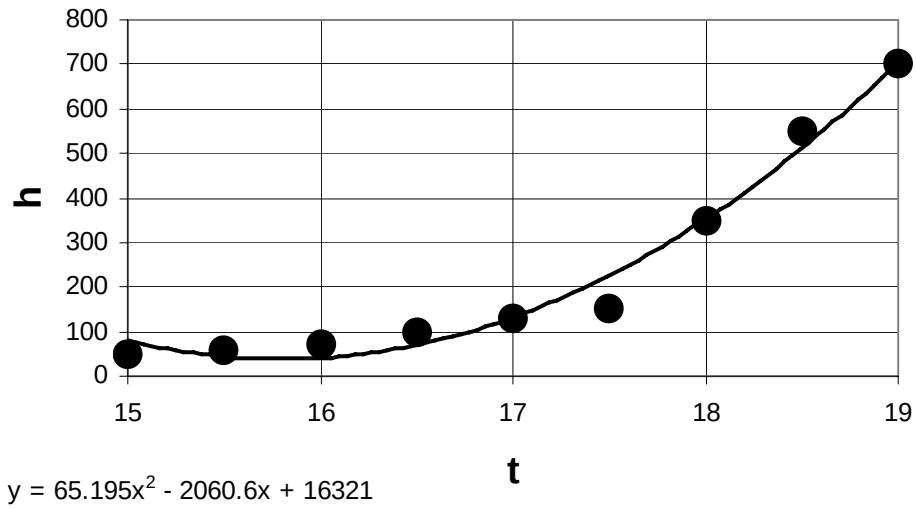
Problem 2 – The data give the height, h, of the eruptive prominence in multiples of 1,000 kilometers from the solar surface, for various times, t, given in hours. For example, at a time of 17 hours, the prominence was 130,000 kilometers above the solar surface. To two significant figures, how high was the prominence at a time of 19.5 hours?

Problem 1 – The table below shows the height versus time data for an eruptive prominence seen on August 6, 1931. Graph the data, and find a polynomial, h(t), that fits this data.

t	**15**	**15.5**	**16**	**16.5**	**17**	**17.5**	**18**	**18.5**	**19**
h	**50**	**60**	**70**	**100**	**130**	**150**	**350**	**550**	**700**

Answer: The best fit degree-2 polynomial is

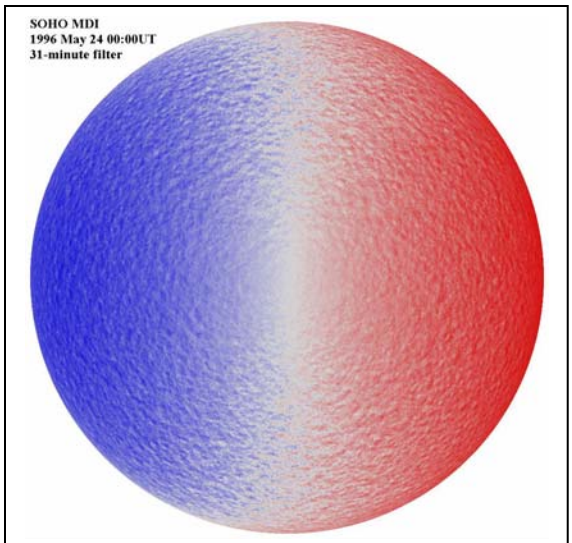
$$h(t)= 65.195t^2 - 2060\,t + 16321$$



Problem 2 – The data give the height, h, of the eruptive prominence in multiples of 1,000 kilometers from the solar surface, for various times, t, given in hours. For example, at a time of 17 hours, the prominence was 130,000 kilometers above the solar surface. To two significant figures, how high was the prominence at a time of 19.5 hours?

Answer: $h = 16.195(19.5)^2 - 2060.6\,(19.5) + 16321$
$= 941.398$
$= 940$ to two significant figures

Since h is in multiples of 1,000 km, the answer will be **940,000 kilometers**.



The rotating sun causes a Doppler Effect. Atoms emitting light on the left half of the sun (east) are 'blue-shifted' as the sun rotates from east to west. (Courtesy NASA /SOHO/ GONG)

When a source of light moves relative to an observer, the frequency of the light waves will be increased if the movement is towards the observer, or decreased if the motion is away from the observer. This phenomenon is called the Doppler Effect, and it is given by the formula:

$$f = f_s \frac{c}{c+V}$$

where V is the speed of the source, fs is the normal frequency of the light seen by the observer when V=0, and f is the 'shifted' frequency of the light from the moving source as seen by the observer. C = 3.0×10^8 meters/sec is the speed of light.

Problem 1 – The Sun rotates at a speed of 2 kilometers/sec at the equator. If you are observing the light from hydrogen atoms at a frequency of fs=4.57108×10^{14} Hertz, A) About what would be the frequency, f, of the blue-shifted eastern edge of the sun? B) What would be the difference in megaHertz between f and fs?

Problem 2 – In 2005, astronomers completed a study of the pulsar B1508+55 which is a rapidly spinning neutron star left over from a supernova explosion. They measured a speed for this dead star of 1,080 km/sec. If they had been observing a spectral line at a frequency of fs=1.4×10^9 Hertz, what would the frequency of this line have been if the neutron star were moving directly away from Earth?

Problem 3 – An astronomer is using a radio telescope to determine the Doppler speed of several interstellar clouds. He uses the light from the J=2-1 transition of the carbon monoxide molecule at a known frequency of fs=230 gigaHertz, and by measuring the same molecule line in a distant cloud he wants to calculate its speed v. A) What is the function f(v)? B) Graph g(v)=f(v)-fs in megaHertz over the interval 10 km/sec < v < 200 km/sec. C) A cloud is measured with a Doppler shift of g(v) = 80.0 megaHertz, what is the speed of the cloud in kilometers/sec?

Problem 1 – Answer: A) $f = 4.57108 \times 10^{14}$ Hertz $(3.0\times10^{8}) / (2.0 + 3.0\times10^{8})$

$f = 4.57108 \times 10^{14}(0.999999993)$

$f = 4.57107997 \times 10^{14}$ Hertz

B) $F\text{-}Fs = 4.57107997 \times 10^{14} - 4.57108 \times 10^{14} = 3{,}048{,}000$ Hertz.

So the frequency of the hydrogen light would appear at **3.048 megaHertz** higher than the normal frequency for this light.

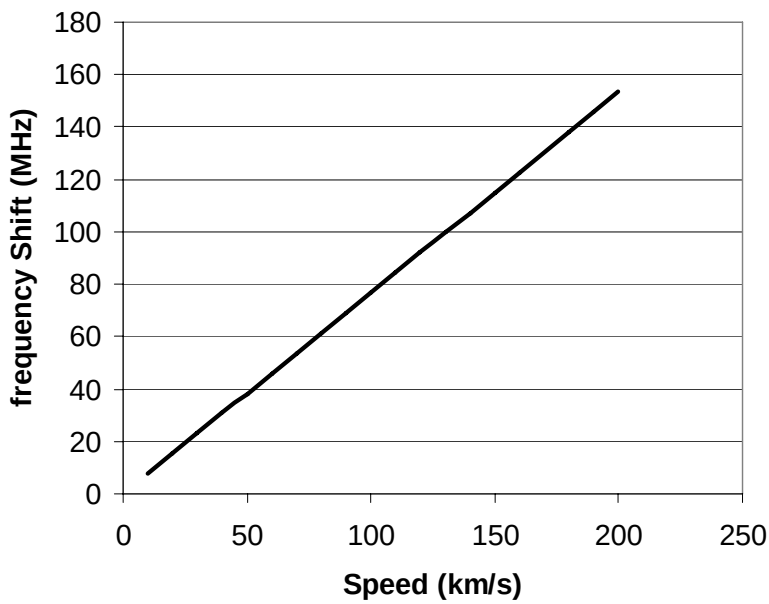
Problem 2 – Answer: $f = 1.4\times10^{9}$ Hertz $\times (1 + 1{,}080/3.0\times10^{5}) =$ **1.405×10^{9} Hertz**.

Problem 3 – A) What is the function f(v)?

Answer: $f(v) = 2.3\times10^{9} (1 + v/300000)$

B) Graph g(v)=f(v)-fs in megaHertz over the interval 10 km/sec < v < 1,000 km/sec.

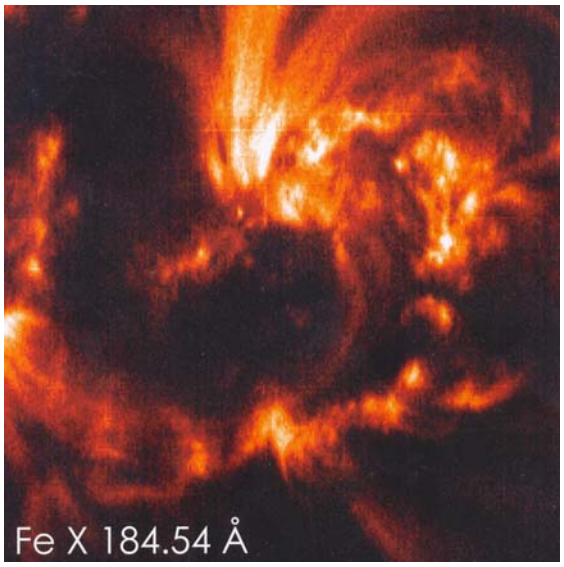
Answer: g(v) = 230 (v/300) megaHertz. or **g(v) = 0.767 v megaHertz**

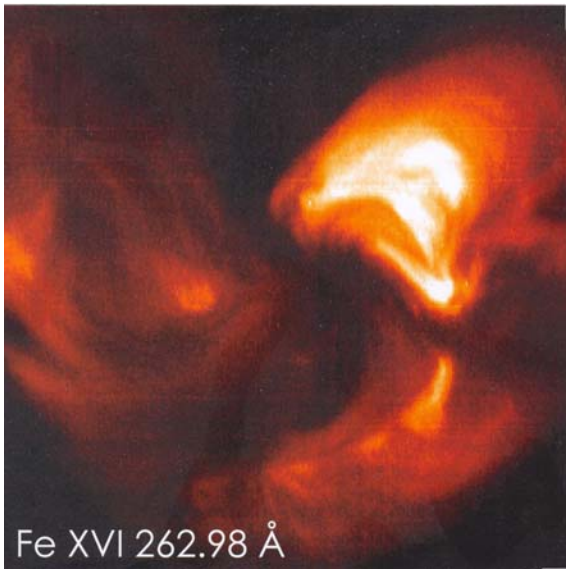


C) A cloud is measured with g(v) = 80.0 megaHertz, what is the speed of the cloud in kilometers/sec?

Answer: From the graph, g(80) = 104 km/sec and by calculation, 80 = 0.767v so v = 80/.767 = **104 km/sec**.

Hot Gas and Cold Gas





The Hinode Extreme Ultraviolet Imaging Spectrometer (EIS) sorts the light from the Sun into a spectrum. It works in a part of the spectrum far beyond the visible light we see, and which can only be studied from space. The atmosphere of Earth absorbs this light, so satellite telescopes have to be placed in orbit to study this light.

Very hot gas near the sun produces this light, and by carefully measuring it, scientists can deduce the exact temperature, density and speed of motion of the gas that produces it. When atoms are heated, they produce specific wavelengths of light, called spectral lines.

The two pictures were taken of the same active region (AR-10940) over a sunspot on February 2, 2007, and are exactly overlapping images. The size of each square image is about 200,000 km on a side.

The top picture is produced by ionized iron atoms (Fe X) in which 9 of the 26 iron electrons have been removed. The light is from a single line at a wavelength of 184.54 Angstroms. The bottom picture is from ionized iron atoms (Fe XVI) in the same region, which have lost 15 of their 26 electrons, and is from the light from a single line at 262.98 Angstroms.

The Fe X emission is produced in plasma with a temperature of 950,000 K. The Fe XVI emission is produced in plasma with a temperature of 2,600,000 K.

Problem 1 - From the information, what is the scale of the images in kilometers per millimeter?

Problem 2 - Of the two gas temperatures, at which gas temperature do you find the smallest clumps, and about how big are they?

Problem 3 - At which temperature do you think the gas is more easily confined by the magnetic fields near this sunspot?

Answer Key:

Problem 1 - From the information, what is the scale of the images in kilometers per millimeter?

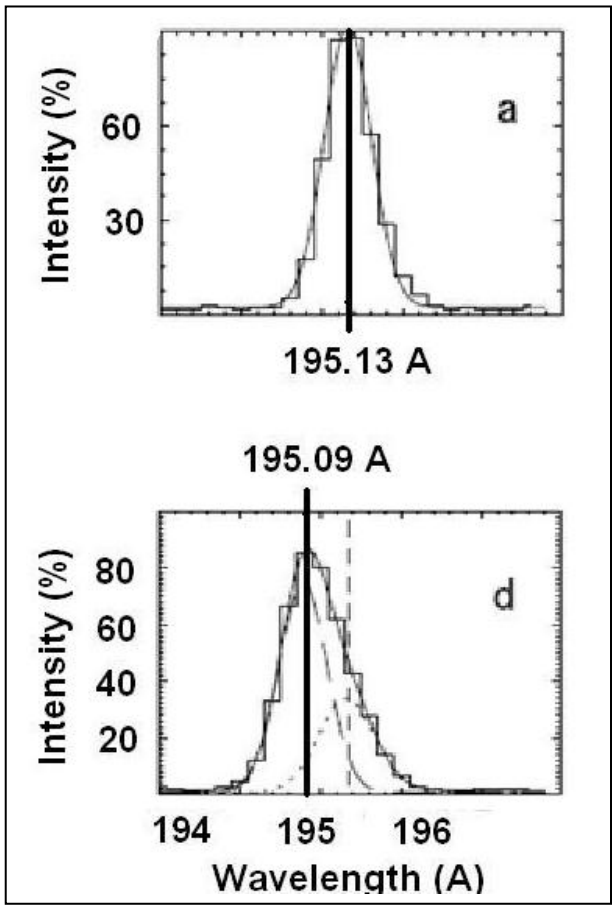
Answer: The images are 74 millimeters on a side, which corresponds to 200,000 km, so the scale is 200,000 kilometers/74 mm= 2,700 km/mm

Problem 2: At what gas temperature do you find the smallest clumps, and how big are they?

Answer: The cold gas revealed by the Fe X light and at a temperature of 950,000 K has the smallest clumps, which can be about 2 millimeters across or 5,400 kilometers in physical size.

Problem 3: At which temperature do you think the gas is more easily confined by the magnetic fields near this sunspot?

Answer: Because the cooler gas has the most clumps, and the smallest ones, compared to the hotter gas, it is easier to confine the cooler gas.



When a fire truck races towards you, the siren sounds at a higher pitch than when it races away. This is called the Doppler Shift, and it can also be used to measure the speed of a gas cloud near the sun.

The Hinode Extreme Ultraviolet Imaging Spectrometer (EIS) sorts the light from the sun into a spectrum. When atoms are heated, they produce specific wavelengths of light, called spectral lines. The wavelengths of some of these lines, such as the one produced by iron atoms, are known precisely. By measuring the wavelengths shift of one of these iron lines, solar physicists can use the Doppler Shift to measure how fast the gas was moving on the sun. Here's how they do it!

The figure to the left shows the intensity of the light produced by a particular iron atom in the sun's spectrum. The top panel shows the light produced by a cloud that was at rest near the solar surface at a wavelength of 195.13 Angstroms. The bottom panel shows the light from a similar plasma cloud that was in motion during a solar flare on December 13, 2006. Notice that the wavelength of the light in the moving cloud is 195.09 Angstroms.

The plots were published by Dr. Shinsuke Imada and his co-investigators in the *Publications of the Astronomical Society of Japan* (v. 59, pp. 759).

Problem 1 - What is the wavelength for the FeXII emission line for: A) the gas at rest: λ(rest)? B) What is the wavelength for the gas in motion: λ(moving)?

Problem 2 - The Doppler Formula relates the amount of wavelength shift to the speed of the gas according to

$$V = 300{,}000 \text{ km/sec} \times \frac{\lambda(\text{rest}) - \lambda(\text{moving})}{\lambda(\text{rest})}$$

From the information in the two panels, what was the speed of the plasma during the solar flare event?

Problem 3 - From the location of the peak of the moving gas, is the gas moving towards the observer (shifted to shorter wavelengths), or away from the observer (shifted to longer wavelengths)?

Problem 4 - From your answer to Problem 3, during a flare, is the heated plasma flowing upwards from the solar surface, or downwards to the solar surface? Explain your answer by using a diagram.

Answer Key:

Problem 1 - What is the wavelength of the FeXII emission line for: A) the gas at rest: λ(rest)? B) the gas in motion: λ(moving)?

Answer: A) λ(rest) = 195.13 angstroms. B) The peak in Panel D is at λ(moving) = 195.09 Angstroms.

Problem 2 - The Doppler Formula relates the amount of shift to the speed of the gas according to

$$V = 300{,}000 \text{ km/sec} \times \frac{\lambda(\text{rest}) - \lambda(\text{moving})}{\lambda(\text{rest})}$$

From the information in the two panels, what was the speed of the plasma during the solar flare event?

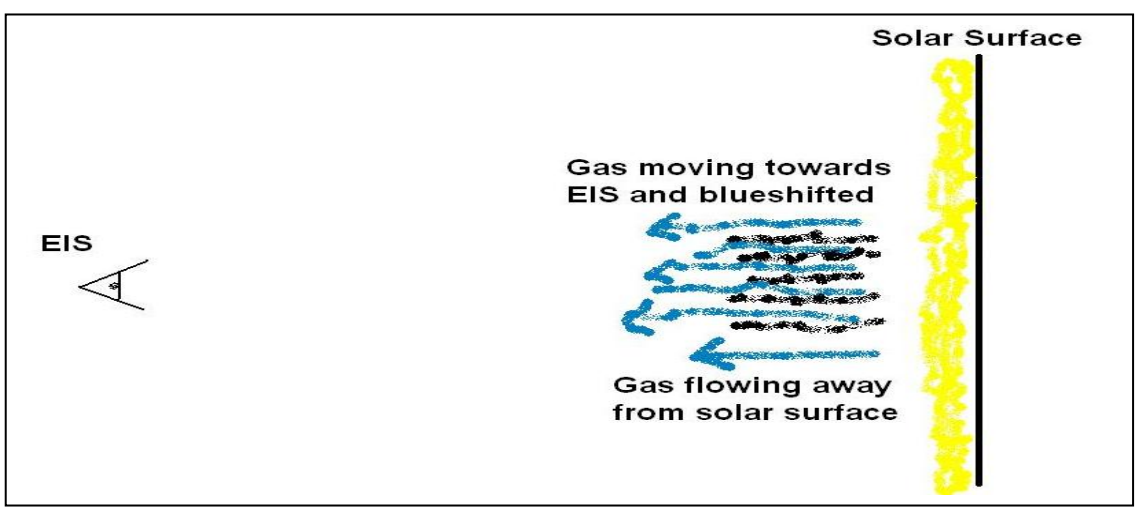
Answer: λ (rest) = 195.13 Angstroms, λ(moving) = 195.09 Angstroms. So

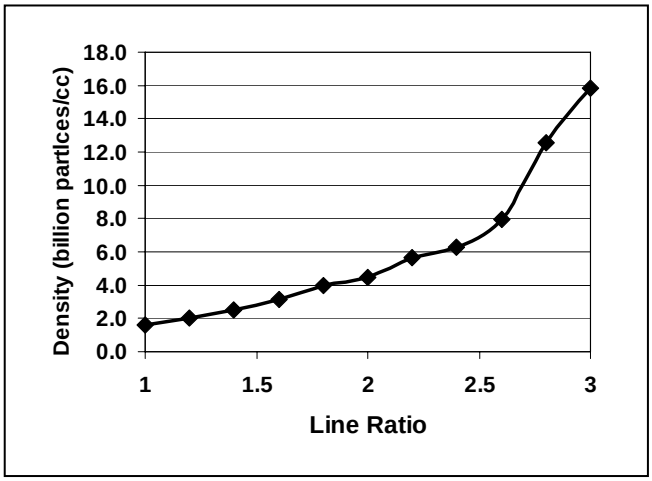
V = 300,000 x (195.13 - 195.09)/195.13

= 61 kilometers/sec

Problem 3 - From the location of the peak of the moving gas, is the gas moving towards the observer (shifted to shorter wavelengths), or away from the observer (shifted to longer wavelengths)? Answer: The peak in panel B is shifted towards smaller, shorter, wavelengths so the gas is 'blue-shifted' and moving towards the observer.

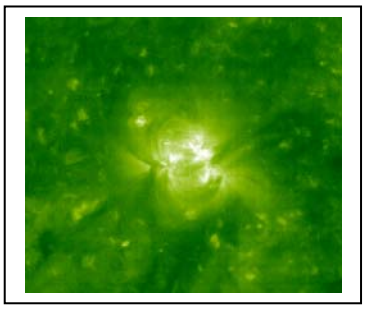
Problem 4 - From your answer to Problem 3, during a flare, is the heated plasma flowing upwards from the solar surface, or downwards to the solar surface? Explain your answer by using a diagram. Answer: The FeXII measurement showed plasma flowing towards the observer. Because the gas was located between the solar surface and the observer, it must have been flowing upwards from the surface.



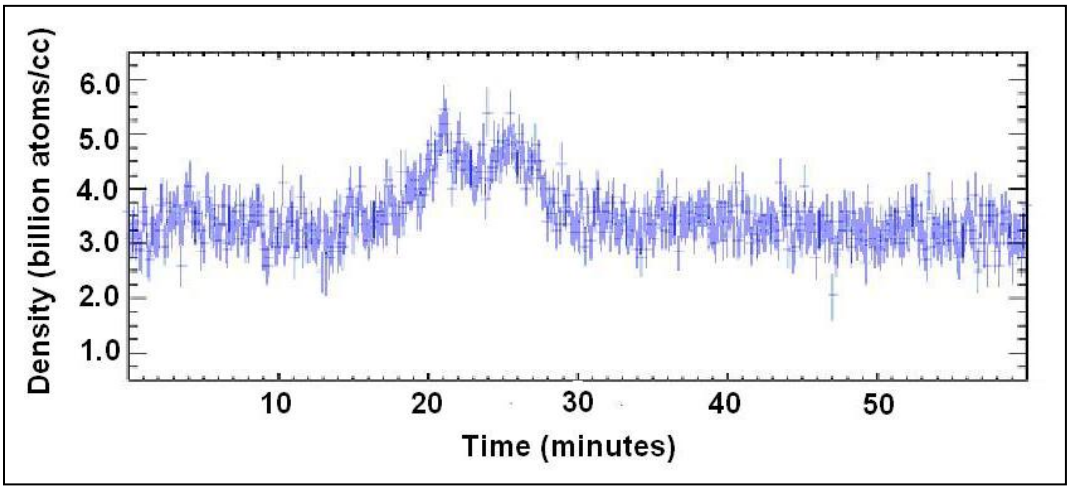


The Hinode EIS instrument can detect the individual 'fingerprint' lines from dozens of elements. Each line has its own unique wavelength. The intensity of an atomic line can be used to learn about the properties of the gas near the solar surface.

Two particular atomic lines produced by the element iron appear at wavelengths of 203.8 and 202.0 Angstroms. The ratio of the intensities of these two lines can be converted into a measurement of the density of the gas producing them, as shown in the figure to the left.



For example, if the intensity of the Fe XII line at 203.8 Angstroms were measured to be 300 units, and the line at 202.0 Angstroms were measured to be 150 units, the ratio yields 300/150 = 2.0. From the figure, a value of '2.0' on the horizontal axis, corresponds to a density of about 4.0 billion atoms per cubic centimeter on the horizontal axis.



Problem 1 - The figures (above) were adapted from an article by Dr. Peter Young at the Rutherford Appleton Laboratory in England. The photograph shows an active region on the sun spotted by Hinode. The graph is the density measured by the EIS instrument near the center of the active region during 60 minutes. A) To the nearest integer, what is the average density of the region during the last 20 minutes of the study? B) About what is the average line ratio that corresponds to the average density?

Problem 2 – A) To the nearest integer, what is the maximum density that was recorded during the 60-minute time period? B) About how many minutes was this plasma density maintained above 4 billion atoms/cc during the flare event?

Answer Key:

Problem 1 - A) To the nearest integer, what is the average density of the region during the last 20 minutes of the study? B) About what is the average line ratio that corresponds to the average density?

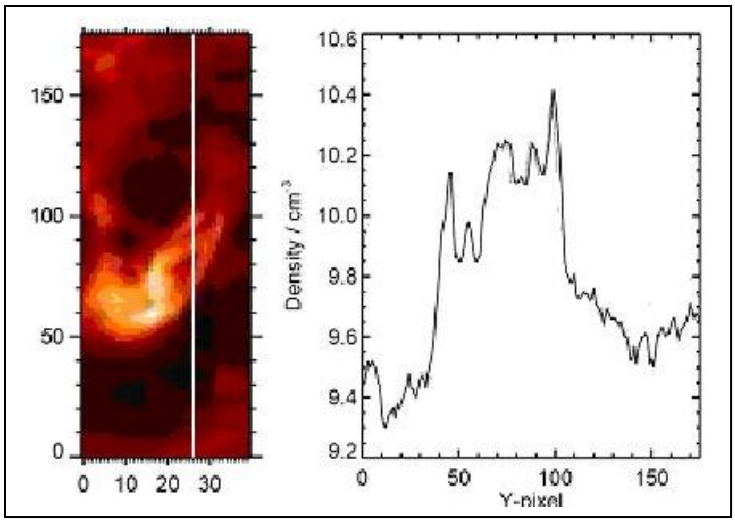
Answer: A) **About 3 billion atoms/cm^3**
B) The average line ratio can be found by drawing a line from the '3.0' on the vertical axis, across to its intersection with the curve, and then drawing a second vertical line down from the intersection point to the horizontal axis. The ratio is about 1.6.

Problem 2 – A) To the nearest integer, what is the maximum density that was recorded during this time period? B) About how many minutes was this plasma density maintained above 4 billion atoms/cc during the flare event?

Answer: A) About 5.0 billion **atoms/cm^3**. Student's estimates may vary.
B) The density was higher than 4 billion particles/cc between about 20-28 minutes or a duration of 8 minutes. Student's estimates may vary.

Density Structure of an Active Region

The Hinode EIS instrument has been used to study many active regions in order to determine how the density of the plasma varies through each region. One of these studies was reported by solar physicist Dr. Peter Young from the Rutherford Appleton Laboratory in England in August 2007. Below-left is an image of a coronal loop with the X and Y axes indicating the pixel number in each direction. The white line is a slice through the data at an X-pixel value of 26, and reveals the density variation in the vertical Y direction shown in the graph on the right. The density is rendered on the vertical axis in terms of the base-10 logarithm of the density value so that '10' means 10^{10} particles/cm^3



The gas densities are found by observing the same region of the sun at two different wavelengths emitted by the iron atom at a temperature near 1.5 million degrees K. *The ratio of the light intensity emitted at these two wavelengths is directly related to the density of the gas producing the light.* This shows how spectroscopy can provide vastly more information about the sun and solar activity, than what you would get from a single image alone.

Problem 1 - About what is the average density of the gas in the dark regions covered by the pixels in the range from Y=10 to Y=30? Convert answers to normal decimal units of density in scientific notation.

Problem 2 - About what is the average density of the gas in the dark regions covered by the pixels in the range from Y=140 to Y=170? Convert answers to normal decimal units of density in scientific notation.

Problem 3 - The dark regions not involved with bright coronal loop are glimpses of the surface of the sun. If higher gas densities tend to be found closer to the solar surface, in which part of the image not including the bright coronal loop may we be looking at a deeper layer of the solar atmosphere?

Problem 4 - About what is the density of the three ribbon-like features at Y= 70, Y=90 and Y=100? Convert answers to normal decimal units of density in scientific notation.

Answer Key:

Problem 1 - About what is the average density of the gas in the dark regions covered by the pixels in the range from Y=10 to Y=30? Convert answers to normal decimal units of density in scientific notation.

Answer: About Log D = 9.4 or D = $10^{9.4}$ = 2.5 x 10^{9} particles/cm^{3}

Problem 2 - About what is the average density of the gas in the dark regions covered by the pixels in the range from Y=140 to Y=170? Convert answers to normal decimal units of density in scientific notation.

Answer: About Log D = 9.6 or D = $10^{9.6}$ = 4.0 x 10^{9} particles/cm^{3}

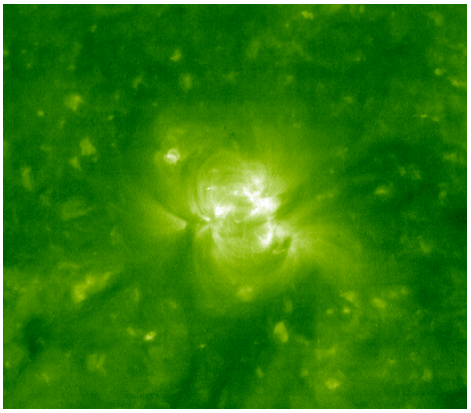
Problem 3 - The dark regions not involved with bright coronal loop are glimpses of the surface of the sun. If higher gas densities tend to be found closer to the solar surface, in which part of the image not including the bright coronal loop may we be looking at a deeper layer of the solar atmosphere?

Answer: The density of the region at Y=0-39 is about $10^{9.4}$ = 2.5 x 10^{9} particles/cm^{3}. The region at Y= 101-170 is about $10^{9.6}$ = 4.0 x 10^{9} particles/cm^{3} so the upper end may be looking closer to the solar surface.

Problem 4 - About what is the density of the three ribbon-like features at Y= 70, Y=90 and Y=100? Convert answers to normal decimal units of density in scientific notation.

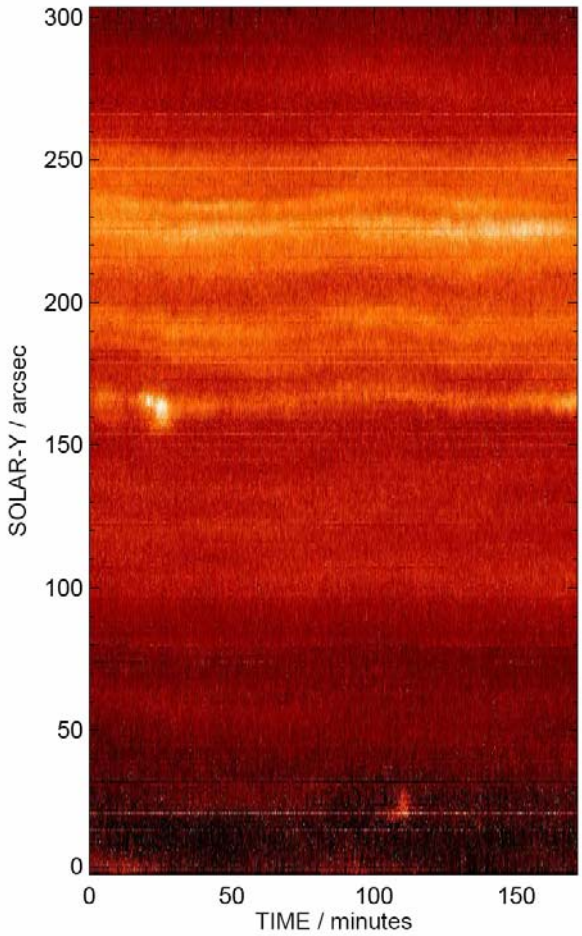
Answer: The log densities are LogD = 10.25, 10.25 and 10.4 respectively, for densities of D = 1.8 x 10^{10} particles/cm^{3} D = 1.8 x 10^{10} particles/cm^{3} and D= 2.5 x 10^{10} particles/cm^{3}

Plasma Speeds in Active Regions



The Hinode EIS instrument can study a single slice of an active region and detect subtle changes in position over time. The image above shows an active region near a sunspot, and reveals its 'bar magnet' magnetic field loops.

The strip plot to the right shows the motion of the plasma sliced vertically along the center line of the active region, over a span of about 200 minutes. The vertical axis gives the location of the plasma in units of arcseconds, where one arcsecond equals a physical distance of 725 kilometers on the solar surface.



Individual features in the upper image appear as bands on the strip plot. The slope of the band is a measure of the speed of motion of the gas.

Problem 1 - The scale of the vertical axis is in angular units of arcseconds. At the distance of the sun, one arcsecond corresponds to 725 kilometers. Select the bright feature near Y=160 arcseconds. What is the change in the Y-position during the period from A) 0-50 minutes? B) 50 - 100 minutes? C) 100-150 minutes? (Hint: Use a millimeter to calculate the scale of the vertical axis in kilometers/mm)

Problem 2 - From your answer to Problem 1, what is the average speed of this feature during each 50-minute period from A to C in kilometers per second?

Problem 3 - About how soon after the start of the data did the flare occur in this feature, and how long did it last? [The flare is the sudden brightening in the data that occurs near Y=160]

Answer Key:

Problem 1 - The scale of the vertical axis is in angular units of arcseconds. At the distance of the sun, one arcsecond corresponds to 725 kilometers. Select the bright feature near Y=160 arcseconds. Select the bright feature near Y=160 arcseconds. What is the change in the Y-position during the period from A) 0-50 minutes? B) 50 - 100 minutes? C) 100-150 minutes?

Answer: Students need to use a millimeter ruler and determine the Y-axis scale of the plot. The total height represents 300 arcseconds or a physical length of 300 x 725 km = 217,500 kilometers. The vertical length of the figure is 116 millimeters, so the scale is 1,875 km/mm

A) The change in the Y-axis location between the beginning and end of this segment is about 1 millimeter or 1,875 kilometers.

B) The change in the Y-axis location between the beginning and end of this segment is about 3 millimeters or 5,625 kilometers.

C) The change in the Y-axis location between the beginning and end of this segment is about 2 millimeter or 3,750 kilometers.

Problem 2 - From your answer to Problem 1, what is the average speed of this feature during each 50-minute period from A to C?

Answer: Each segment is 50 minutes long or 3000 seconds. Dividing the distances in Problem 1 by 3000 seconds you get A) 0.625 kilometers/sec; B) 1.87 kilometers/sec and C) 1.25 kilometers/sec.

Problem 3 - How soon after the start of the data did a flare occur in this feature? How long did it last?

Answer: The flare event is located in the strip plot between Y = 160 and 170 arcseconds. On the horizontal axis, 50 minutes equals 18 millimeters so the scale is 2.8 minutes/mm. The flare occurred about 7 mm or 19.6 minutes after the start of the data, and lasted 3 mm or 8.4 minutes.

A note from the Author:

Hi again!

This booklet will introduce you to the Hinode satellite, and a few of its many discoveries. Many of the problems involve the basic skill of measuring the size of an image, and converting the measurement into an actual physical unit (kilometers, seconds, etc). Most students can successfully perform this activity by grade-5 when they have become familiar with decimal-math and division.

Calculating the scale of an image is a critical operation in just about any scientific investigation. You know all those pretty pictures that NASA loves to show you of planets, stars and galaxies? Well before an astronomer can make any sense of them, he has to be able to figure out how many millimeters on the image/photograph corresponds to a meter, kilometer or even a light-year in actual physical units. Otherwise, you have no clue how big the object is that you are investigating! Also, it is a fun exercise to see what the smallest detail in an image is in physical units.

The Hinode satellite creates many images of the sun at various sizes to study phenomena as big as the sun, or as small as a micro-flare. The X-ray and optical imager are good for establishing a size to things, and the movies give us a sense for how quickly or slowly things occur. But the satellite can do much more than this!

The Extreme Ultraviolet Imaging Spectrometer dissects the ultraviolet light from the sun, and is able to use the 'fingerprints' of various atoms producing light, to work out the density, temperature and speed of the various phenomena being studied. You cannot get this information by just looking at a picture of the sun. Solar scientists need this kind of information in order to mathematically model how a phenomenon changes in time, how the gases flow, and how magnetic energy heats the various gases found near sunspots and other active regions on the sun.

Ironically, spectroscopy is the unsung hero of solar and astrophysics. Students that view dazzling pictures of nebula, galaxies, planets and solar storms instinctively enjoy these images because they represent an image, like a photograph of a house, mountain or lake. It is very easy to explain what the image represents because we have many familiar examples of the same kind of data in our family photo albums!

Spectroscopy is different. It is a technology that sorts the electromagnetic spectrum by wavelength. Each atom produces its own 'fingerprint' lines of light that resemble a bar code. Scientists can read this bar code, and even take a picture of the sun by the light from one particular 'bar'. The relative intensities of the different spectral bars or 'lines', can be used to calculate the density and temperature of the gas that is producing it. Some of the problems in this book will give students a taste of how this kind of information is used to more clearly understand how solar flares and other phenomena on the sun are 'put together'.

Enjoy!!

Sincerely,
Dr. Sten Odenwald
NASA Astronomer

Internet Resources

The Hinode Mission education page - A variety of resources that explain solar flares, space weather, and many of the basic solar science terms encountered in this math guide.

http://solarb.msfc.nasa.gov/for_educators/index.html

Space Math @ NASA – A collection of over 200 individual math problems covering nearly all aspects of solar science, astronomy, astrophysics and engineering.

http://spacemath.gsfc.nasa.gov

The human and technological impacts of solar storms and space weather - A guide to the many issues that occur in studying how the sun affects the many human and technological systems we rely upon, and how they can be disturbed or damaged by solar storms.

http://www.solarstorms.org

NOAA space weather forecasting center – This website gives up to the minute forecasts of solar storms and flares, and is used by thousands of agencies and institutions across the country and world.

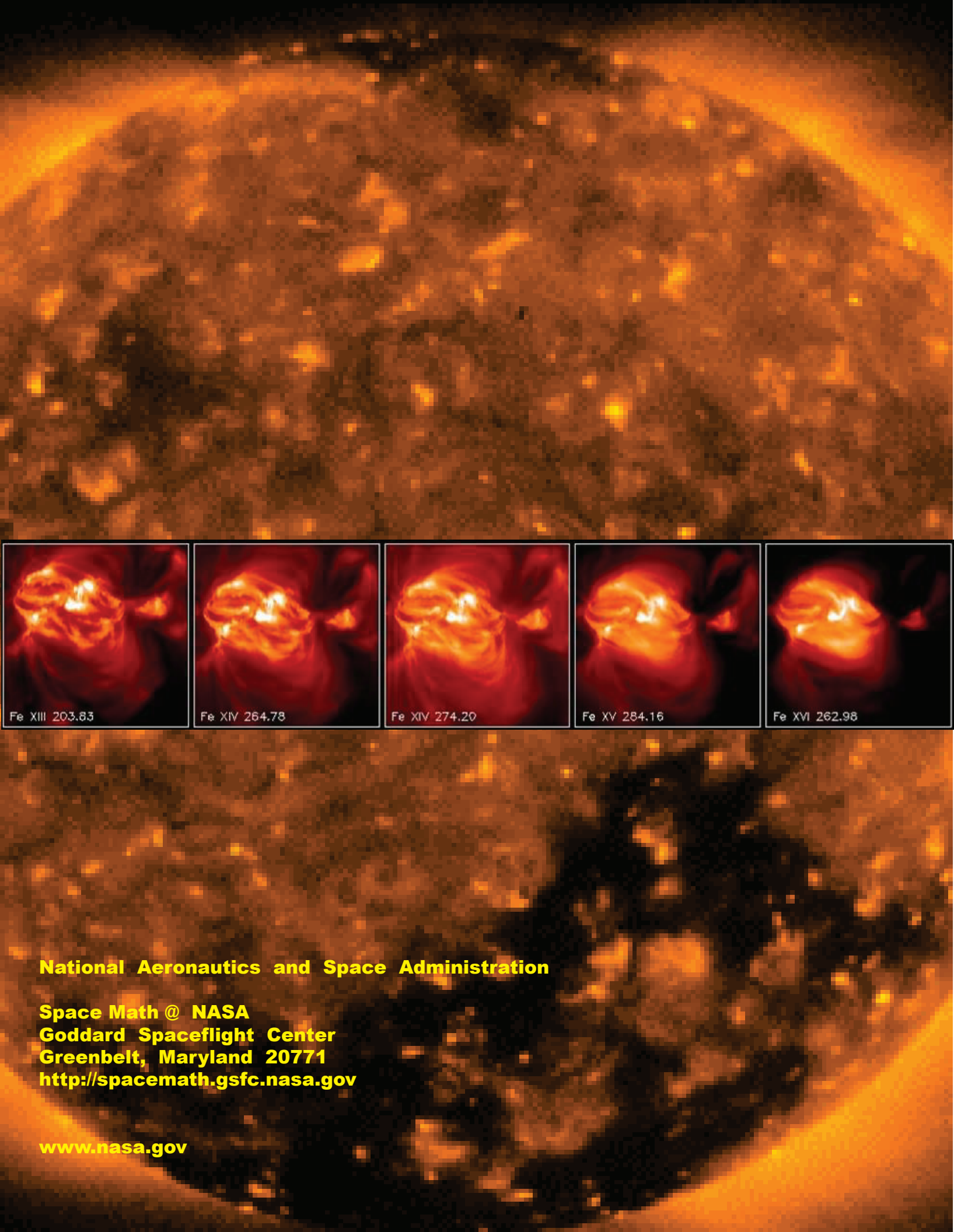
http://www.noaa.sec.gov/SWN

NASA Student Observation Network:Tracking a Solar Storm – This is a hands-on educational resource and tool that allows students to monitor space weather and make their own forecasts.

http://son.nasa.gov/tass/index.htm

Elementary Space Mathematics Primer – This is a short math guide that covers workin g with positive and negative numbers, scientific notation, graph analysis, and simple algebra, in the context of authentic astronomy mathematics problems.

http://image.gsfc.nasa.gov/poetry/MathDocs/spacemath.html



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